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Benchmark study of rock slope stability through generalized Hoek-Brown criterion: a case study of an open pit in Sudan

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ABSTRACT

The non-linear Generalized Hoek-Brown (GHB) failure criterion for rock mass is widely accepted and has been applied in a large number of open pits slope designs. This paper proposes new equations for estimating the maximum confining stress from the (GHB) parameters and geometrical properties of the slope in the case where the strength ratio is critical $(SR)_{crit} = (\sigma_{ci}/\gamma H)_{crit}$ and the factor of safety (FOS)=1. This maximum confining stress can be used to calculate the global equivalent Mohr-Coulomb (MC) parameters. It was found that, compared to the calculation with the limit analysis method (LAM), the discrepancies do not exceed 5% and remain in most cases less than 1%. Hence, the estimation of the (FOS) is much more improved, because the comparison of the literature's results with the (LAM) led to a difference up to 21%. For any value of (FOS ≠1), an iterative method has been proposed to evaluate $(SR)_{crit}$. The comparison between the results-driven from this method and those of (LAM) showed a good agreement, which proves its accuracy. A case study has been conducted in an open pit located in Sudan to evaluate the discrepancy of the (FOS) provided by different methods using limit equilibrium method (LEM) with Rocscience Slide software and using the (LAM) given in the form of charts.

Keywords: *Factor of safety, Limit analysis method, Limit equilibrium method, Open-pit, Slope stability.*

1. Introduction

Various methods can be used to study rock slope stability. The approach of equivalent homogeneous and continuous medium is commonly used when the fracture density is high i.e. the spacing between the two adjacent discontinuity surfaces is too small compared to the overall dimension of the rock structures [1].

Several failure criteria have been suggested by different researchers to describe the strength envelope of rock masses, such as the power-law criterion presented by Hobbs (1966) [2] and the nonlinear failure criterion, which is based on Griffith crack theory [3] that has been proposed by Ladanyi (1974) [4]. The nonlinear Hoek-Brown (HB) is widely accepted. This criterion was developed by Hoek and Brown (1980) [5]. It takes into account the properties of the intact rock and discontinuities. In 2002, the Generalized Hoek-Brown (GHB) criterion was presented by Hoek et al. (2002) [6]. Compared with the linear Mohr-Coulomb (MC) criterion, which includes two parameters, the cohesion (c') and the internal friction angle (φ'), the non-linear (GHB) criterion depends on three parameters, namely (m_b), (s) and (a). These parameters depend on:

-The Geological Strength Index (GSI). (GSI) was used to estimate the rock mass strength for different geological conditions, because Bieniawski's rock mass rating (RMR) system [7] and the Q-system [8] were found to be unsuitable for poor rock masses.

-The parameter (D) is a factor, which represents the degree of disturbance. It ranges from 0 for undisturbed in situ rock masses to 1 for disturbed rock mass properties.

-The uniaxial compressive strength of the intact rock material (σci).

-The material constant (m_i).

It is difficult to apply directly the non-linear (GHB) criterion to the stability analysis of rock slopes with the limit equilibrium method (LEM). The rock mass strength parameters are usually converted into the equivalent (MC) parameters, as proposed by Hoek et al. (2002) [6]. In this case, the cohesion (c') and the angle of friction (φ′) are constant along any given slip surface. Moreover, the general numerical solution can be used to assess the equivalent (MC) shear strength parameters from the (GHB) criterion as suggested by Kumar (1998) [9]. In this case, (c′) and (φ′) will vary along any given slip surface, depending on the stress level. Hence, accurate results are achieved for the slope stability studies.

Instead of (LEM), several research teams have attempted to apply the limit analysis method (LAM) to investigate the rock slope stability. Collins et al. (1988) [10] proposed the "tangential technique" to evaluate the stability of an infinite and homogeneous rock slope with the original Hoek - Brown (HB) failure criterion (a = 0.5). The method was generalized to the (GHB) failure criterion by Yang et al. (2004) [11]. The effect of the exponent (a) on the stability of the rock slopes was then investigated. Li et al. (2008) [12] used the numerical limit analysis

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method to produce stability charts for rock slopes. They proposed the non-dimensional stability number (N_{crit}) = $\sigma_{ci}/\gamma H\, FOS_{LAM}$ which is expressed as a function of the term $(SR) = (\sigma_{ci}/\gamma H)$. The use of (SR) is a significant innovation for the rock slope stability analysis because when the values of the input parameters (GSI), (m_i), (D), and (β) are determined, the (FOS) is related only to the (SR) of that slope [13]. The accuracy of the method was tested and approved except in some cases for which the difference between (LEM) and (LAM) remains high and can reach 21% [12-14].

In this research, special attention will be given to the equivalence established between the linear criterion (MC) and the non-linear (GHB) criterion [6, 12]. New equations to assess this equivalence are proposed. Through the case study of an open pit located in Sudan, the (FOS)'s discrepancies of rock mass slope have been evaluated according to different assumptions and methods.

2. The Generalized Hoek–Brown failure criterion (GHB)

In this paper, the latest version of (GHB) failure criterion [6] is used (Eq. 1). Relations between (GSI), (D), (m_b), (s), and (a) are introduced to provide a smoother transition between very poor quality rock masses (GSI ≤ 25) and stronger rocks as detailed in Eqs. 2, 3 and 4. A disturbance factor (D) to account for stress relaxation and blast damage is also introduced.

$$\sigma_1' = \sigma_3' + \sigma_{ci}(m_b \frac{\sigma_3'}{\sigma_{ci}} + s)^a \quad (1)$$

Where (σ_1') and (σ_3') are the major and minor effective principal stresses, and (m_b), (s), and (a) are the material constants that can be related to the (GSI) and rock damage.

$$m_b = m_i\, exp\,(\frac{GSI-100}{28-14\,D}) \quad (2)$$

$$s = exp\left(\frac{GSI-100}{9-3D}\right) \quad (3)$$

$$a = \frac{1}{2} + \frac{1}{6}(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}}) \quad (4)$$

The unconfined compressive strength is given by Eq. 5:

$$\sigma_c = \sigma_{ci} s^a \quad (5)$$

And the tensile strength by Eq. 6:

$$\sigma_t = -s\frac{\sigma_{ci}}{m_b} \quad (6)$$

2.1. Limit equilibrium method with local equivalent (MC) parameters

The (GHB) failure criterion has been used successfully for the design approaches that use limit equilibrium solutions. Since most geotechnical engineering software is still written in terms of the (MC) failure criterion, it is necessary to determine equivalent friction angles and cohesive strengths for each rock mass and stress range.

Based on a generic form of Balmer's equations [15] and the general solution proposed by Kumar (1988) [9], Shen et al. (2012) [16] proposed a new approximate analytical solution to estimate the local (MC) shear strength parameters from the (GHB) criterion for a highly fractured rock mass:

$$\varphi' = \sin^{-1}\left(1 - 2\,/2 + am_b\left(m_b\,\frac{\sigma_3'}{\sigma_{ci}} + s\right)^{a-1}\right) \quad (7)$$

$$\sigma_3'/\,\sigma_{ci} = \sigma_n'\,/\sigma_{ci} - (\,m_b\,\frac{\sigma_3'}{\sigma_{ci}}\, +\, s\,)^a/2 + am_b(\,m_b\,\,\frac{\sigma_3'}{\sigma_{ci}}\, +\, s\,)^{a-1}) \quad (8)$$

$$\tau' = (\sigma_{ci}\cos{(\varphi')}\,/2 + (\,1\,+\,\frac{\sin{(\varphi'\,)}}{a})^{\,a})\left(m_b\,\frac{\sigma_n'}{\sigma_{ci}} + s\right)^a \quad (9)$$

$$c' = \tau' - \sigma_n' \tan{(\varphi')} \quad (10)$$

For given normal stress (σ_n'), the acceptable value for (σ_3'/σ_{ci}) presented in Eq. 8 must be solved iteratively. The internal friction angle (φ') and cohesion (c') can be directly calculated from Eqs. 7, 9 and 10, and represent the local (MC) strength parameters of any point in the rock mass provided by locating the tangent of the (GHB) envelope under a normal stress value of (σ_n') as shown in Fig. 1.

This method has been used in different software to calculate rock slope stability with (LEM). For instance, when the (GHB) criterion is selected, the software Slide [17] calculates a set of instantaneous equivalent (MC) parameters based on the normal stress at the base of each slice.

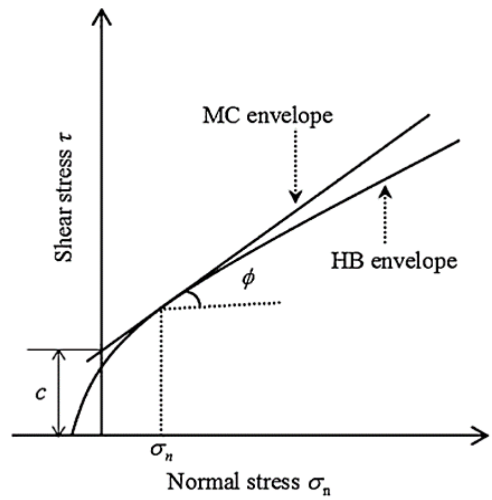

Fig. 1. Relationships between the local (MC) envelope and (HB) envelope in the normal and shear stress plane [16]

2.2. Limit equilibrium method with global equivalent (MC) parameters

As proposed by Hoek et al. (2002) [6], a useful method can be used to convert the rock mass strength parameters into the global equivalent (MC) parameters. For a range of minor principal stress values defined by ($\sigma'_t \le \sigma'_3 \le \sigma'_{3max}$), the fitting process is illustrated in Fig. 2. In this case, (c') and (φ') are constant along any given slip surface and are estimated, by balancing the areas above and below the (MC) plot over a range of minor principal stress values, as expressed ssin Eqs. 11, 12, and 13.

$$c' = \frac{\sigma_{ci}\,[(1+2a)\,s + (1-a)\,m_b\sigma_{3n}'](s + m_b\,\sigma_{3n}')^{a-1}}{(1+a)(2+a)\sqrt{1 + \frac{6\,a\,m_b\,(s + m_b\,\sigma_{3n}')^{\,a-1}}{(1+a)(2+a)}}} \quad (11)$$

$$\varphi' = \sin^{-1}\left(\frac{6\,am_b\,(s+m_b\,\sigma_{3n}')^{a-1}}{2(1+a)(2+a) + 6\,a\,m_b(s+m_b\,\sigma_{3n}')^{a-1}}\right) \quad (12)$$

where

$$\sigma_{3n}' = \frac{\sigma_{3max}'}{\sigma_{ci}} \quad (13)$$

The value of (σ_{3max}'), which is the upper limit of confining stress, is determined for each specific problem. For slope stability, the following equation is used to estimate σ_{3max}') (Eq. 14).

$$\frac{\sigma_{3max}'}{\sigma_{cm}} = 0.72\left[\frac{\sigma_{cm}}{\gamma H}\right]^{-0.91} \quad (14)$$

where (γ) is the material unit weight and (H) is the height of the slope. For the stress range, ($\sigma'_t \le \sigma'_3 \le \sigma'_{ci/4}$), the compressive strength of rock mass (σ_{cm}) can be determined by using Eq. 15.

$$\sigma_{cm} = \sigma_{ci}\,\frac{m_b + 4s - a\,(m_b - 8s)\,(\frac{m_b}{4} + s)^{a-1}}{2(1+a)(2+a)} \quad (15)$$

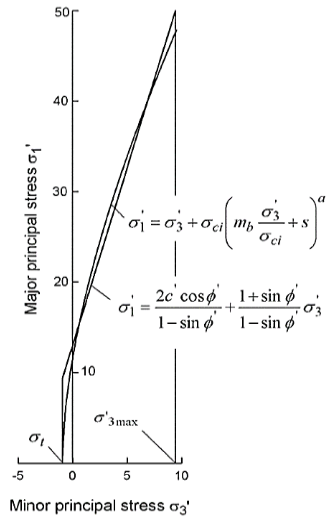

Fig. 2. Relationships between major and minor principal stresses for Hoek-Brown (HB) and equivalent Mohr-Coulomb (MC) criteria [6]

2.3. Limit analysis method with local equivalent (MC) parameters Analysis of Brazilian tensile strength

For non-linear (GHB) failure criterion, Yang et al. (2004) [11] proposed the "generalized tangential technique", as an extension of the "tangential technique" developed by Collins et al. (1988) [10], to evaluate the stability of an infinite and homogeneous rock slope. The strength of a tangential line exceeds or equals the line of a non-linear failure criterion (Fig. 1). Thus, the strength of the tangential line will lead to an upper bound on the actual load. The expression of equivalent (MC) parameters is the same compared to Eqs. 7, 8, 9, and 10. The objective function is achieved by equating the work rate of external forces to the internal energy dissipation rate. The objective function depends on (c′), (φ′), and both of the geometrical parameters of the slope and the sliding surface. The optimization of this function provides a least upper bound for the critical value of $(SR)_{crit} = (\sigma_{ci} / \gamma H)_{crit}$. Based on this method, Li et al. (2008) [12] provided stability charts for rock slopes for undisturbed rock slope (D = 0). Similar stability charts, with (D = 0.7) and (D = 1.0), were also proposed by Li, Merifield et al. (2011) [18], in order to examine the effects of disturbance on rock slope stability. Seismic stability charts were also proposed by Li et al. (2009) [19] to account for the seismic effects on rock slope stability.

3. The enhancement of the global equivalent (MC) parameters' estimation

The agreement between the global equivalent (MC) and (GHB) criterion needs an accurate identification of the stress field at any point in each step of the calculation. Li et al. (2008) [12] have shown that the difference in estimated (FOS) using the global (MC) equivalent parameters (Eq. 15) and the (LAM) using (GHB) criterion was as high as 64% (Table 1). This difference mainly lies in estimating a suitable minor principal stress. In order to achieve accurate equivalent (MC) parameters, they proposed new equations to evaluate the maximum confining stress. Therefore, they used Eq. 16 for steep slopes and Eq. 17 for gentle slopes, to estimate with accuracy the (FOS) by using (LEM) with Slide Software.

For steep slope (β ≥ 45°):

$$\frac{\sigma'_{3max}}{\sigma_{cm}} = 0.20\left[\frac{\sigma_{cm}}{\gamma H}\right]^{-1.07} \tag{16}$$

For gentle slope (β < 45°):

$$\frac{\sigma'_{3max}}{\sigma_{cm}} = 0.41\left[\frac{\sigma_{cm}}{\gamma H}\right]^{-1,23} \tag{17}$$

However, this adjustment remains inaccurate especially for some cases where the error can reach 21% [12] (Table 1).

In this paper, another adjustment to estimate with accuracy (σ'_{3max}) has been proposed. We started from the idea that for given input parameters (GSI), (m_i), (D), and (a), the (FOS) depends only on the $(SR) = (\sigma_{ci} / \gamma H)$. Therefore, the same calculations of Li et al. (2008) [12] were used by fixing, for several slopes and a set of parameters (GSI), (m_i), (D = 0) and (β) of (GHB), a critical nondimensional parameter $(\sigma_{ci}/\gamma H)_{crit}$ that leads to a (FOS) equal to 1 with (LAM). In each considered case, this critical value of $(SR)_{crit} = (\sigma_{ci}/\gamma H)_{crit}$ can be estimated from the charts established by Li et al. (2008) [12]. Fig. 3 presents an example of these charts for (D = 0) and β = 45°.

These different cases were calculated with (LEM) using Slide that is based on non-linear (GHB) criterion. Thus, local (MC) parameters were estimated according to Eqs. 7, 8, 9, and 10. The calculations helped to estimate the maximum normal stress applied on the slip surface and consequently the value of (σ'_{3max}) for each case. The variable ($Y = m_b\, \sigma'_{3max} / \sigma_{ci}$) was plotted as a function of ($X = (\sigma_{ci} / \gamma H)_{crit}$) in Figs. 4-(a) and (b) for (D = 0) and (D = 1) respectively. The linear regressions, performed at the logarithmic scale, provided a correlation coefficient of $R^2 = 0.99$. This is valid for a large range of ($\sigma'_{3max}/\sigma_{ci}$) from lower to higher confining stress. For a given $(SR)_{crit}$, Eqs. 18 and 19 are used to calculate ($\sigma'_{3max(crit)}$) for (D = 0) and (D = 1) respectively. The global equivalent parameters (c'_{crit}) and (φ'_{crit}) are then calculated using Eqs. 11, 12, and 13.

$$m_b\,\frac{\sigma_{3max}}{\sigma_{ci}} = 0.18\left[\frac{\sigma_{ci}}{\gamma H}\right]^{-1.74} \tag{18}$$

$$m_b\,\frac{\sigma_{3max}}{\sigma_{ci}} = 0.12\left[\frac{\sigma_{ci}}{\gamma H}\right]^{-1.74} \tag{19}$$

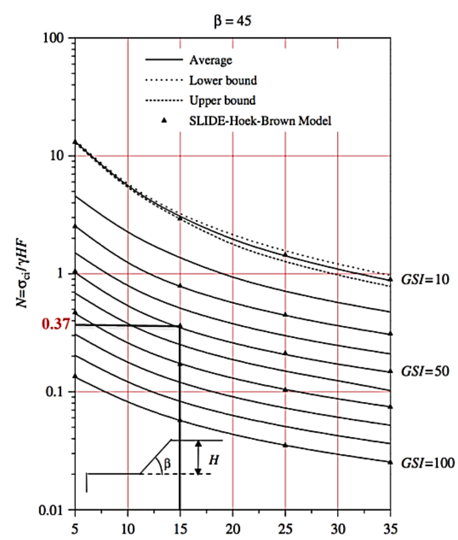

Fig. 3. Slope stability charts with (D=0) and (β= 45°) [12]

For (D = 0), and based on our suggested method, the (FOS) is calculated for each case with (MC) criterion using the global equivalent parameters (c'_{crit}) and (φ'_{crit}) (Table 1). It was found that, compared to a calculation with the (LAM), the difference does not exceed 5% and remains, in most cases, less than 1%. Eq. 14 and both of Eqs. 16 and 17, led to a difference up to 64% and 21% respectively. Consequently, the estimation of the (FOS) is improved with the adjustment that we have proposed.

It is worth recalling that the Eqs. 18 and 19 are only valid for $(SR)_{crit}$ and a (FOS) = 1. For a general case (FOS ≠ 1), iterative calculations have to be performed to achieve the $(SR)_{crit}$ as shown in the algorithm presented in Fig. 5.

Table 1. Comparisons of safety factors between the Non-linear (GHB) strength parameters and the equivalent (MC) parameters for (D = 0)

			Limit analysis [12]		SLIDE-limit equilibrium using equivalent Mohr-Coulomb (MC) parameters							
			Non-linear Hoek-Brown		Non linear Hoek-Brown		Linear (MC) Eq. 14 [6]		Linear (MC) Eqs. 16, 17 [12]		Linear (MC) Eq. 18 The current method	
β	GSI	mi	$(\sigma_{ci} / \gamma H)_{crit}$	(FOS)	(FOS)	%Diff	(FOS)	%Diff	(FOS)	%Diff	(FOS)	%Diff
75	70	5	1.703	1.000	0.988	-1.2%	1.081	8.0%	1.025	2.0%	0.998	-0.2%
75	70	15	1.169	1.000	1.002	0.2%	1.287	29.0%	1.081	8.0%	1.028	2.8%
75	70	25	0.890	1.000	1.005	0.5%	1.350	35.0%	1.124	12.0%	1.034	3.4%
75	70	35	0.717	1.000	1.016	1.6%	1.394	39.0%	1.156	16.0%	0.999	-0.1%
75	50	5	4.980	1.000	0.997	-0.3%	1.154	15.0%	1.036	4.0%	0.998	-0.2%
75	50	15	2.988	1.000	1.004	0.4%	1.336	34.0%	1.119	12.0%	1.004	0.4%
75	50	25	2.156	1.000	1.018	1.8%	1.425	43.0%	1.148	15.0%	0.997	-0.3%
75	50	35	1.668	1.000	1.024	2.4%	1.450	45.0%	1.174	17.0%	0.997	-0.3%
75	30	5	15.011	1.000	1.001	0.1%	1.248	25.0%	1.047	5.0%	0.995	-0.5%
75	30	15	8.576	1.000	1.016	1.6%	1.459	46.0%	1.136	14.0%	1.023	2.3%
75	30	25	5.824	1.000	1.025	2.5%	1.510	51.0%	1.173	17.0%	0.997	-0.3%
75	30	35	4.327	1.000	1.033	3.3%	1.516	52.0%	1.194	19.0%	0.997	-0.3%
75	10	5	93.721	1.000	1.004	0.4%	1.224	22.0%	1.018	2.0%	1.011	1.1%
75	10	15	53.362	1.000	1.023	2.3%	1.504	50.0%	1.126	13.0%	1.000	0.0%
75	10	25	35.186	1.000	1.035	3.5%	1.605	61.0%	1.185	19.0%	0.999	-0.1%
75	10	35	24.994	1.000	1.046	4.6%	1.642	64.0%	1.210	21.0%	0.999	-0.1%
45	70	5	0.469	1.000	1.001	0.1%	1.038	4.0%	1.001	0.0%	0.997	-0.3%
45	70	15	0.176	1.000	1.012	1.2%	1.080	8.0%	1.002	0.0%	1.020	2.0%
45	70	25	0.108	1.000	1.017	1.7%	1.060	6.0%	1.007	1.0%	1.028	2.8%
45	70	35	0.077	1.000	1.019	1.9%	1.061	6.0%	1.009	1.0%	1.029	2.9%
45	50	5	1.046	1.000	1.004	0.4%	1.045	4.0%	1.001	0.0%	0.992	-0.8%
45	50	15	0.369	1.000	1.009	0.9%	1.065	6.0%	1.004	0.0%	1.004	0.4%
45	50	25	0.222	1.000	1.020	2.0%	1.066	7.0%	1.010	1.0%	1.023	2.3%
45	50	35	0.158	1.000	1.021	2.1%	1.044	4.0%	1.011	1.0%	1.022	2.2%
45	30	5	2.593	1.000	1.011	1.1%	1.066	7.0%	0.999	0.0%	0.992	-0.8%
45	30	15	0.829	1.000	1.018	1.8%	1.070	7.0%	1.007	1.0%	1.001	0.1%
45	30	25	0.480	1.000	1.021	2.1%	1.076	8.0%	1.010	1.0%	1.013	1.3%
45	30	35	0.334	1.000	1.024	2.4%	1.085	9.0%	1.011	1.0%	1.018	1.8%
45	10	5	13.585	1.000	1.014	1.4%	1.087	9.0%	1.000	0.0%	0.998	-0.2%
45	10	15	3.155	1.000	1.023	2.3%	1.106	11.0%	1.005	0.0%	1.002	0.2%
45	10	25	1.552	1.000	1.023	2.3%	1.107	11.0%	1.009	1.0%	1.003	0.3%
45	10	35	0.969	1.000	1.026	2.6%	1.079	8.0%	1.010	1.0%	1.003	0.3%
30	70	5	0.218	1.000	1.018	1.8%	0.985	-2.0%	1.011	1.0%	1.006	0.6%
30	70	15	0.075	1.000	1.023	2.3%	0.996	0.0%	1.028	3.0%	1.035	3.5%
30	70	25	0.045	1.000	1.024	2.4%	1.004	0.0%	1.035	3.0%	1.050	5.0%
30	70	35	0.032	1.000	1.025	2.5%	1.010	1.0%	1.040	4.0%	1.050	5.0%
30	50	5	0.461	1.000	1.020	2.0%	0.993	-1.0%	1.014	1.0%	0.999	-0.1%
30	50	15	0.153	1.000	1.024	2.4%	1.003	0.0%	1.026	3.0%	1.019	1.9%
30	50	25	0.091	1.000	1.025	2.5%	1.024	2.0%	1.032	3.0%	1.030	3.0%
30	50	35	0.065	1.000	1.026	2.6%	1.008	1.0%	1.036	4.0%	1.043	4.3%
30	30	5	1.057	1.000	1.022	2.2%	1.001	0.0%	1.012	1.0%	0.995	-0.5%
30	30	15	0.323	1.000	1.026	2.6%	1.003	0.0%	1.026	3.0%	1.007	0.7%
30	30	25	0.185	1.000	1.026	2.6%	1.005	0.0%	1.031	3.0%	1.013	1.3%
30	30	35	0.129	1.000	1.027	2.7%	1.004	0.0%	1.035	3.0%	1.020	2.0%
30	10	5	4.363	1.000	1.023	2.3%	1.002	0.0%	1.006	1.0%	1.002	0.2%
30	10	15	0.943	1.000	1.025	2.5%	1.007	1.0%	1.023	2.0%	1.003	0.3%
30	10	25	0.460	1.000	1.026	2.6%	0.996	0.0%	1.033	3.0%	1.004	0.4%
30	10	35	0.286	1.000	1.026	2.6%	1.004	0.0%	1.040	4.0%	1.003	0.3%

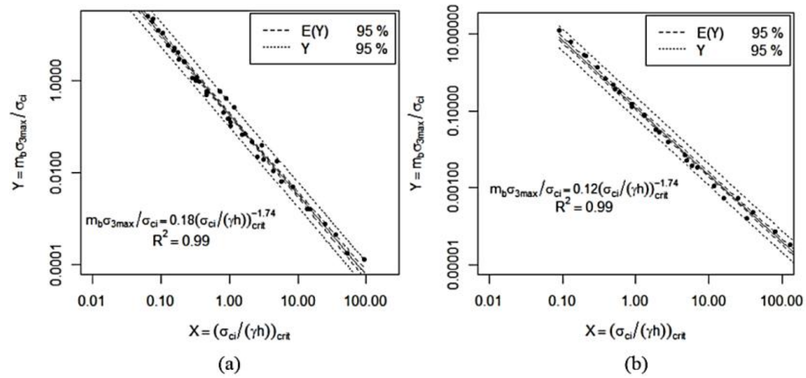

Fig. 4. Relationship for the calculation of ($\sigma'_{3max(crit)}$) between equivalent Mohr-Coulomb and Hoek-Brown parameters. (a)- D = 0. (b)- D = 1

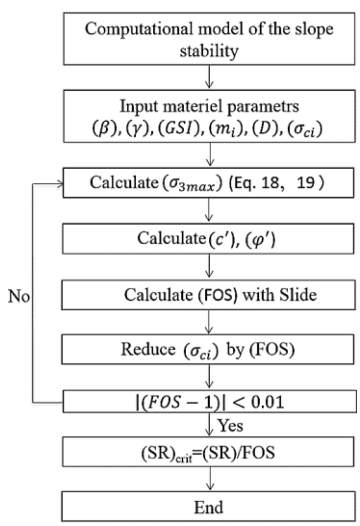

Fig. 5. Iterative procedure to estimate $(SR)_{crit}$

The iterations are established from a first global equivalent evaluation of (MC) parameters, namely (c′) and (φ′). This allows calculating a first (FOS) with the software Slide. The (*SR*) is thus divided by the calculated (FOS), which helps to calculate the new values of (σ_{3max}), (c′), and (φ′), besides a new value of the (FOS). The iterations continue until the condition (|FOS−1| ≤ 0.01) is satisfied. The converged state represents the configuration for which our suggested equation is a valid solution.

For instance, let us consider the following case, where (β = 45°), (γ = 23 kN/m³), (*H = 45m*), (GSI = 50), (m_i = 15), (D = 0) and (σ_{ci} = 10 MPa). The different values calculated at each step of the iteration procedure are presented in Table 2. It was found that the achieved $(SR)_{crit}$ was equal to 0.37, which is the same value calculated by Li et al. (2008) [12] using (LAM) (Fig. 3). It can be concluded that both of the methodologies are equivalent.

4. Case study of an open pit in Sudan

The quarry in the north of Sudan has been considered, as a case study, to assess the rock mass and to provide slope angles for the design of the open pit.

The geotechnical surveys detected a single formation of meta-sediment, the upper part is altered over 45 m. The geotechnical model comprised 45 m of weathered material, underlain by 90 m of fresh rock. No groundwater was encountered during the geotechnical drilling. The pole and contour plot of geological structures recorded in boreholes indicate that the predominant structure feature (foliation) in both footwall and hangingwall dips consistently eastwards at an average angle of 65°. The foliation planes are traversed by moderately dipping (45°) northerly structures. The geotechnical logging was done concurrently with the drilling. The material properties, based on laboratory tests and rock mass classification, are summarized as presented in Table 3.

First of all, the comparison between the classical analyses based on (LEM) and the charts cited above has been carried out. Secondly, the accuracy of our estimation of the global equivalent (MC) parameters, using Eq. 19, has been evaluated.

Table 2. $(SR)_{crit}$ assessment for (β = 45°), (γ=23 kN/m³), (H = 45m), (GSI=50), (m_i = 15), (D = 0) and (σ_{ci} = 10MPa)

Iterations	(σ_{ci}) (MPa)	(SR)	(c') (kPa)	(φ') (°)	(FOS)
#1	10	9.66	58.42	66.84	3.146
#2	3.17	3.07	27.83	58.41	2.209
#3	1.43	1.39	25.17	48.97	1.634
#4	0.88	0.85	26.19	42.14	1.365
#5	0.64	0.62	27.28	37.63	1.218
#6	0.52	0.51	28.06	34.76	1.133
#7	0.46	0.45	28.56	32.95	1.083
#8	0.43	0.41	28.89	31.81	1.058
#9	0.40	0.39	29.11	31.01	1.031
#10	0.39	0.38	29.24	30.57	1.018
#11	0.38	0.38	29.31	30.32	1.012
#12	0.38	0.37	29.36	30.15	1.008

4.1. Limit equilibrium method with local equivalent (MC) parameters (Rocscience software Slide)

Different inter-ramp slope angles (IRA) were applied and analyzed to determine which slope angle provide the desired minimum factor of safety (FOS) of 1.3 against shear failure. The Rocscience software Slide was used for the limit equilibrium analyses with the non-linear (GHB) criterion. Using the material proprieties presented in Table 3 and an inter-ramp angle of 45° in the weathered zone, 55° in the fresh meta-sediments, the (FOS) of the slope against shear failure was 1.433, as presented in Fig. 6. For any subsequent comparison, this $(FOS)_{LEM} = F1 = 1.433$ is taken as the reference value.

4.2. Limit analysis with the charts of Li et al. (2011) [18]

The charts of Li et al. (2011) [18] were used to determine the $(FOS)_{LAM}$ in the same configuration and with the same parameters that were used in the previous models (Fig. 7). These parameters provide:

$$(SR) = \left(\frac{\sigma_{ci}}{\gamma H}\right) = \frac{17.5}{0.023\times45} = 16.9 \qquad (20)$$

The values (m_i = 10), (GSI = 42) and (D = 1) are used to determine the value of $(FOS)_{LAM}$ (Fig.7). Based on the linear interpolation between the two lines of (GSI = 40) and (GSI = 50), the value of $N_{crit} = \left(\frac{\sigma_{ci}}{\gamma H}\right)_{crit} = 6.32$ is obtained. Therefore, the value of $(FOS)_{LAM} = F2 = \frac{(SR)}{(SR)_{crit}}$

for D = 1 is 2.67 and $(SR)_{crit} = 6.32$ for $(GSI) = 42$.

The safety factor F_2=$(FOS)_{LAM}$ is defined by Li et al. (2008, 2011) [12, 18] as the ratio of *(SR)* to $(SR)_{crit}$ $(F_2 = (FOS)_{LAM} = (SR)/(SR)_{crit})$. While $F1 = (FOS)_{LEM}$ is obtained from the (LEM), which is a result of Slide software and defined as a function of the ratio of the resisting force (f_R) to the driving force (f_D) ($F1 = (FOS)_{LEM} = f_R/f_D$). Therefore, as mentioned by Shen et al. (2013) [13], these factors cannot be directly compared. Then, we performed a calculation with Slide by using the mechanical parameters corresponding to $(SR)_{crit}$ ((γ) and (H) were kept constant, and (σ_{ci}) was divided by F2).

If the two methods are equivalent, the calculated safety factor using Slide software with $(SR)_{crit}$ should be equal to 1. The achieved final value was 0.998; hence, the assessed error margin between the two methods is 0.2%. It can be concluded that the (LAM) is in good agreement with the (LEM).

Table 3. Meta-Sediments GSI Rating [20]

	Weathered Meta-Sediments		Fresh Meta-Sediments	
	value	rating	value	rating
σ_{ci} (MPa)	10-25	2	50-100	7
RQD (%)	24	3	45	8
$Jcond_{89}$	Slightly Rough – Moderately to highly weathering	20	Slightly Rough – Highly weathering	25
GSI(1.5$Jcond_{89}$+RQD/2) [21]	-	42	-	60
m_i	10		10	
Disturbance Factor D	1		1	
Unit Weight (kN/m³)	23		26	

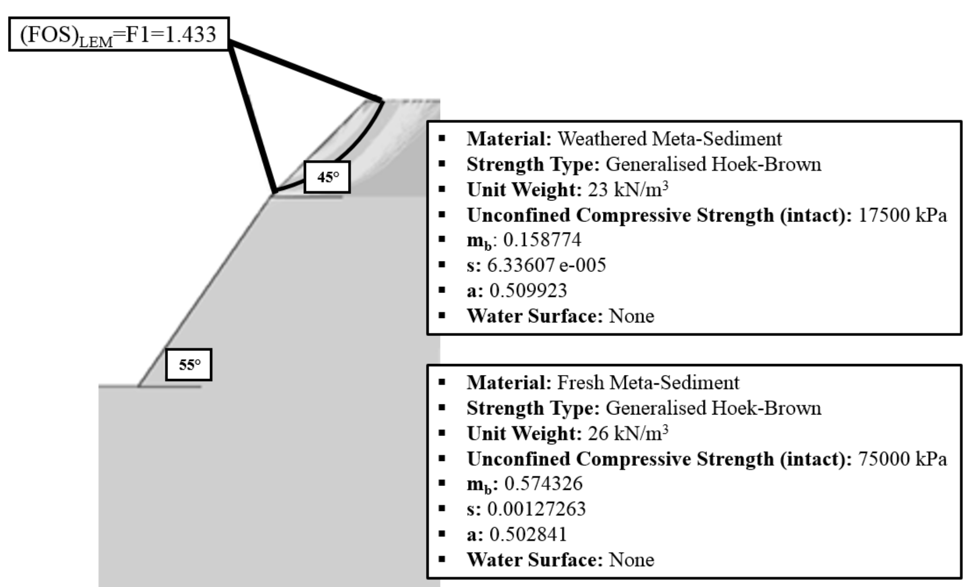

Fig. 6. Meta-Sediments - Weathered Rock at 45° IRA, Fresh Rock at 55° (Bishop Simplified Method with Non-linear (GHB) and local equivalent (MC) parameters)

4.3. Limit equilibrium method using global equivalent (MC) parameters with the current method

We proceed in the same way as the previous example presented in section 3. The different values calculated at each step of the iteration procedure are presented in Table 4. It was found that the achieved $(SR)_{crit}$ was equal to 6.29 and $F3 = \frac{(SR)}{(SR)_{crit}}$ was equal to 2.69, which is almost the same value calculated by Li et al. (2011) [18] using (LAM) (Fig. 7). Hence, it can be concluded that our suggested method is in good agreement with (LAM).

It seems that, in this case, our improvement is not noticeable, since Eq. 16 also provides satisfactory results for (β = 45°). It is important to know that the study developed above shows that for steep slopes greater than 45°, our proposed method is in good agreement with the non-linear (GHB) criterion. For instance, it was found that for (β = 75°), (GSI = 10), (m_i = 35), both Eq. 14 and Eq. 16 lead to a difference of 64% and 21% respectively, while the difference with our current method (Eq. 18) is -0.1% as shown in Table. 1.

Table 4. $(SR)_{crit}$ assessment of the case study of an open pit in Sudan

Iterations	(σ_{ci}) (MPa)	(SR)	(c') (kPa)	(φ') (°)	(FOS)
#1	17.5	16.908	35.96	41.13	1.452
#2	12.05	11.64	35.14	35.93	1.259
#3	9.57	9.249	35.29	32.67	1.156
#4	8.28	8.00	35.56	30.64	1.097
#5	7.54	7.29	35.77	29.36	1.061
#6	7.11	6.87	35.92	28.54	1.039
#7	6.84	6.61	36.03	28.02	1.026
#8	6.67	6.44	36.10	27.67	1.015
#9	6.57	6.35	36.14	27.47	1.01
#10	6.51	6.29	36.17	27.34	1.007

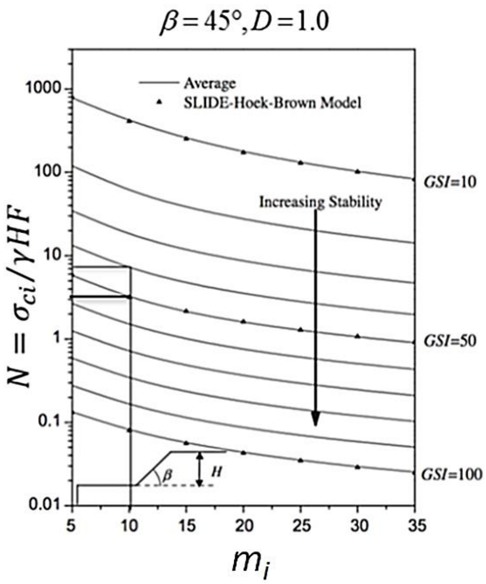

Fig. 7. (FOS) curves for stability analysis based on charts of Li et al. (2011) [18]: Limit analysis method with (D = 1)

5. Conclusion

In this paper, two equations have been established to estimate the upper limit of confining stress (σ_{3max}), from the (GHB) parameter (m_b), the height of the slope (H), and the intact rock mass properties (σ_{ci}) and (γ), for (D = 0) and (D = 1). These equations create a nexus between (σ_{3max}/(σ_{ci})) and $(SR)_{crit}$ and remain useful only in the case where (FOS) = 1. For the given materials and geometrical properties of the slope, the (LAM) provides the critical non-dimensional parameter $(SR)_{crit}$ such that the collapse has just occurred (FOS = 1). Therefore, for different values of (GSI), (D), and (m_i), and by fixing (γ) and (H), (σ_{ci}) can be defined to estimate (σ_{3max}) using our suggested new equations. Based on the equations previously presented in subsection 2.2 and the calculated (σ_{3max}), the global (MC) shear strength parameters can be calculated. Subsequently, the $(FOS)_{LEM}$ was calculated using Rocscience software Slide for different theoretical cases. It was found that, compared to the calculation with the (LAM), the discrepancies do not exceed 5% and remain, in most cases, less than 1%. Hence, the estimation of the (FOS) is much more improved, because the comparison of the achieved results of [6] and [12] led to a difference up to 64% and 21% respectively.

For the general case (FOS ≠ 1), an iterative method has been proposed for each slope characterized by (β) and (H) and for each rock mass characterized by (GSI), (m_i) and (D), the $(SR)_{crit}$ was determined. Subsequently, it led to an identical value of (LAM)'s result.

A case study has been conducted in an open pit located in Sudan to evaluate the discrepancy of (FOS) provided by different methods. The results showed that the (LAM) is in good agreement with the (LEM), using non-linear (GHB) with local equivalent (MC) parameters. The difference, in terms of (FOS), between (LAM) and (LEM) with non-linear (GHB) is 0.2%. In addition, our method provides (FOS)'s nearly equal to the (LAM)'s safety factor, which proves its accuracy.

The charts can be adopted as useful tools for the preliminary rock slope stability analysis. For real topographies, rather than flat upper surfaces and underground water flows, the use of software such as Slide is compulsory. Therefore, the calculation can be used to perform (LEM) analysis either with non-linear (GHB) criterion or linear global equivalent (MC) criterion that is based on our developed method.

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