

# Analytical model for studying the effect of weak bedding plane on wellbore stability

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## ABSTRACT

Optimum mud weight estimation in wellbore is one of the most important steps to prevent instability. In wellbore stability studies, media (rock) is usually assumed to be isotropic but errors occur when weak bedding planes cause the rocks to be anisotropic. In this study the effect of weak bedding plane in stability of wellbore was studied. Also, the effect of bedding plane parameters on stability of vertical and horizontal wellbore was investigated. Through the use of the geometric relations of bedding plane and wellbore, new equations were presented to calculate the attack angle. Sensitivity analysis on the dip and dip direction of weak bedding plane in the vertical and horizontal wellbore were also performed. On the basis of the porous elasticity theory and Jaeger theories, an analytical model was proposed to analyze the wellbore stability with regard to dip and dip direction of weak bedding plane. A code in MATLAB was written based on analytical model and effect of dip and dip direction of weak bedding plane can be reviewed. By using real data from a wellbore, a comparative analysis was carried out between the new analytical model and the intact rock failure model. Minimum drilling mud weight was calculated in two phases, without weak bedding planes and with weak bedding planes. Results show that the existence of weak bedding planes causes more instability in the wellbore in some azimuths and deviations. Dip and dip direction of weak bedding planes have a great impact on the wellbore stability and in the horizontal wellbore according to dip and dip direction, the optimum wellbore trajectory is different. By applying the code, geomechanical engineers can calculate the amount of mud weight based on the dip and dip direction of the weak bedding plane.

**Keywords:** Wellbore stability, Weak bedding plane, In-situ stress, Analytical model and determination of mud weight.

## 1. Introduction

Wellbore stability is very important in the oil and gas industry. Problems of the industry include: wellbore collapse, tight wellbore, stuck pipe, poor wellbore cleaning, wellbore enlargement, plastic flow, sand production, fracturing and lost circulation. These drive up the drilling costs, which are related to wellbore stability. These problems are mainly due to the imbalance created between rock stresses and its strength at drilling time. In wellbore stability studies, rock is usually assumed to be isotropic but errors occur when the presence of weak bedding planes causes rocks to be transversely anisotropic. Shale layers have (made up) over 75% of the drilled formations, and over 70% of the wellbore instability is related to shale layers [1]. In shale formations, weak bedding planes can be very effective on wellbore instability and causes problems such as stuck pipe due to the failure of weak bedding planes. Therefore, a model of wellbore stability which considers weak bedding plane is needed. Also, the effect of weak bedding planes parameters on wellbore stability should be investigated.

The first research on wellbore stability was carried out by Bradley [2] based on the assumption of linear elastic isotropic rock without considering the anisotropic characteristics of the rock. Aadnoy [3] presented a model to studying the cracks and instability of the wellbores in the anisotropic stress field with considering anisotropic elastic properties, tensile and shear strength depending on direction. His study concluded that ignoring the effects of anisotropy would cause problems in the wellbore.

In order to investigate the compressive strength of transverse anisotropic rocks in different directions and under various lateral

pressures, the researchers have proposed different failure criteria in the past. These criteria can be classified into continuous and discrete/discontinuous categories. By modifying Mohr-Coulomb criteria, Jaeger proposed two criteria to analyze failure anisotropic rocks. Jaeger's primary criterion is the plane of weakness theory. This theory, which is a discontinuous criterion, describes the rock failure strength consisting of a single plane of weakness or a single plane of weakness system. The second theory of Jaeger is continuously variable shear strength that describes failure strength with variable cohesion but constant internal friction angle. McLamore and Gray [4] developed the second theory of Jaeger by modifying the continuous variables of the internal friction angle. In addition, the continuous failure criterion for the transverse isotropic rock is provided by Hoek and Brown [5] and Ramamurthy [6], which generally simulate the failure strength but requires a wide range of experiments and a large number of interpretations of the graph. Nova [7] presented a general failure criterion to describe the failure strength of transverse isotropic rocks under realistic three-dimensional stress conditions. Also, Tien et al. [8] worked on the mechanism and failure states of anisotropic rocks. They performed a compressive strength test on transverse anisotropic rocks in different directions and different lateral pressures. The results of their experiments showed that failure can be categorized as follows and based on the failure criterion, first sliding failure along discontinuities and second sliding failure across discontinuities.

Many studies have been performed on the effect of weak plane on wellbore stability. Ong and Roegiers [9] presented a three-dimensional

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analysis model including stress concentration, internal pressure of wellbore, and thermal induced stresses. The results of this model show that due to the theory of single plane of weakness, high inclined wellbores are affected by the rock's anisotropic, in-situ stresses difference and thermal conditions. Also, McLellan and Cormier [10] discovered that the wellbore stability in shale with bedding plane affected the reaction of bedding plane, in-situ stresses, rock fractures, wellbore trajectory, rock properties, and others factors. Okland and Cook [11], in a study on the field of the North Sea, found that when the wellbore was parallel or roughly parallel to the bedding plane, the instability of the wellbore would be much more severe. Aadnoy et al. [12] also presented a model to analyze the single plane of weakness theory. They used their model in an inclined wellbore drilled in shale strata, which included a large number of weak bedding planes. They showed that the existence of weak bedding plane in rock makes the wellbore more unstable and gives the following results: 1) the critical parameters are planes of weakness in rock strength, the relative normal stress values on the borehole, and, the relative angle between the borehole and bedding plane and 2) for highly-layered rocks, the critical angle between wellbore and weak bedding plane is 10 to 30°, for angles of zero and 90° wellbore are more stable. Dokhani et al. [13] designed an experiments and developed a model to evaluate the effect of weak bedding plane on pore pressure. Their experimental results showed that the distribution of pore flow was significantly affected by weak bedding plane. Also, Lu et al. [14] examined the influence of pore flow on wellbore stability while considering weak bedding planes and showed that pore pressure caused the wellbore to become more unstable. Lee et al. [15] used mathematical relationships and transferring stresses on the weak plane and considered the two failure modes along the weak planes and inner intact rock. A model was developed to investigate the wellbore stability based on the Mohr-Coulomb criteria for rocks with weak planes. Attack angle is one of the factors that can help to choose optimum wellbore trajectory (mud weight) to prevent slipping and shearing failures.

Zhou et al. [16] studied wellbore stability in horizontal wellbore with and without weak bedding plates. They found that the minimum mud weight of drilling was greatly increased by considering weak bedding plates. Ding et al. [17] used an analytical model to reviewed the stability of a horizontal wellbore. They found that when the angle of weak bedding plates with the wellbore is low or high, failure does not occur and the stability of the wellbore depends on the strength of intact rock. Moreover, for intermediate angles, failure occurs in the weak bedding plate. Hung et al. [18] investigated the effect of the anisotropic shale layer on wellbore stability and mode of failure and found that weak bedding plates had a great effect on wellbore stability.

In this research, the effect of weak bedding plane in the stability of wellbore has been studied. Also, the effect of bedding plane parameters on the stability of vertical and horizontal wellbore has been investigated.

## 2. Stresses around wellbore

The first step in order to analyze the wellbore stability is calculating stresses around the wellbore. It is assumed that the main stresses in the environment (before the drilling of the wellbore) are  $\sigma_v$  (vertical stress),  $\sigma_H$  (maximum horizontal stress) and  $\sigma_h$  (minimum horizontal stress). These stresses are related to coordinate system  $x'y'z'$  that  $z'$  axis is parallel to  $\sigma_v$ ,  $x'$  axis is parallel to  $\sigma_H$  and  $y'$  is parallel to  $\sigma_h$  (Fig. 1).

In order to make calculation simpler, in-situ stresses are replaced by the coordinate system  $xyz$ . In the coordinate system  $xyz$ , the  $z$  axis is parallel to the well axis and the  $y$ -axis points toward the highest point on the wellbore periphery, while the  $x$ -axis is oriented 90° counterclockwise in the same plane as the  $y$ -axis. This transformation can be obtained by a rotational matrix in Eq. (1) where  $\alpha$  is the azimuth angle around the  $z$  axis and  $i$  is the inclination angle around the  $y$  axis (Fig. 2).

$$\begin{bmatrix} \cos \alpha \sin i & \sin \alpha \cos i & -\sin i \\ -\sin \alpha & \cos \alpha & 0 \\ \cos \alpha \sin i & \sin \alpha \sin i & \cos i \end{bmatrix} \quad (1)$$

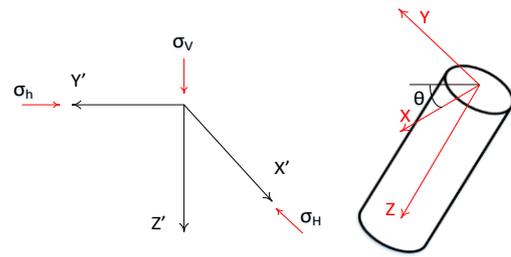


Fig. 1. Coordinate system of in-situ stresses.

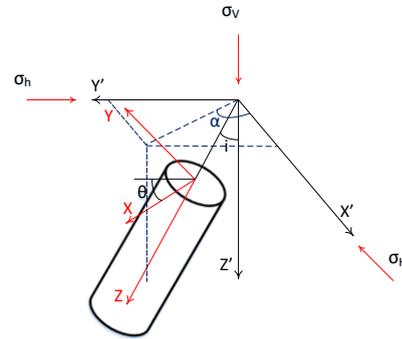


Fig. 2. Stresses transformation system for inclined wellbore.

By using the transport equations of axis, the stresses in the coordinate system  $xyz$  are expressed in Eqs. (1) [19].

$$\begin{aligned} \sigma_x^0 &= (\sigma_H \cos^2 \alpha + \sigma_h \sin^2 \alpha) \cos^2 i + \sigma_v \sin^2 i \\ \sigma_y^0 &= \sigma_H \sin^2 \alpha + \sigma_h \cos^2 \alpha \\ \sigma_z^0 &= (\sigma_H \cos^2 \alpha + \sigma_h \sin^2 \alpha) \sin^2 i + \sigma_v \cos^2 i \\ \sigma_{xy}^0 &= 0.5(\sigma_h - \sigma_H) \sin 2\alpha \cos i \\ \sigma_{yz}^0 &= 0.5(\sigma_h - \sigma_H) \sin 2\alpha \sin i \\ \sigma_{xz}^0 &= 0.5(\sigma_H \cos^2 \alpha + \sigma_h \sin^2 \alpha - \sigma_v) \sin 2i \end{aligned} \quad (1)$$

Wellbore drilling changes the stresses expressed by Eq. (1) near the drilling site; to simplify the calculation, the stresses created around the wellbore after drilling express in  $r_{\theta z}$  cylindrical coordinates. In the cylindrical coordinate system, the stresses around the well with the desired direction are calculated by Eq. (2) [19].

$$\begin{aligned} \sigma_{rr} &= \left( \frac{\sigma_x^0 + \sigma_y^0}{2} \right) \left( 1 - \frac{a^2}{r^2} \right) + \left( \frac{\sigma_x^0 - \sigma_y^0}{2} \right) \left( 1 + 3 \frac{a^4}{r^4} - 4 \frac{a^2}{r^2} \right) \cos 2\theta \\ &\quad + \sigma_{xy}^0 \left( 1 + 3 \frac{a^4}{r^4} - 4 \frac{a^2}{r^2} \right) \sin 2\theta + P_w \frac{a^2}{r^2} \\ \sigma_{\theta\theta} &= \left( \frac{\sigma_x^0 + \sigma_y^0}{2} \right) \left( 1 + \frac{a^2}{r^2} \right) - \left( \frac{\sigma_x^0 - \sigma_y^0}{2} \right) \left( 1 + 3 \frac{a^4}{r^4} \right) \cos 2\theta \\ &\quad - \sigma_{xy}^0 \left( 1 + 3 \frac{a^4}{r^4} \right) \sin 2\theta - P_w \frac{a^2}{r^2} \\ \sigma_{zz} &= \sigma_x^0 - \nu \left[ 2(\sigma_x^0 - \sigma_y^0) \frac{a^2}{r^2} \cos 2\theta + 4\sigma_{xy}^0 \frac{a^2}{r^2} \sin 2\theta \right] \\ \sigma_{r\theta} &= \left( \frac{\sigma_x^0 - \sigma_y^0}{2} \right) \left( 1 - 3 \frac{a^4}{r^4} + 4 \frac{a^2}{r^2} \right) \sin 2\theta \\ &\quad + \sigma_{xy}^0 \left( 1 - 3 \frac{a^4}{r^4} + 4 \frac{a^2}{r^2} \right) \sin 2\theta \\ \sigma_{\theta z} &= (-\sigma_{xz}^0 \sin \theta + \sigma_{yz}^0) \cos \theta \left( 1 + \frac{a^2}{r^2} \right) \\ \sigma_{rz} &= (\sigma_{xz}^0 \cos \theta + \sigma_{yz}^0) \sin \theta \left( 1 - \frac{a^2}{r^2} \right) \end{aligned} \quad (2)$$

Where  $r$  is the distance from the center of the wellbore along the radius,  $a$  is the wellbore radius,  $P_w$  is the mud pressure,  $\theta$  is the angle between  $ox$  of  $xyz$  and some radial vector of wellbore in the clockwise direction and  $\nu$  is the Poisson's ratio. In a linear elastic material, the greatest concentration of stress occurs in the wellbore wall, so the first failure is observed at the wellbore wall. By placing  $r = a$  in Eq. (2),

stresses at the wellbore wall are presented in Eq. (3) [19].

$$\begin{aligned} \sigma_{rr} &= P_w \\ \sigma_{\theta\theta} &= \sigma_x^0 + \sigma_y^0 - 2(\sigma_x^0 - \sigma_y^0)\cos 2\theta - 4\sigma_{xy}^0\sin 2\theta - P_w \\ \sigma_{zz} &= \sigma_z^0 - \theta(2(\sigma_x^0 - \sigma_y^0)\cos 2\theta + 4\sigma_{xy}^0\sin 2\theta) \\ \sigma_{\theta z} &= 2(-\sigma_{xz}^0\sin\theta + \sigma_{yz}^0\cos 2\theta) \\ \sigma_{r\theta} &= \sigma_{rz} = 0 \end{aligned} \quad (3)$$

In order to investigate the stability of wellbore, it is necessary to obtain the effective principal stresses (which can be done through the Mohr-Coulomb criteria). Effective principal stresses at wellbore are calculated using the following equations:

$$\begin{aligned} \sigma_1 &= \frac{1}{2}(\sigma_\theta + \sigma_z) + \sqrt{\frac{1}{4}(\sigma_\theta - \sigma_z)^2 + \sigma_{\theta z}^2} - \alpha_s \times P_p \\ \sigma_2 &= \frac{1}{2}(\sigma_\theta + \sigma_z) - \sqrt{\frac{1}{4}(\sigma_\theta - \sigma_z)^2 + \sigma_{\theta z}^2} - \alpha_s \times P_p \\ \sigma_3 &= P_w - \alpha_s \times P_p \end{aligned} \quad (4)$$

Where  $\sigma_1$  and  $\sigma_3$  are the maximum and minimum principal stresses, MPa.  $\alpha_s$  is the effective stress coefficient;  $P_p$  is pore pressure in MPa.

### 3. Rock failure criteria

In order to investigate the effect of layering on the wellbore stability, typically the Jaeger theory (which is described below) is used. Based on Jaeger theory [20], failure occurs along layering of intact rock. Based on the Mohr-Coulomb criterion, the condition of failure on weak bedding plane can be expressed by Eq. (6) [20].

$$\tau = c_w + \sigma_n \tan \phi_w \quad (6)$$

Where  $c_w$  is cohesion of the weak bedding plane,  $\phi_w$  is the internal frictional angle. Normal stress and shear stress acting on the weak bedding plane are calculated by the following equation:

$$\begin{aligned} \sigma &= \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\beta \\ \tau &= \frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\beta \end{aligned} \quad (7)$$

Where  $\beta$  is the angle between the major principal stress direction and the weak bedding plane (attack angle). By substituting Eq. 6 into Eq. (6), failure criterion for weak bedding plane is expressed in Eq. (8)

$$\sigma_1 = \sigma_3 + \frac{2(c_w + \sigma_2 \tan \phi_w)}{(1 - \tan \phi_w \times \cot \beta) \sin 2\beta} \quad (8)$$

Based on Fig. 3 and geometric relations, the failure occurs along the weak bedding plane when  $\beta_1 < \beta < \beta_2$ , where  $\beta_1$  and  $\beta_2$  are calculated by following equations [20]:

$$\begin{aligned} \beta_1 &= \frac{\phi_w}{2} + 0.5 \sin^{-1} \left\{ \left[ \frac{(\sigma_1 + \sigma_3 + 2c_w \times \cot \phi_w)}{\sigma_1 + \sigma_3} \right] \times \sin \phi_w \right\} \\ \beta_2 &= \frac{\pi}{2} + \frac{\phi_w}{2} - 0.5 \sin^{-1} \left\{ \left[ \frac{(\sigma_1 + \sigma_3 + 2c_w \times \cot \phi_w)}{\sigma_1 + \sigma_3} \right] \times \sin \phi_w \right\} \end{aligned} \quad (9)$$

When  $\beta < \beta_1$  or  $\beta > \beta_2$  failure occurs in intact rock. Based on the Mohr-Coulomb criterion the condition of failure on intact can be expressed by Eq. (10).

$$\sigma_1 = \frac{2C \cos \phi}{1 - \sin \phi} + \sigma_3 \sin \frac{1 + \sin \phi}{1 - \sin \phi} \quad (10)$$

Where  $C$  is the cohesion strength of intact rock;  $\phi$  is the internal frictional angle of intact rock.

### 4. Attack angle

According to the Jaeger criteria, the attack angle ( $\beta$ ) is defined as angle between maximum principal stress and normal direction of weak bedding plane. To calculate the attack angle, the angle between the

minimum principal stress direction ( $\sigma_3$ ) and the normal direction of weak bedding plane is determined, whose complement is  $\beta$  (Fig. 4).

The minimum principal stress unit vector ( $S_{\pi}$ ) in the cylindrical coordinates is equal to:

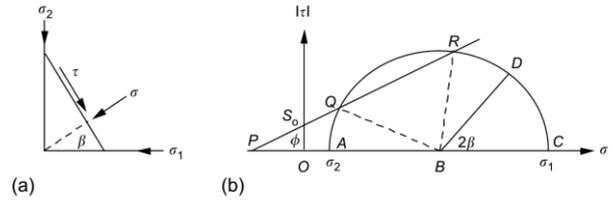


Fig. 3. (a) Plane of weakness with outward normal vector oriented at angle  $\beta$  to the direction of maximum principal stress. (b) Situation described on a Mohr diagram.

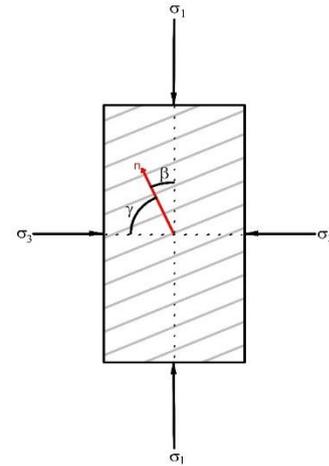


Fig. 4. Schematic of weak bedding planes.

$$\vec{c} = \vec{r} \quad (11)$$

According to Fig. 2, the normal direction of weak bedding plane vector ( $\alpha$ ) in the coordinate system  $x'y'z'$  will be equal to:

$$a = a_1 i + a_2 j + a_3 k \quad (12)$$

Where  $a_1 = \cos \theta_{dd} * \sin \theta_{dip}$ ,  $a_2 = -\sin \theta_{dd} * \sin \theta_{dip}$ ,  $a_3 = \cos \theta_{dip}$ ;  $\theta_{dip}$  is the dip angle of weak bedding plane;  $\theta_{dd}$  is the dip direction of the weak bedding plane related to maximum principal stress.

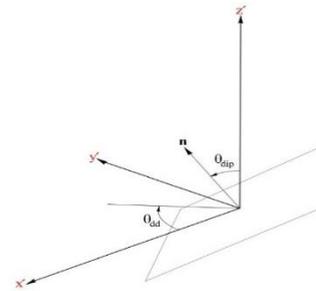


Fig. 2. The normal direction of weak bedding plane in the coordinates  $x'y'z'$ .

To calculate the normal direction of weak bedding plane in the cylindrical coordinates around the wellbore, it is necessary to calculate the rotational matrix ( $R$ ) using the following equation:

$$R = R_{oz'} R_{oy'} R_{oz} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos i & 0 & -\sin i \\ 0 & 1 & 0 \\ \sin i & 0 & \cos i \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

The normal direction of bedding plane in the cylindrical coordinates around the wellbore using a rotational matrix (13) is in Eq. (14):

$$\vec{b} = b_1\vec{r} + b_2\vec{\theta} + b_3\vec{z}$$

$$\vec{b} = \vec{a}R$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos i & 0 & -\sin i \\ 0 & 1 & 0 \\ \sin i & 0 & \cos i \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where values  $b_1$ ,  $b_2$  and  $b_3$  can be calculated using by following equations:

$$\begin{aligned} b_1 &= \cos \theta_{dd} \sin \theta_{dip} (\cos i \cos \alpha \cos \theta - \sin \alpha \sin \theta) \\ &\quad - \sin \theta_{dd} \sin \theta_{dip} (\sin \alpha \cos i \cos \theta \\ &\quad \quad + \cos \alpha \sin \theta) + \cos \theta_{dip} (\sin i \cos \theta) \\ b_2 &= \cos \theta_{dd} \sin \theta_{dip} (\cos i \cos \alpha \sin \theta + \sin \alpha \sin \theta) \\ &\quad - \sin \theta_{dd} \sin \theta_{dip} (\sin \alpha \cos i \sin \theta \\ &\quad \quad - \cos \alpha \cos \theta) + \cos \theta_{dip} (\sin i \sin \theta) \\ b_3 &= -\cos \theta_{dd} \sin \theta_{dip} \sin i \cos \alpha + \sin \theta_{dd} \sin \theta_{dip} \sin \alpha \sin i \\ &\quad + \cos \theta_{dip} \cos i \end{aligned}$$

It is worth pointing out that the size of vector  $\vec{b}$  will also be equal to one because it does not change with the rotation of the vector. In order to calculate the angle between stress and the normal direction of bedding plane using the concept of Dot Product, we have:

$$\gamma = \cos^{-1} \frac{|\vec{b} \cdot \vec{c}|}{|\vec{b}| \cdot |\vec{c}|} = \cos^{-1} |b_1|$$

As  $\beta$  is complimentary of the  $\gamma$ , we have:

$$\beta = 90 - \gamma$$

### 5. Computational algorithm for wellbore stability

Based on Jaeger's failure criterion and considering stresses around the wellbore, an analytical model based on the equations was developed to calculate the minimum drilling mud weight to prevent shear failure, whereby a deep bedding formation could be studied.

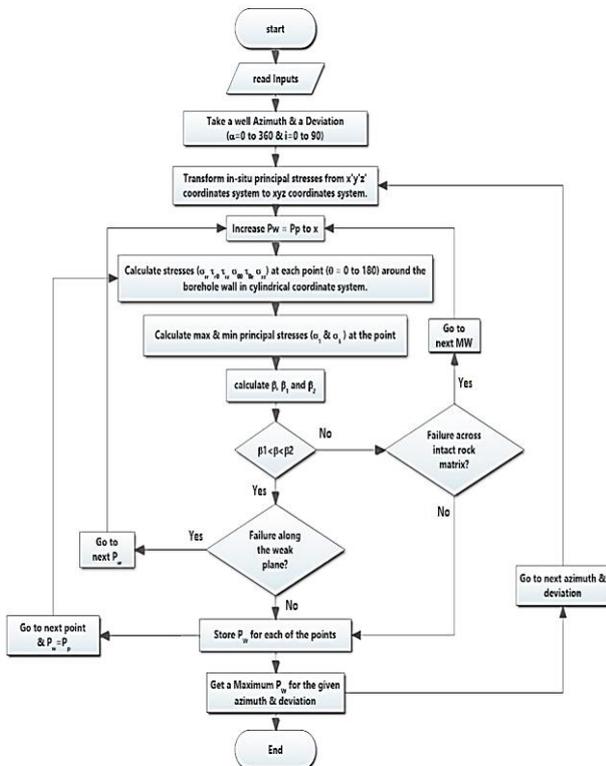


Fig. 6. shows the computational algorithm to determine the minimum drilling mud weight

At the first stage, input parameters including magnitude and direction of stresses, strength parameters of intact rock and weak bedding plane, pore pressure as wellbore as dip and dip direction of the weak bedding plane have been introduced in the analytical model. Then, mud pressure has initialized equally to the pore pressure and thereby the stresses around borehole have been calculated. In the next step,  $\beta$ ,  $\beta_1$  and  $\beta_2$  are calculated. If the condition  $\beta_1 < \beta < \beta_2$  satisfies, the failure criterion of weak bedding plane are going to be used otherwise the intact rock failure criteria will be considered in the analysis. If the maximum effective principal stress is greater than the anticipated strength of the failure criteria, the amount of mud pressure is increased and the calculations will be performed again. It continues so that the stress is equal to or less than the rock strength. This process is performed for all angles  $\theta$  from zero to  $180^\circ$  and thereby the highest mud pressure to prevent borehole breakout instability has been calculated and would be selected. According to the algorithm, a code in MATLAB has been written and the effect of dip and dip direction of weak bedding plane could be reviewed.

### 6. Wellbore stability analysis

Wellbore stability analysis in a strike-slip fault had been carried out for the case of Pedernales field in Venezuela according to the code. The information on this field is presented in Table 1.

#### 6.1. Effects of drilling direction

Wellbore stability base on the geomechanical properties of the field has been carried out in two phases: (1) intrinsic (intact) rock without weak bedding plane and (2) with weak bedding plane. Fig. 7 shows the minimum drilling mud weight in different directions for the intact rock failure model (IRFM). According to this form, the variations in drilling mud weight are from 1.13 to 1.51 gr/cm<sup>3</sup> and the highest stability is in the azimuths of zero and  $180^\circ$  relative to the horizontal maximum (requires the lowest mud weight), and the highest instability for this case study is in the azimuths of  $90^\circ$  and  $270^\circ$  degrees.

Fig. 8 shows the minimum drilling mud weight required to be stable for the model considering weak bedding plane (MWBP) in different directions. As can be seen, variations in drilling mud weight are from 1.16 to 1.92 gr/cm<sup>3</sup> and show that the weak bedding planes generally causes instability at the wellbore. The optimum drilling azimuth is in the direction of up-dip ( $\alpha=180^\circ$ ). This is consistent with other studies that have been done in this field [11, 15, 21, 22]. As shown in Fig. 7, the worst case in terms of stability is in azimuths  $90^\circ$  and  $270^\circ$  at an angle of  $90^\circ$  from the vertical (horizontal wellbore).

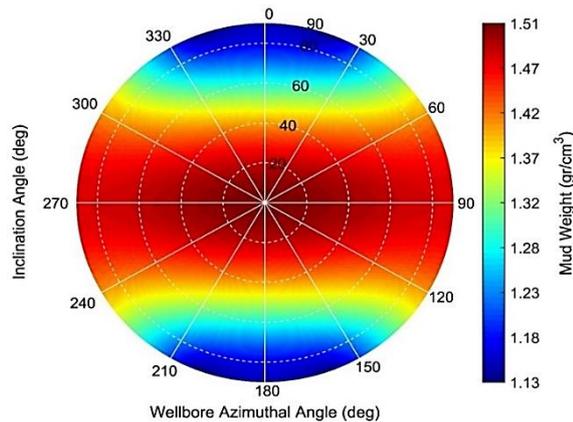
Table 1: Field data for analysis of wellbore stability in the Pedernales field of Venezuela [23].

Wellbore depth (m)	1676
Poisson's ratio ( $\theta$ )	0.3
Pore pressure ( $P_p$ , MPa)	17.72
Vertical stress ( $\sigma_v$ , MPa)	37.21
Maximum horizontal stress ( $\sigma_H$ , MPa)	45.42
Minimum horizontal stress ( $\sigma_h$ , MPa)	34.86
Cohesion of rock matrix (C, MPa)	8.27
Internal friction angle of rock matrix ( $\phi$ , Degree)	31
Dip angle of bedding planes ( $\theta_{dip}$ , Degree)	45
Dip direction of weak bedding planes ( $\theta_{dd}$ , Degree)	zero
Cohesion of weak bedding planes ( $C_w$ , MPa)	2.07
Internal friction angle of weak bedding planes ( $\phi_w$ , Degree)	26.6

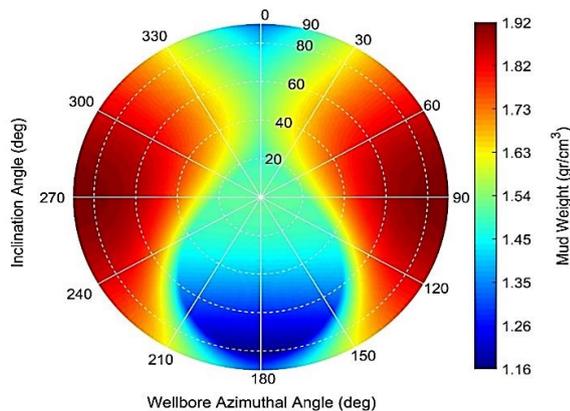
#### 6.2. Effects of weak bedding plane on a vertical wellbore

Fig. 9 shows the variation of wellbore collapse mud weight on dip 0 to  $90^\circ$  and the dip direction 0 to  $360^\circ$  in the vertical wellbore. As shown in this Figure, mud weight varies from 1.16 to 1.92 gr/cm<sup>3</sup> depending on

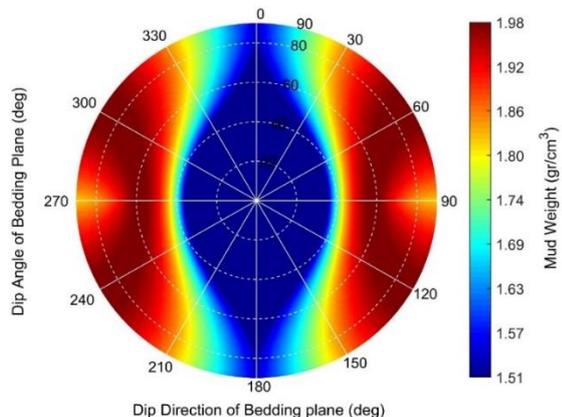
the dip and dip direction of weak bedding planes. When the dip angle of weak bedding planes is zero to about  $40^\circ$ , the mud weight is the same and the change in dip direction of weak bedding planes has no effect on the minimum drilling mud weight. By comparing the mud weight in these slopes with the amount of mud weight for IRFM in the vertical wellbore (Figure 7,  $i = 0^\circ$ ), it can be concluded that on a slope of  $0$  to  $40^\circ$ , layering does not have an effect on stability of the wellbore. The best mode in terms of stability is in the dip directions of zero and  $180^\circ$  and the worst mode in terms of stability is at dip angle of  $90^\circ$ .



**Fig. 7.** Variation of wellbore collapse mud weight for IRFM with  $\alpha=0$  to  $360^\circ$  and inclination angle ( $i=0$  to  $90^\circ$ ).



**Fig. 8.** Variation of wellbore collapse mud weight for MWBP with  $\alpha=0$  to  $360^\circ$  and inclination angle ( $i=0$  to  $90^\circ$ ).



**Fig. 9.** Variation of wellbore collapse mud weight for MWBP with  $\theta_{dd}=0-360^\circ$  and  $\theta_{dip}=0-90^\circ$  in vertical wellbores.

### 6.3. Effects of weak bedding plane on a horizontal wellbore

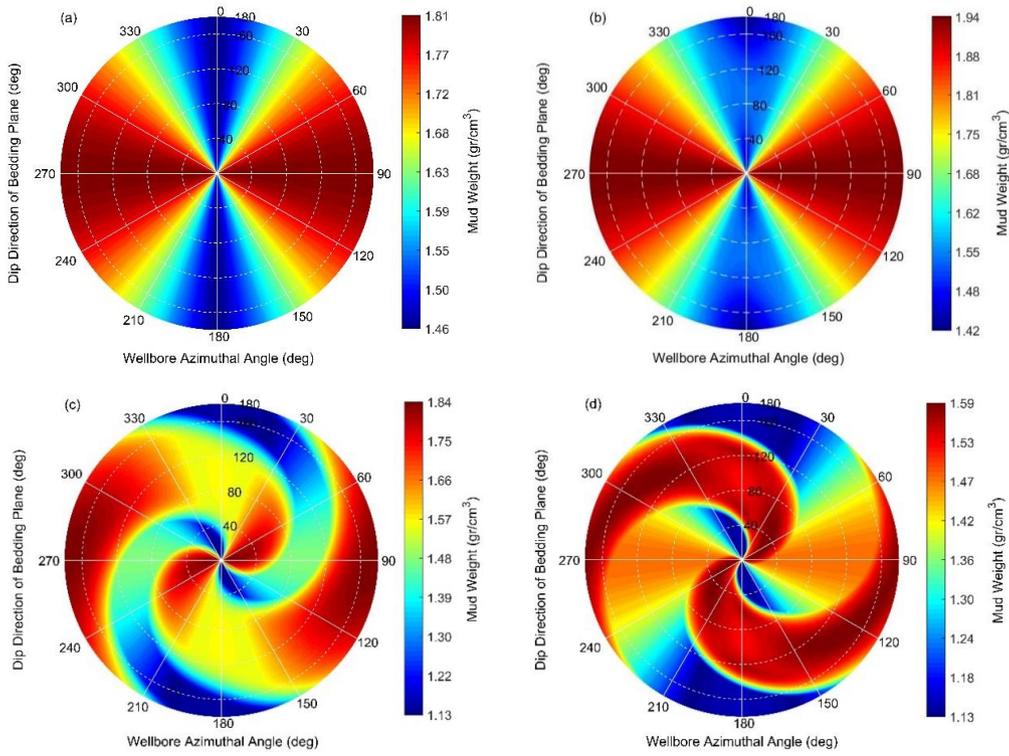
Fig. 10 shows the variations of the wellbore collapse mud weight in the dip directions zero to  $180^\circ$  and the azimuths zero to  $360^\circ$  when dip angle of bedding planes is  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$  and  $90^\circ$ . This Figure illustrates several different scenarios: when dip angle is zero mud weight varies from  $1.46$  to  $1.81$   $\text{gr}/\text{cm}^3$ , when dip angle is  $30^\circ$  mud weight varies from  $1.42$  to  $1.94$   $\text{gr}/\text{cm}^3$ ; when dip angle is  $60^\circ$  mud weight varies from  $1.13$  to  $1.84$   $\text{gr}/\text{cm}^3$  (the most variations), and when dip angle is  $90^\circ$  mud weight varies from  $1.13$  to  $1.59$   $\text{gr}/\text{cm}^3$ . The best mode in terms of stability is at dip angle of  $90^\circ$  whereas the lowest stability is at dip angle of  $30^\circ$ . At dip angles of zero and  $30^\circ$  in different azimuths, the change in dip direction does not have a significant effect on the stability of the wellbore, but at dip angles of  $60^\circ$  and  $90^\circ$  at different wellbore azimuths, changing the dip direction of weak bedding plane the mud weight fluctuates. When dip angle of bedding planes is zero and  $30^\circ$ , the optimum drilling azimuth is in the direction of maximum horizontal stress (less mud weight) and the worst mode in terms of stability is in direction of minimum horizontal stress (more mud weight). When dip angle of bedding planes is  $60^\circ$  and  $90^\circ$ , the best drilling azimuth varies according to the dip direction of bedding planes.

Fig. 11 shows the variation of wellbore collapse mud weight for MWBP with  $\alpha=0$  to  $360^\circ$  and  $\theta_{dd}=0$  to  $90^\circ$  in the horizontal wellbores when a:  $\theta_{dip}=0^\circ$ , b:  $\theta_{dip}=30^\circ$ , c:  $\theta_{dip}=60^\circ$  and d:  $\theta_{dip}=90^\circ$ . The optimum drilling azimuth in the dip direction of zero for all dip angles is in direction of maximum horizontal stress. In the dip directions of  $30^\circ$ ,  $60^\circ$  and  $90^\circ$  degrees, when dip direction is  $0$  to about  $60^\circ$ , the best drilling azimuth is in direction of maximum horizontal stress and in another dip angles depending on dip direction of weak bedding plane, the optimum drilling azimuth is different. For example, in dip direction of  $90^\circ$  when dip angle is  $60^\circ$  to  $90^\circ$ , the best drilling azimuth is in direction of minimum horizontal stress.

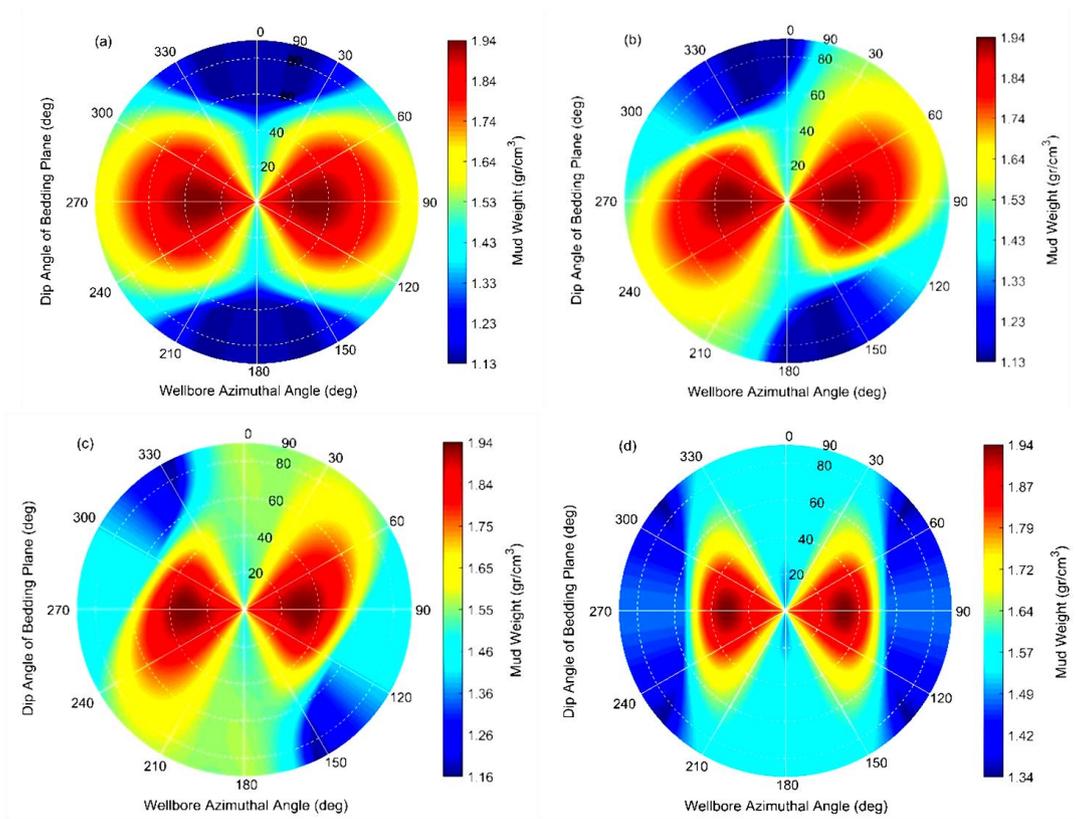
In order to analyze the effects of dip and dip direction of weak bedding plane on wellbore stability, Fig. 12 illustrates the variation of wellbore collapse mud weight for MWBP with  $\theta_{dd}$  ( $0$  to  $360^\circ$ ) and  $\theta_{dip}$  ( $0$  to  $90^\circ$ ) in the horizontal wellbores at different condition; a:  $\alpha=0^\circ$ , b:  $\alpha=30^\circ$ , c:  $\alpha=60^\circ$  and d:  $\alpha=90^\circ$ . The lowest mode in terms of stability is in the azimuth zero degree and the worst mode in terms of stability in the azimuth is  $90^\circ$  relative to the horizontal stress. In a horizontal wellbore, when the wellbore trajectory is perpendicular to the dip direction of weak bedding plane, the wellbore is unstable, and when the wellbore trajectory is parallel to dip direction of weak bedding plane, the wellbore is more stable. When the wellbore trajectory is perpendicular to the dip direction of weak bedding plane, the maximum stability at dip angles is greater than  $55^\circ$ .

## 7. Conclusion

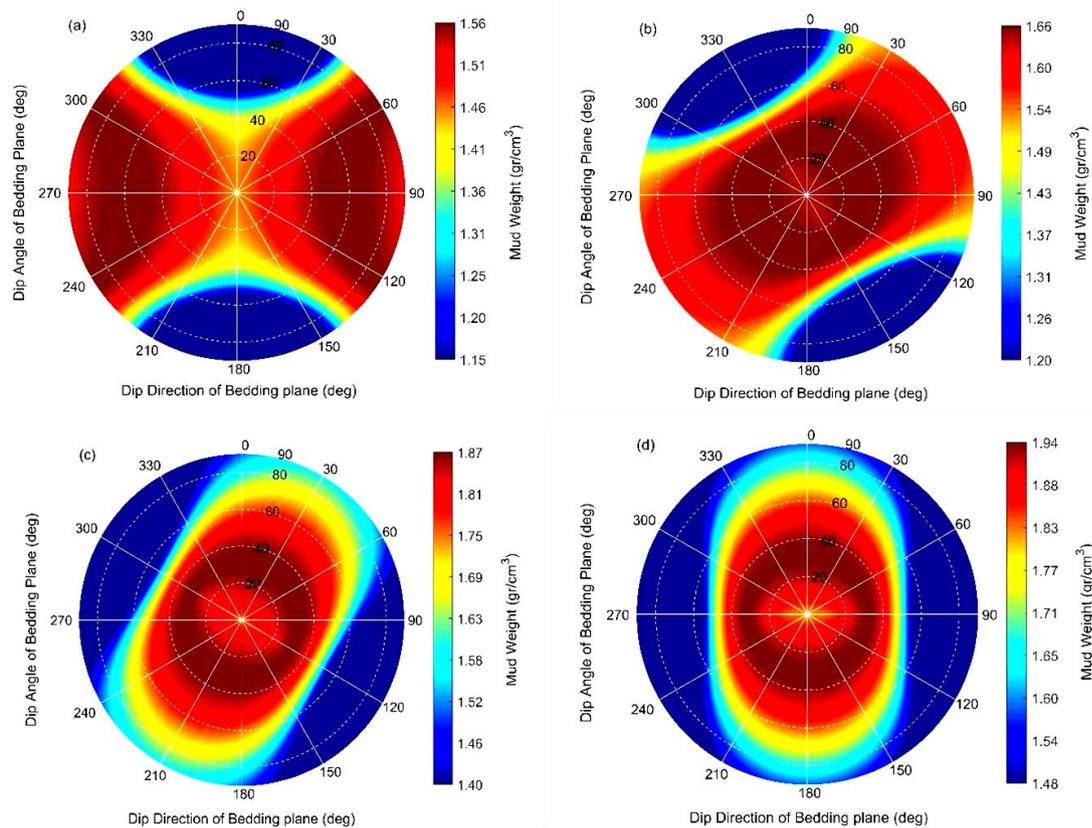
In this study, the effect of weak bedding plane on the stability of wellbore was reviewed. Based on the investigation of the stresses around the wellbore and study of the geometric relations of weak bedding plane, new equations are proposed to obtain attack angle ( $\beta$ ) and a new wellbore stability analytical model with considering weak bedding planes were presented. A code in MATLAB was written based on the analytical model and the effect of dip and dip direction of weak bedding plane was considered and used to calculate the amount of drilling mud weight. Using field data, the stability of the wellbore in the strike-slip fault regime was studied by considering the weak bedding plane and also, regardless of the weak bedding plane. A sensitivity analysis was performed on the effective parameters in vertical and horizontal wellbores. The results showed that the presence of weak bedding planes leads to more instability of the wellbore, and the wellbores in the up-dip drillings are more stable than down-dip and cross-dip drillings. Also, the dip and dip direction of the weak bedding plane and the direction of the wellbore has a great effect on the stability of the wellbore. In the vertical wellbore, when dip direction is zero or  $180^\circ$  and the dip angle is less than  $40^\circ$ , wellbore is more stable. In a horizontal wellbore, when the wellbore trajectory is parallel to the dip direction of weak bedding plane relative



**Fig. 10.** Variation of wellbore collapse mud weight for MWBP with  $\alpha=0$  to  $360^\circ$  and  $\theta_{dd}=0$  to  $90^\circ$  in the horizontal wellbores when a:  $\theta_{dip}=0^\circ$ , b:  $\theta_{dip}=30^\circ$ , c:  $\theta_{dip}=60^\circ$  and d:  $\theta_{dip}=90^\circ$ .



**Fig. 11.** Variation of wellbore collapse mud weight for MWBP with  $\alpha=0$  to  $360^\circ$  and  $\theta_{dip}=0$  to  $90^\circ$  in horizontal wellbores, a:  $\theta_{dd}=0^\circ$ , b:  $\theta_{dd}=30^\circ$ , c:  $\theta_{dd}=60^\circ$  and d:  $\theta_{dd}=90^\circ$ .



**Fig. 3.** Variation of wellbore collapse mud weight for MWBP with  $\theta_{\text{dip}}=0$  to  $360^\circ$  and  $\theta_{\text{dip}}=0$  to  $90^\circ$  in horizontal wellbores, a:  $\alpha=0$ , b:  $\alpha=30^\circ$ , c:  $\alpha=60^\circ$  and d:  $\alpha=90^\circ$ .

to the maximum horizontal stress wellbore is more stable. By applying the code geomechanical engineers could calculate the amount of drilling mud weight based on the dip and dip direction of weak bedding plane.

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