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Photogrammetry and Monte Carlo Simulation based statistical characterization of rock mass discontinuity parameters

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ABSTRACT

Discontinuities within the rock mass are present in a wide range of networks. Their characterization and analysis exist with considerable diversity. Prior research appraises the significance of mechanical discontinuities and their effect on geotechnical structures and deficient with integral discontinuities. The variability and uncertainty related to rock mass discontinuity parameters such as spacing, persistence, and aperture size cannot be present in a single value; it exhibits variability between specific range values. The use of a statistical method to present the discontinuity parameters provides a basis for Monte Carlo (MC) based stochastic modeling of discontinuity parameters to evaluate the stability of rock mass. The road cut slope of Bukit Merah, Malaysia, was investigated using close-range photogrammetry. Details of high precision rock mass discontinuities (mechanical and integral) parameters such as spacing, persistence, and aperture were captured. To evaluate the best fit distribution for discontinuity parameters, the Chi-Square test, Modified Kolmogorov Smirnov (K-S), and Anderson-Darling tests were employed. According to the findings, the discontinuity spacing is subjected to a lognormal distribution. In contrast, discontinuity persistence and aperture size followed log-logistic distribution. Furthermore, the Monte Carlo simulation (MCS) is a promising approach for assessing the variability and uncertainty of discontinuity parameter relationships.

Keywords: *Mechanical discontinuity, Integral discontinuity, Statistical characterization, Monte Carlo simulation, Frequency distribution.*

1. Introduction

Discontinuities in rock mass develop in the form of joints, faults, bedding planes, cleavage, lineation, foliation, and fractures exhibiting a significant effect on the stability of rock slope (Shah et al. 2020, Zhang et al. 2020). Discontinuity is a flaw or plane of weakness or mechanical breaks in a rock mass. The discontinuities are divided into integral or incipient and mechanical discontinuity. Integral discontinuities are intrinsic heterogonies within an intact rock, such as mineralogical bands or some series of partial breaks. These integral discontinuities change into mechanical discontinuities with an increase in weathering. An existing integral discontinuity can propagate into mechanical discontinuity depending on the number of mechanisms such as the stresses condition, local climate, fracture toughness, and geometry of integral discontinuity (Ghamgosar et al. 2015, Tating et al. 2015, Tating et al. 2019, Hack 2020, Shah et al. 2020). Generally, the discontinuities in rock mass exhibit random nature and set form (Wang et al. 2020). The properties of discontinuity in the rock mass are characterized by spacing, aperture size, persistence, and orientation. The discontinuity, geometrical information is derived from the exposed discontinuities at the rock mass outcrop (Salvini et al. 2020). Discontinuities present in rock mass play a preeminent role in the engineering structure's instability. Therefore, it is important to characterize discontinuities to evaluate the properties of the rock mass (Devkota et al. 2009).

Variability and uncertainty-related discontinuities are unavoidable in the geotechnical field due to the heterogeneous nature of the rock mass. Unfortunately, limited information is available for the assessment of the influence of discontinuities on rock mass properties. Therefore, the probabilistic analysis must describe the random character of discontinuity. A statistical approach was first employed to describe and incorporate the inherent uncertainty and variability related to the strength of brittle material based on Weibull distribution (Weibull 1939). Consequently, probabilistic analysis of the rock mass characteristic gained attention. Earlier researchers used statistical and probability approaches to estimate the best fit distribution for joint spacing and they revealed joint spacing follows exponential distribution (Devkota et al. 2009, Sari 2009, Stavropoulou 2014). However, according to Wong et al. (2018) and Bao et al. (2019), the lognormal distribution is the best fit distribution for joint spacing.

Early researchers used statistical and probability approaches to evaluate rock strength and deformability. At the same time, stochastic models are used to deal with uncertainties due to the stochastic nature of the rock mass geometry and variability associated with mechanical properties (Weibull 1939, Park and West 2001, Sari 2009). In literature, there are a limited number of studies that consider the stochastic estimation of the discontinuity parameters such as aperture size and persistence; mostly, researchers focused on spacing and orientation of the discontinuities. Additionally, for field measurements, the scan line and statistical window approaches are usually adopted to trace the length and spacing of discontinuities. However, it is independent of the type of method is chosen for measurement because the measurement scope is limited. Therefore, the discontinuity parameter measurement is limited. Furthermore, integral or incipient discontinuities are frequently ignored because they are either not properly appreciated in the site surveys or impossible to measure in the field. Hence, it is important to consider integral discontinuities because mechanical discontinuities are formed from incipient discontinuities due to an increase in weathering

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or change in the stress environment around the excavation. Furthermore, image-based strategies for discontinuity parameters produce images with limited sensing range to flow depth, causing uncertainty (Shah et al. 2021). Several scholars advocate the statistical method of analyzing uncertainty.

In the geotechnical sector, researchers are now presenting probabilistic methods that use the concept of probabilistic modelling to cope with inherent uncertainty. In general, the rock profile is extremely complicated; it frequently yields findings that differ from those expected during study and design. These results can be ascribed to neglecting the incipient discontinuities. Shang et al. (2018) devised a method for determining the tensile strength of incipient discontinuities on a laboratory scale. The specimens were tested according to the design scheme, and the results were compared to the results of the uniaxial tensile and Brazilian tests, as recommended by the International Society of Rock Mechanics (ISRM). They discovered that the scheme produces slightly inferior results when compared to traditional ISRM approaches. Shang (2020) evaluated the uniaxial tensile strength (UTS) of integral discontinuities of cylindrical specimens. The author concluded that the integral discontinuities have variation-related UTS and can have a high tensile strength when compared to the parent rock. Following the assessment of the strength of incipient discontinuities, the inherent uncertainty associated with discontinuity parameters has an impact on geotechnical design. To deal with the inherent uncertainty, many researchers adopted the Monte Carlo simulation. Sari et al. (2010) carried out the stochastic analysis of Ankara andesite by using Monte Carlo simulation to evaluate the possible range of the Hoek and Brown system. The authors unveiled that the suggested framework allows for a simple and efficient assessment of the variation in rock mass properties. Shah et al. (2021) suggested a Monte Carlo simulation-based approach for integrating the inherent uncertainty associated with particle shape descriptor distributions. The authors revealed that the suggested framework is capable of incorporating the inherent uncertainties associated with particle shape.

In this study, the photogrammetry method is adopted to consider the integral discontinuities in statistical characterization. In this method, firstly, the rock slope is divided into four windows. For further details regarding discontinuity parameters, each window is delineated into several blocks. Close range photogrammetry (CRP) is used to observe the discontinuity parameters for each block using digital images. Furthermore, to evaluate the microcracks, characterization of integral discontinuities and existing cracks was done based on image analysis using scanning electron microscopy. Discontinuity persistence, aperture size, and spacing are analyzed from the digital image using a manual particle primary method in ImageJ. Moreover, this study focused on the use of the Monte Carlo simulation to incorporate the uncertainties relating to rock mass discontinuity parameters.

2. Materials and Methods

2.1. Study area description

The study area is located in Bukit Merah Lake-town, Malaysia shown in Figure 1. Naturally, Sandstone rock contains microstructures such as integral discontinuities and microcracks. In Bukit Merah slope, the bedded sandstone and chert are inter-bedded with shale. The lower part of the slope comprises sandstone while in the upper part shale and silt are dominated. Sandstone bedding is varying from 60 to 100 cm while shale and silt range from a few centimeters to 8 cm. Based on observation of geological features, structures such as fault and fold are not observed. Load casts are seen at the bottom of the bedded sandstone. Sandstone is composed of very fine to medium size particles 0.125 mm to 0.25 mm. The particles of sandstone have low sphericity and angular texture (Usop 2014).

2.2. Methodology

In this research, the rock mass was firstly delineated into four windows. Further, for detailed geological mapping, each window is divided into several blocks (see Figure 2). Close range photogrammetry (CRP) is used to observe the discontinuity parameters for each block using a digital image taken by a 48-megapixel camera. Discontinuity persistence, aperture size, and spacing are analyzed from a digital image using a manual method. Images with discontinuities having clear and observable boundaries were reported. The reported photographs were analyzed and processed using ImageJ, an open architecture program. The "set measurement" dialogue box command in ImageJ was used to calculate the parameters of discontinuities. The microfracture analysis was carried to evaluate the presence of microfracture.

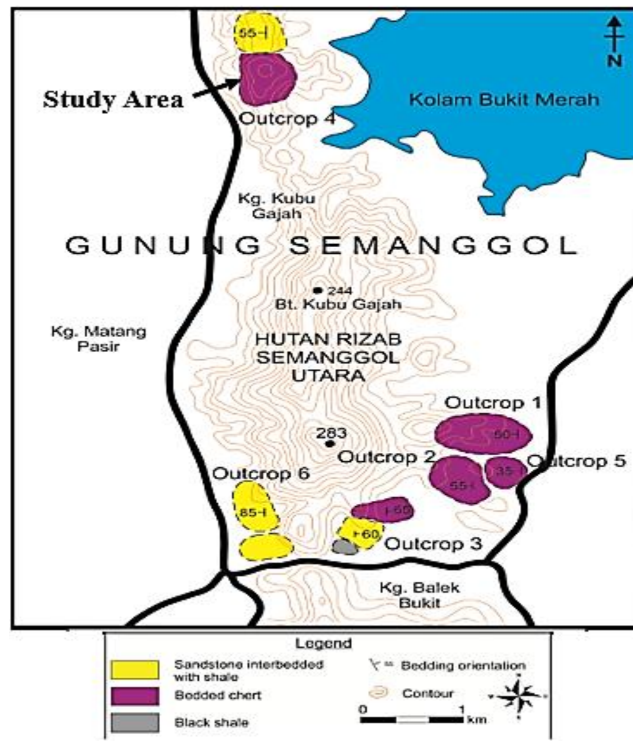

Figure 1: Lithology of the study area based on observation during geological mapping (Usop, 2014).

2.3. Monte Carlo simulation

The MCS is a stochastic simulation technique used to generate models of possible outcomes based on the previous fit distribution of input parameters. Firstly, discontinuity parameters were modeled with specific probability distributions (given in Table 1) and were used as input parameters to generate thousands of random values to model possible outcomes. The SimulAr and Microsoft excel are a comprehensive set of analysis tools that implements MCS by characterizing uncertainty to generate possible outcomes. It evaluates different types of probability distributions for input parameters and selects the best-fitted distribution of the data as well as produces statistics. For shape descriptors, all the input parameters were considered as random variables. The uncertainty of the input parameters can be depicted by the relative frequency histogram, which can be acquired from the micrograph using ImageJ software. When the distributions were acquired for the parameters, the distributions of spacing, persistence, and aperture size, MCS was used to model the probability of output parameters. The discontinuities' input parameter values would be randomly sampled from their distributions. While probability distributions of the discontinuity's parameters were obtained from MCS.

This section focused on technical aspects of how MCS incorporates inherent uncertainty into discontinuity's parameters based on the reference model. MCS builds a model of possible outcomes and then calculates results repeatedly using a distinct set of random numbers from the reference model. According to the nature of uncertainties, MCS calculates up to thousands and ten thousand based on a specified range. For example, an uncertain variable x has a specific distribution. The CDF F(x) provides the probability P that the variable X will be less than or equal to. i.e.

$$F(x) = P(X \leqslant x) \tag{1}$$

CDF F(x) ranges from 0 to 1. Now, reverse equation 1, to find inverse function.

G(F(x)) can be written as;

$$G(F(x)) = x \tag{2}$$

Equation 2 is used to generate random values r from reference distribution between 0 and 1. Then these random values are fed into equation 3 to generate the probability distribution of outcomes.

$$G(r) = x \tag{3}$$

These random numbers were generated from the reference model. SimulAr and Microsoft Excel are programs that process the data in the spreadsheet; it specifically involves the input parameters uncertainty to generate outputs presenting all possible outcomes. The package provides simple and instinctive execution of a Mont Carlo simulation to allow users to evaluate the changing effect in the discontinuity parameters. Latin Hypercube sampling was used to simulate the model to 10000 iterations. This alludes that for every run, the simulation generates 10000 possible outcomes of input parameters that were sampled randomly from the defined distribution.

2.4. Chi-Square test

A Chi-Square test is used to test the hypothesis.

H_o = The data that follow a specific type of distribution with cumulative distribution function F(x).

H_a = The data that does not follow a specified distribution with cumulative distribution function F(x). The data for the Chi-Square test is divided into k bins.

$$x^2 = \sum_{i=1}^{k} (o_i - E_i)^2 / E_i \tag{4}$$

Where o_i is the observed frequency and E_i is the expected frequency for the bin i.

$$E_i = N(F(Y_u) - F(Y_l)) \tag{5}$$

Where F is the CDF, Y_u is the upper limit, Y_l is the lower limit, and N is the sample size.

2.5. Modified Kolmogorov-Smirnov (K-S) Goodness-of-Fit Test

The K-S test hypothesis can be measured as

H_o= The data that follow a specific type of distribution with cumulative distribution function F(x).

H_a= The data that does not follow a specified distribution with cumulative distribution function F(x).

$$D = \max_{1 \le i \le N} \left(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i) \right) \tag{6}$$

Where F is the theoretical CDF of the reference distribution that is tested for adequacy.

2.6. Anderson-Darling test

The intend of the Anderson-Darling test is to estimate the adequacy of the type of distribution with sample data. This test is the alternative to the Kolmogorov-Smirnov Goodness-of-Fit Test and gives more importance to the tail end. The Anderson-Darling test is defined as:

H_o = The data that follow a specific type of distribution with cumulative distribution function F(x).

H_a = The data that does not follow a specified distribution with cumulative distribution function F(x).

A^2=-N-S, Where:

$$S = \sum_{i=1}^{N} \frac{(2i-1)}{N} [\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))] \tag{7}$$

F is the CDF and Y_i is the order of the data.

Table 1: Typical distribution and probability density function (PDF) and parameters.

Distribution	Probability density function	Parameters
Normal	$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < +\infty$	μ, σ
Lognormal	$f(x, \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), 0 < x < +\infty$	μ, σ
Exponential distribution	$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x \geq 0. \end{cases}$	$\lambda > 0$, rate, or inverse scale
Loglogistic	$f(x; \alpha, \beta) = \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left(1+\left(\frac{x}{\alpha}\right)^{\beta}\right)^2}, x > 0, \alpha > 0, \beta > 0.$	$\alpha > 0$scale $\beta > 0$shape
Weibull	$f(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$\lambda \in (0, +\infty)$ scale $k \in (0, +\infty)$ shape
Gamma	$f(x; \alpha, \beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$ for $x > 0$ $\alpha, \beta > 0.$	$\alpha > 0$scale $\beta > 0$shape

3. Results and Discussion

The primary objective of this study was to characterize the discontinuity parameters for sandstone outcrop in Bukit Merah, Malaysia. The principal elements of this research are; discontinuity parameters measurement using photogrammetry and ImageJ software, statistical modeling of spacing, persistence and aperture size of discontinuities, Monte Carlo simulation to incorporate uncertainty relating discontinuity parameters. For the determination of best fit distribution before and after MCS, the Anderson-Darling, Modified Kolmogorov-Smirnov, and Chi-Square tests were used.

3.1. Integral discontinuities

Integral discontinuities are intrinsic heterogonies within an intact rock, such as mineralogical bands or some series of partial breaks. These mineralogical bands and breaks may produce due to weathering, which alters and dissolved the existing minerals and subsequently results in mechanical discontinuities due to loading and unloading during the weathering process. Consequently, the image analysis tool is implemented to evaluate the integral as well as mechanical discontinuities for each block. Furthermore, the data-related spacing and persistence of discontinuities are obtained using ImageJ software.

3.2. Fitting distribution types to discontinuity's parameters

SimulAr and Microsoft Excel were used to fit the distribution to discontinuity's parameters. Modified Kolmogorov-Smirnov, Anderson-Darling, and Chi-square approaches were adopted for fitting the distribution of sampling data and their parameter results are given in Table 2. The provided figures show a histogram with the best fit model and the cumulative probability distribution of discontinuity's parameters. Table 3 summarizes the results of the statistics of discontinuities parameters from the study area.

3.2.1. Persistence

Discontinuity persistence is one of the important parameters that define the spacing and block size of the rock mass. Furthermore, intact rock between two adjacent persistence discontinuities exhibits a negative impact on the strength of the slope. Unfortunately, persistence is difficult to measure because usually discontinuity is not fully visible on the outcrop. Therefore, various researchers developed their methodology to measure discontinuity persistence by measuring the exposed traced length of discontinuity in a specific area. In this study, joint persistence is measured according to the new concept of measuring discontinuity on outcrop as the length of exposed discontinuity. Further, the persistence is generated as a random variable to incorporate uncertainty using MCS. In this study, we estimate the probability distribution of discontinuity persistence according to discontinuity length data rather than measuring persistence data. The goodness of fit results indicates that log-logistic distribution (see Figure 3) is the best fit distribution for discontinuity persistence.

3.2.2. Aperture

Aperture is the perpendicular distance between adjacent walls of open discontinuity filled with water or air. Therefore, the aperture differentiates from the width of the filled discontinuity. In real apertures of discontinuity varies over the length of discontinuity, and aperture has an obvious impact on the strength of rock mass. More importantly, the discontinuity aperture influences the shear strength of discontinuity that is an important parameter to consider the stability of rock slope. According to ISRM (Brown 1981), discontinuity aperture can only be measured roughly through direct observation of exposed open discontinuity. The histogram of discontinuity aperture is best represented by a log-logistic distribution shown in Figure 4. The goodness of fit results also validates the followed distribution and random nature of the discontinuity aperture.

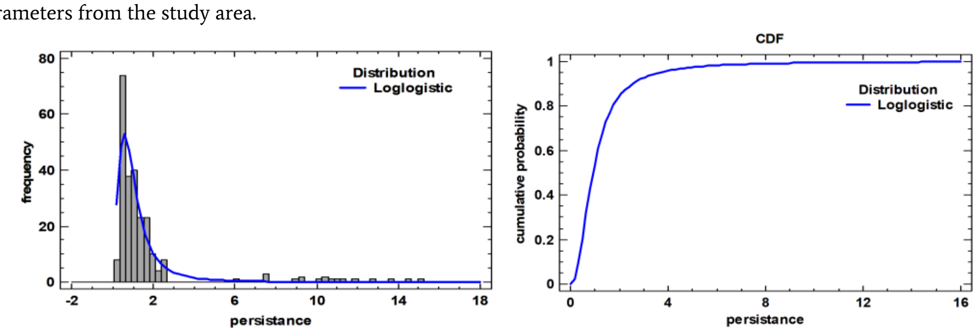

Figure 3: Best fit probability distribution for discontinuity persistence.

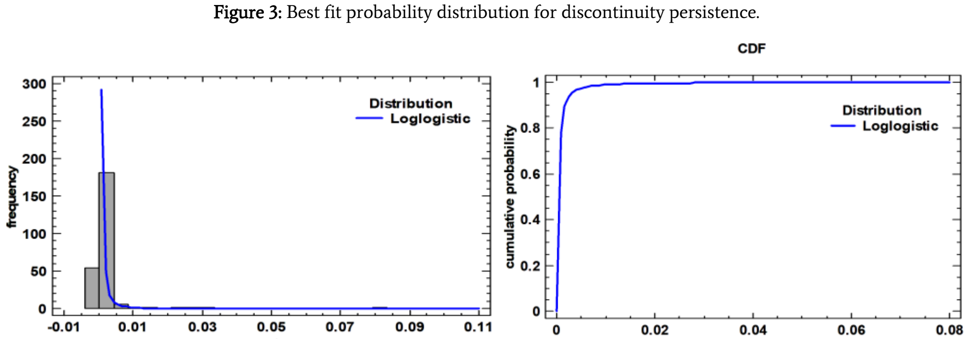

Figure 1: Distribution followed by discontinuity aperture.

3.2.3. Spacing

Rock mass discontinuity spacing is one of the important parameters that define the rock mass quality. Discontinuity spacing is defined as the perpendicular distance between two adjacent discontinuities. Several researchers represented spacing as a random variable and followed a type of distribution (Priest and Hudson 1976, Hudson and Priest 1983). Most of the researchers concluded that discontinuity spacing followed negative exponential and lognormal distribution (Park and West 2001, Park et al. 2005, Devkota et al. 2009, Cai 2011, Bao et al. 2019). According to Priest and Hudson (1976), if discontinuity spacing originates in uniform column jointing basalt or bedded sandstone, the most appropriate distribution type will be a normal distribution. Although it is similar to our study area where bedded sandstone followed negative exponential distribution appropriately. In our studies, we concluded that spacing followed lognormal distribution, which proves the results of Cai (2011) shown in Figure 5.

3.3. Monte Carlo simulation application

Monte Carlo simulation is an approach used to incorporate the uncertainty-related data and visualize the potential outcomes. When experienced with uncertainty during estimation or forecasting, rather than substituting uncertain data with a single average number, MCS might provide a better alternative solution. In this research, MCS was used for probabilistic evaluation of variations observed in the rock mass discontinuity's parameters.

The resulting histogram and distribution with CDF for each discontinuity parameter are given in Figure 6. The histogram provided by MCS show that spacing followed the lognormal distribution, aperture followed log-logistic while persistence best fit with the log-logistic distribution. The model calculates the average values of spacing, persistence, and aperture of Bukit Merah sandstone as before simulation 0.77, 1.67, 0.0013 and after simulation 0.772, 1.018, and 0.00042, respectively. The summary of statistics pre and post-simulation is presented in Table 3. The spacing ranges from 0.02 to 2.78, with a mean value of 0.78m. While the persistence of discontinuity ranges from 0.18 to 15.14 with mean values of 1.67 and aperture size ranges from 0.0000012 to 0.08 with a mean value of 0.0013.

From Table 3 and Table 2 it can notice that the statistical values for each discontinuity parameters changed after simulation. Therefore, it can be seen from the stochastic estimation of discontinuities parameters of Bukit Merah sandstone is not present a single value, but it exhibits variability between specific range values. The information acquired from MCS is still significant and provides additional evidence related to variability and uncertainty associated with discontinuity parameters.

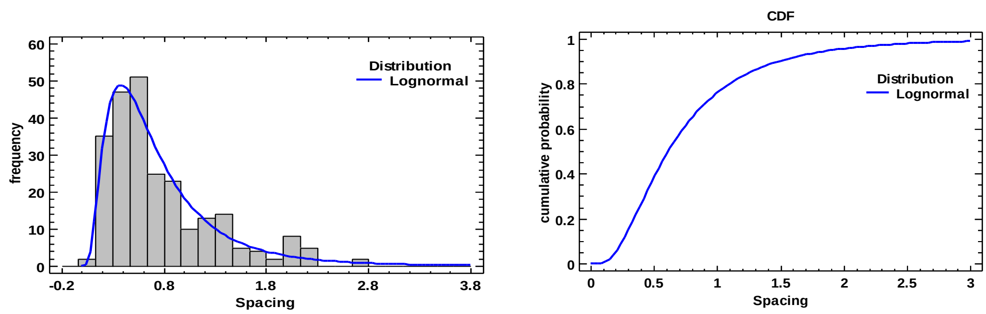

Figure 2: Joint spacing best fit distribution with CDF.

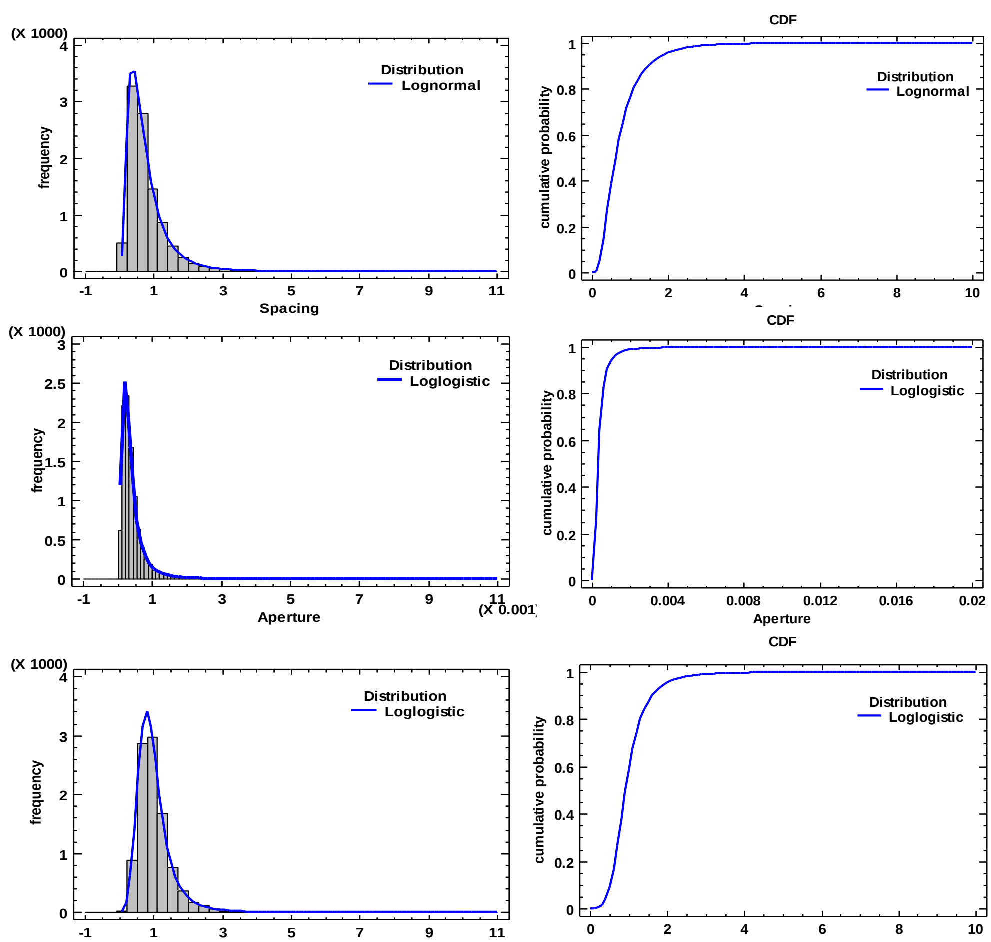

Figure 3: Histogram with best fit distribution and CDF for discontinuity parameters generated from simulation runs.

Table 2: Goodness of fit test results with their parameters pre and post-simulation.

Discontinuity parameters		Chi-Square test			Modified K-S			Anderson-Darling A^2	
		Chi-square	d.f	P-value	D	Modified form	P-value	A^2	P-value
Pre-simulation	Spacing	26.6	13	0.0143	0.05	0.78	>=0.10	0.602	>=0.10
	Persistence	67.2	6	1.5	0.082	1.29	<0.10	2.7	<0.05
	Aperture	11.02	1	0.00089	0.15	2.33	<0.01	9.5	<0.01
Post-simulation	Spacing	99.36	97	0.415	0.0073	0.73	>=0.10	0.53	>=0.10
	Persistence	8.7	16	0.93	0.003	0.3	>=0.10	0.07	>=0.10
	Aperture	51.4	41	0.13	0.003	0.29	>=0.10	0.09	>=0.10

Table 3: Summary Statistics before and after simulation.

Statistics	Discontinuity Parameters					
	Pre-simulation			Post-simulation		
	Aperture	Persistence	Spacing	Aperture	Persistence	Spacing
Average	0.0013	1.67	0.77	0.00042	1.018	0.772
Standard deviation	0.0059	2.61	0.533	0.00050	0.574	0.606
Coeff. of variation	474.33	155.9	69.2	118.243%	56.35%	78.53%
Minimum	0.0000012	0.18	0.02	0.0000026	0.058	0.037
Maximum	0.08	15.14	2.78	0.0185493	12.8537	9.74279
Stnd. skewness	10.4	3.40	1.37	428.939	183.7	125.233
Stnd. kurtosis	126.89	11.2	1.54	4898.41	1108.37	399.191

The population of microfractures present at sample localities of the study area exhibits varieties of discontinuity parameters. The revealed fractures are microscopic in nature and discovered by scanning electron microscopy. The fracture mechanics deals with the phenomena of microfracture initiation and coalescence provoked macro-scale fractures. Therefore, the macroscopic approach lack describing the entire fracture process in the rock mass. This study also put light on the microscopic fracture presence and its importance regarding the geotechnical study. Unfortunately, this research includes insufficient details to evaluate the microfractures parameters, but it put light on the importance of microfractures in a rock mass.

The simulation results unearth that Monte Carlo simulation is a strong tool for the deterministic techniques for analyzing the discontinuity parameters statistics. In comparison to the deterministic approaches, the MCS provides a thorough probability distribution of and an extremely broad range of discontinuity parameters values. A detailed explanation and analysis can also be possible with the MCS.

The deterministic technique, on the other hand, only delivers a limited quantity of analytic information and a great deal of uncertainty, making inferences subjective. Furthermore, when compared to the deterministic technique, the statistics of discontinuity parameters values derived using the Monte Carlo simulation are more reliable. The discontinuity parameters statistics (average, median, maximum, minimum, standard deviation, coefficient of variation, standard skewness, and standard kurtosis) were obtained using the Monte Carlo simulation from a large number (1000 in this case) of simulated samples. In contrast to deterministic analysis, the average and median of discontinuity parameter values are calculated instantly from input parameter statistics. The most important component of MCS is that it is included, the effects of discontinuity parameters standard deviations.

4. Conclusion

This study characterizes rock mass discontinuity parameters using photogrammetry and stochastic modelling. Close-range photogrammetry with digital photography is utilized to capture the discontinuity parameters, and stochastic modelling is utilized to assess the system's variability, with an emphasis on the weathered Bukit Merah sandstone's characteristic of discontinuity parameters.

- Close range photogrammetry can be applied to obtain high accuracy, the surface exposer, or outcrop of discontinuities in the study area. The obtained results indicate that the possibility of describing the study area with approximate blocks and accurately computing the discontinuity parameter data.
- According to the goodness of fit tests, discontinuity spacing is subjected to lognormal distribution while discontinuity persistence and aperture size follow the log-logistic distribution.
- Variability in the mechanical behavior of discontinuity could be easily assessed if an adequate approach is applied that considered the frequency distribution of the discontinuity parameters. This is because discontinuities parameters have a wide range of uncertainty and variability and cannot be described in a single value as input and output parameters. Consequently, it is suggested to use probabilistic analysis where discontinuity parameters are scattered.
- The proposed approach to this variability problem provides a viable means to capture the uncertainty and variability-related discontinuity parameters that cannot appraise during experimental work. Compare to the deterministic approach; these simulation results will help the engineer to get more rational design parameters for engineering structures in a rock mass.

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