

Fastest Modified Model of Hardy Cross Method for Ventilation Network Analysis of Mines (Second Conflation Model)

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ABSTRACT

Ventilation network design is done in manual and computerized methods. A computerized method is based on mathematical approximate methods. Several algorithms were presented in mathematical approximate methods for analyzing water distribution and ventilation networks. Hardy Cross method is the most common model of the mathematical approximate method for analyzing ventilation networks in mine. For faster convergence to at the final result of Hardy Cross method were presented other models such as Wang model, conflation model, and Newtonian models (First, third and sixteenth). An initial review of the Hardy Cross method and its modified models is performed in this paper. Then first, third, and sixteenth modified models of Newtonian are presented for more accurate analysis of ventilation networks in mines. Finally, the second conflation model will be presented as the fastest modified model of the Hardy Cross method to achieve the final result.

Keywords: Ventilation, Hardy Cross, Conflation model, Newtonian models

1. Introduction

Ventilation network design of mines is done in manual and computerized methods. The computerized method is based on mathematical approximate methods. If the purpose of ventilation network analysis is the effective investigation of one or more fans in an underground space network, then it is better used mathematical approximate methods. Several algorithms were presented in mathematical approximate methods for analyzing water network distribution [1-22] and ventilation networks [23-42]. The first mathematical equation was proposed by Hardy Cross for calculating of flow intensity error of each ring [1]. Although this method was proposed for analyzing water distribution networks, this equation was later used by Wang for analyzing ventilation networks and was corrected by him [23, 38]. Also, for faster convergence to the final result of the Hardy Cross method were presented other models such as the spatial ring method [33], conflation method [39], and Newtonian methods [18]. In mathematical approximate methods should be distributed hypothetical flow intensity with hypothetical directions in each branch of ventilation network according to node law. Then, using ventilation network fans and mathematical approximate methods are calculated the error quantity of each ring in the ventilation network and then airflow intensity is corrected for each branch. Correction operations of airflow intensity based on mathematical approximate methods must be repeated several times until of calculation error of each ring is become less or equal to calculation accuracy. Using mathematical approximate methods for solving large-scale networks by humans is almost impossible, so using the computer for solving them is essential. Numerous computerized software is provided for analyzing ventilation networks, one of the most famous them is the Ventsim software. This software was designed based on the Hardy Cross method [34-39].

2. Review of the Hardy Cross Method and its corrected models

2.1. Hardy Cross Method (HCM)

One of the most common mathematical approximate methods for analyzing ventilation networks in mines is the Hardy Cross method. The famous Hardy Cross method was presented in 1936 and it is calculated according to Equation (1). In this method, the error quantity of flow intensity is calculated in each ring [1].

$$\Delta Q = -\frac{\sum \pm \Delta P_i}{2 \sum R_i Q_i} = -\frac{\sum \pm R_i Q_i^2}{2 \sum R_i Q_i} \quad (1)$$

ΔQ : Ring flow intensity error (m³/s)

Q_i : Flow intensity of each branch (m³/s)

R_i : Resistance of each branch (KMurgue)

ΔP_i : Pressure loss of each branch (mmH₂O)

The problem-solving stages of the Hardy Cross method are as follows:
Stage 1:

According to the node law, for each branch of the ventilation network is assumed hypothetical flow intensity.

Stage 2:

Identification of useful rings in the ventilation network by equation 2 and selection hypothetical directions for them.

$$N_R = N_B - N_U \quad (2)$$

N_R : Number of rings

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N_B : Number of branches

N_U : Number of underground nodes

Stage 3:

Calculation of Pressure loos in each branch was calculated according to equation 3.

$$\Delta P = R Q^2 \tag{3}$$

Stage 4:

Calculation of error quantity of flow intensity in each ring was calculated according to equation (1). If the airflow direction in a branch with airflow direction in its ring was the same, therefore the pressure loos of its branch has appeared with a positive sign in the Hardy Cross equation. In otherwise it has appeared with a negative sign in this equation. Also, the denominator sign in this equation will always appear with a positive sign.

Stage 5:

Calculation of new flow intensity of ventilation network branches, at this stage has investigated the effect of each ring in each branch and then the positive or negative sign of each ring is determined and finally, the flow intensity of each branch with the error quantity of its ring are summed with together. If the new flow intensity of a branch was negative therefore the flow direction of a branch for the next iteration must be reversed.

Stage 6:

The above operations from the third stage to the fifth stage must be repeated several times until the calculation error of each ring becomes less or equal to the calculation accuracy.

For a better understanding of the Hardy Cross method pay attention to figure 1. Two fans with productive pressure 80 (kg/m²) and 100 (kg/m²) exist in this model. This hypothetical model consists of three surface nodes (a, c, d) and an underground node (b). At first, this network is simulated with Ventsim software and then its results are compared with the manual Hardy Cross method. The final results of the Ventsim software simulation for flow intensity distribution of each branch in the ventilation network are presented in figure 2.

Flow intensity analysis of each branch in mathematical approximate methods needs intensity of hypothetical flows in ventilation network. This intensity of hypothetical flows along with useful rings of the network is presented in figure 3. According to figure 3, mathematical calculations of the first iteration in the Hardy Cross method are as follows and its results are presented in figure 4. Also, the results of other stages are reported in table 1. If the accuracy of calculations is assumed 10⁻⁵, therefore iteration number of calculations in this method has become 24. The final results of this method are completely the same as the results of Ventsim software. According to stage 5, if airflow intensity in a branch became negative, therefore airflow direction in that branch before the next stage must be inversed and its quantity has become positive. In other words, in this method, the airflow direction in each branch of the ventilation network is variable for all iterations.

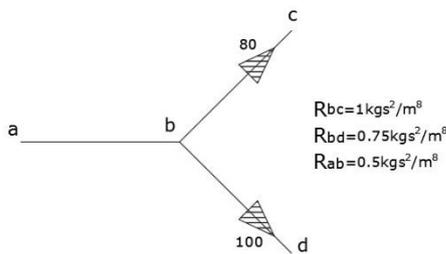


Figure 1. Hypothetical ventilation network [38]

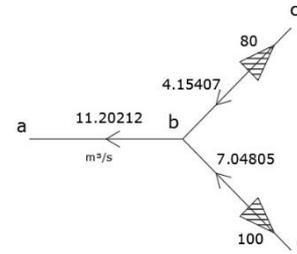


Figure 2. Airflow intensity distribution with using of Ventsim software

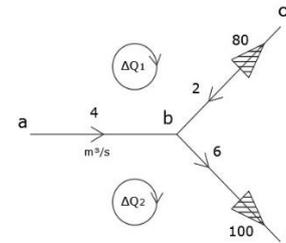


Figure 3. The intensity of hypothetical flows

$$\delta Q_1 = -\frac{1 \times 2^2 - 0.5 \times 4^2 - 80}{2 \times (1 \times 2 + 0.5 \times 4)} = 10.5$$

$$\delta Q_2 = -\frac{0.5 \times 4^2 + 0.75 \times 6^2 + 100}{2 \times (0.5 \times 4 + 0.75 \times 6)} = -10.38462$$

$$Q'_{bc} = Q_{bc} + \delta Q_1 = 2 + 10.5 \rightarrow Q'_{bc} = 12.5 \text{ m}^3 / \text{s}$$

$$Q'_{ab} = Q_{ab} - \delta Q_1 + \delta Q_2 = 4 - 10.5 - 10.38462 \rightarrow Q'_{ab} = -16.88462 \text{ m}^3 / \text{s}$$

$$Q'_{bd} = Q_{bd} + \delta Q_2 = 6 - 10.38462 \rightarrow Q'_{bd} = -4.38462 \text{ m}^3 / \text{s}$$

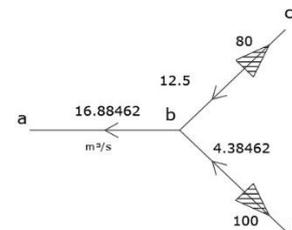


Figure 4. The calculation results of the first iteration in the Hardy Cross method

Table 1. The results of the Hardy Cross method

Description	Q_{bc}	Q_{ab}	Q_{bd}	δQ_1	δQ_2
Iteration 1	12.5	-16.884615	-4.384615	10.5	-10.38462
Iteration 2	7.276241	9.232893	1.956652	-5.223759	2.427963
Iteration 3	6.621770	13.057880	6.436110	-0.654471	-4.479459
Iteration 4	4.754876	10.472348	5.717472	-1.866894	0.718639
Iteration 5	4.882797	11.684227	6.801431	0.127920	-1.083959
Iteration 10	4.168146	11.183225	7.015080	-0.049903	0.008763
Iteration 15	4.155717	11.203138	7.047421	0.000424	-0.002244
Iteration 20	4.154099	11.202076	7.047977	-0.000112	1.9E-05
Iteration 24	4.154070	11.202115	7.048045	-0.00001	0.000002

2.2. Wang model of Hardy Cross Method (WM)

The correction of Wang is related to the fourth stage of the Hardy Cross method. In this method, the error quantity of the flow intensity in each ring is calculated according to equation (4). This method was presented by Wang in 1982 [23].

$$\Delta Q = -\frac{\sum b_{ki}(R_i|Q_i|Q_i - P_{ni} - P_{Fi})}{2\sum b_{ki}^2 R_i |Q_i|} \quad (4)$$

b_{ki} : Fundamental matrix element of ring (-1, 0, 1)

P_{ni} : Natural ventilation pressure

P_{Fi} : Productive pressure of the fan

For a better understanding of Wang's model of the Hardy Cross method pay attention to figure 3. According to figure 3, mathematical calculations of the first iteration in this method are as follows and its results are presented in figure 5. Also, the results of other stages are reported in table 2. If the accuracy of calculations is assumed 10^{-5} , therefore the iteration number of calculations in this method is 24. The final results of this method are completely the same as the results of Ventsim software. In this method, the airflow direction in each branch of the ventilation network is fixed for all iterations.

$$\delta Q_1 = -\frac{1 \times |2| \times 2 - 0.5 \times |4| \times 4 - 80}{2 \times (1 \times |2| + 0.5 \times |4|)} = 10.5$$

$$\delta Q_2 = -\frac{0.5 \times |4| \times 4 + 0.75 \times |6| \times 6 + 100}{2 \times (0.5 \times |4| + 0.75 \times |6|)} = -10.38462$$

$$Q'_{bc} = Q_{bc} + \delta Q_1 = 2 + 10.5 \rightarrow Q'_{bc} = 12.5 \text{ m}^3/\text{s}$$

$$Q'_{ab} = Q_{ab} - \delta Q_1 + \delta Q_2 = 4 - 10.5 - 10.38462 \rightarrow Q'_{ab} = -16.88462 \text{ m}^3/\text{s}$$

$$Q'_{bd} = Q_{bd} + \delta Q_2 = 6 - 10.38462 \rightarrow Q'_{bd} = -4.38462 \text{ m}^3/\text{s}$$

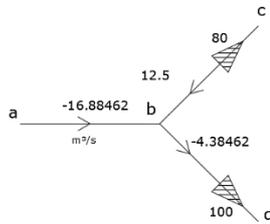


Figure 5. The calculation results of the first iteration of the Wang model

Table 2. The results of the Wang model

Description	Q_{bc}	Q_{ab}	Q_{bd}	δQ_1	δQ_2
Iteration 1	12.5	-16.884615	-4.384615	10.5	-10.38462
Iteration 2	7.276241	-9.232893	-1.956652	-5.223759	2.427963
Iteration 3	6.621770	-13.057880	-6.436110	-0.654471	-4.479459
Iteration 4	4.754876	-10.472348	-5.717472	-1.866894	0.718639
Iteration 5	4.882797	-11.684227	-6.801431	0.127920	-1.083959
Iteration 10	4.168146	-11.183225	-7.015080	-0.049903	0.008763
Iteration 15	4.155717	-11.203138	-7.047421	0.000424	-0.002244
Iteration 20	4.154099	-11.202076	-7.047977	-0.000112	1.9E-05
Iteration 24	4.154070	-11.202115	-7.048045	-0.000001	0.000002

2.3. Conflation model of Hardy Cross Method (CM)

The most modified models of the Hardy Cross method have represented that their corrections were done for the fourth and fifth stages of the Hardy Cross method. Also, these two stages are independent of each other. But in the conflation model that was presented by Elahi in 2015, these two stages are dependent on each other. In other words, after calculating the first ring error quantity should be performed flow intensity correction of each branch of the first ring and then the second ring error quantity is calculated [39].

For a better understanding of the conflation model of the Hardy Cross method pay attention to figure 3. According to figure 3, mathematical calculations of the first iteration in this method are as follows and its results are presented in figure 6. Also, the results of other stages are reported in table 3. If the accuracy of calculations is assumed 10^{-5} , therefore the iteration number of calculations in this method

becomes 14. The final results of this method are completely the same as the results of Ventsim software. According to stage 5, if airflow intensity in a branch became negative, therefore airflow direction in that branch before the next stage must be inverted and its quantity becomes positive. In other words, in this method, the airflow direction in each branch of the ventilation network is variable for all iterations.

$$\delta Q_1 = -\frac{1 \times 2^2 - 0.5 \times 4^2 - 80}{2 \times (1 \times 2 + 0.5 \times 4)} = 10.5$$

$$Q'_{bc} = Q_{bc} + \delta Q_1 = 2 + 10.5 \rightarrow Q'_{bc} = 12.5 \frac{\text{m}^3}{\text{s}}$$

$$Q'_{ab} = Q_{ab} - \delta Q_1 = 4 - 10.5 \rightarrow Q'_{ab} = -6.5 \frac{\text{m}^3}{\text{s}}$$

$$\delta Q_2 = -\frac{-0.5 \times 6.5^2 + 0.75 \times 6^2 + 100}{2 \times (0.5 \times 6.5 + 0.75 \times 6)} = -6.83065$$

$$Q'_{ab} = Q_{ab} - \delta Q_2 = 6.5 + 6.83065 \rightarrow Q'_{ab} = 13.33065 \frac{\text{m}^3}{\text{s}}$$

$$Q'_{bd} = Q_{bd} + \delta Q_2 = 6 - 6.83065 \rightarrow Q'_{bd} = -0.83065 \frac{\text{m}^3}{\text{s}}$$

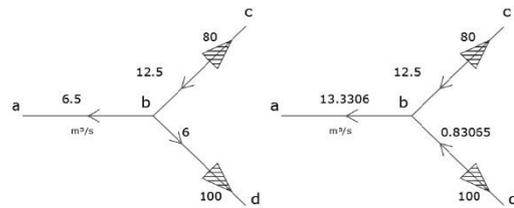


Figure 6. The calculation results of the first iteration of the conflation

Table 3. The results of the conflation model

Description	Q_{bc}	Q_{ab}	Q_{bd}	δQ_1	δQ_2
Iteration 1	12.5	13.330645	-0.83065	10.5	-6.830645
Iteration 2	8.192662	14.746446	6.55378	-4.307338	-5.723139
Iteration 3	5.113860	11.654572	6.54071	-3.078802	0.013072
Iteration 4	4.471053	11.361613	6.89056	-0.642807	-0.349848
Iteration 5	4.247772	11.248181	7.00041	-0.223281	-0.109849
Iteration 10	4.154277	11.202220	7.04794	-0.000501	-0.000258
Iteration 14	4.154069	11.202119	7.04805	-0.000004	-0.000002

2.4. Newtonian models of Hardy Cross Method

These methods were presented by Moosavian and Jaefarzadeh for water distribution networks in 2014 [18, 43-44].

2.4.1. First-Order Newtonian model (FONM)

The first-order Newtonian model of the Hardy Cross method was presented based on various matrixes and is calculated according to equations 5-10 [18].

$$M_{ij}(k;L) = \begin{cases} +1 & \text{if the flow of pipe k in loop L is clockwise} \\ 0 & \text{if pipe k is not in loop L} \\ -1 & \text{if the flow of pipe k in loop L is counterclockwise} \end{cases} \quad (5)$$

$$A_{ij} = \begin{bmatrix} R_1 |Q_1|^{n-1} & 0 & 0 & 0 \\ 0 & R_2 |Q_2|^{n-1} & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & R_{NP} |Q_{NP}|^{n-1} \end{bmatrix} \quad (6)$$

$$D_{11} = \begin{bmatrix} nR_i |Q_i|^{n-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & nR_{NP} |Q_{NP}|^{n-1} \end{bmatrix} \quad (7)$$

$$M_{31} = M_{13}^T \quad (8)$$

$$\Delta Q^{(t)} = -\frac{M_{31} A_{11}^{(t)} \text{diag}(Q^{(t)})}{M_{31} D_{11}^{(t)}} \quad (9)$$

$$Q^{(t+1)} = Q^{(t)} + M_{13} \Delta Q^{(t)} \quad (10)$$

For a better understanding of this Newtonian model of the Hardy Cross method pay attention to figure 3. According to figure 3, the first stage of mathematical calculations of this method is as follows:

$$M_{31} = \begin{bmatrix} K_{bc} & K_{ab} & K_{cd} \\ K_{bc} & K_{ab} & K_{cd} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow M_{13} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4.5 \end{bmatrix}, \text{diag}(Q^{(0)}) = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, D_{11} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$M_{31} D_{11}^{(0)} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 4 & -4 & 0 \\ 0 & 4 & 9 \end{bmatrix} \rightarrow M_{31} D_{11}^{(0)} = \begin{bmatrix} 4+|-4|+0 \\ 0+4+9 \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$$M_{31} A_{11}^{(t)} \text{diag}(Q^{(t)}) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4.5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 35 \end{bmatrix}$$

$$\begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \end{bmatrix} = -\frac{\begin{bmatrix} -4 \\ 35 \end{bmatrix}}{\begin{bmatrix} 8 \\ 13 \end{bmatrix}} = \begin{bmatrix} 0.5 \\ -2.6923 \end{bmatrix}$$

$$\begin{bmatrix} Q_{bc} \\ Q_{ab} \\ Q_{cd} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.5 \\ -2.6923 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0.8077 \\ 3.3077 \end{bmatrix}$$

It is observed that the first stage results of this Newtonian method with the Hardy Cross method are not equal. Therefore, the correction of this Newtonian method is necessary for the analysis of ventilation networks.

2.4.2. Third-Order Newtonian model (TONM)

The third-order Newtonian model of the Hardy Cross method was presented based on various matrixes and is calculated according to equations 5-8 and 11-15 [18].

$$M_1 = M_{31} D_{11}^{(t)} \quad (11)$$

$$M_2 = M_{31} A_{11}^{(t)} \text{diag}(Q^{(t)}) \quad (12)$$

$$y^{(t)} = Q^{(t)} - M_{13} \left(\frac{M_2}{M_1} \right) \quad (13)$$

$$M_3 = M_{31} A_{11}^{(t)} \text{diag}(y^{(t)}) \quad (14)$$

$$Q^{(t+1)} = Q^{(t)} - M_{13} \left(\frac{M_2 + M_3}{M_1} \right) \quad (15)$$

According to the results of section 2.4.1, the correction of this Newtonian method is necessary for the analysis of ventilation networks.

2.4.3. Sixteenth-Order Newtonian model (SONM)

The Sixteenth-order Newtonian model of the Hardy Cross method was presented based on various matrixes and is calculated according to equations 5-8 and 16-25 [18].

$$M_1 = M_{31} D_{11}^{(t)} \quad (16)$$

$$M_2 = M_{31} A_{11}^{(t)} \text{diag}(Q^{(t)}) \quad (17)$$

$$y^{(t)} = Q^{(t)} - M_{13} \left(\frac{M_2}{M_1} \right) \quad (18)$$

$$M_3 = M_{31} A_{11}^{(t)} \text{diag}(y^{(t)}) \quad (19)$$

$$z^{(t)} = y^{(t)} - M_{13} \left(\frac{M_3 (2M_2 - M_3) / (2M_2 - 5M_3)}{M_1} \right) \quad (20)$$

$$M_4 = M_{31} D_{11}^{n(t)} \quad (21)$$

$$M_5 = M_{31} A_{11}^{n(t)} \text{diag}(z^{(t)}) \quad (22)$$

$$x^{(t)} = z^{(t)} - M_{13} \left(\frac{M_5}{M_4} \right) \quad (23)$$

$$M_6 = M_{31} A_{11}^{m(t)} \text{diag}(x^{(t)}) \quad (24)$$

$$Q^{(t+1)} = x^{(t)} - M_{13} \left(\frac{M_6 (2M_5 - M_6) / (2M_5 - 5M_6)}{M_4} \right) \quad (25)$$

According to the results of section 2.4.1, the correction of this Newtonian method is necessary for the analysis of ventilation networks.

3. Newtonian corrected models of Hardy Cross Method

3.1. First-Order Newtonian corrected model (FONCM)

This new method is calculated based on equations (26 and 27). Therefore, for a better understanding of the first-order Newtonian corrected model of the Hardy Cross method pay attention to figure 3. According to figure 3, mathematical calculations of the first iteration in this method are as follows and its results are completely the same as in figure 5. Also, the results of other stages are completely the same as in table 2. If the accuracy of calculations is assumed 10^{-5} , therefore the iteration number of calculations in this method becomes 24. The final results of this method are completely the same as the results of Ventsim software. In this method, the airflow direction in each branch of the ventilation network is fixed for all iterations.

$$\Delta Q^{(t)} = -\frac{M_{31} A_{11}^{(t)} Q^{(t)} - M_{31} (P_{Fan} + P_{Natural})}{|M_{31}| D_{11}^{(t)} \times M} \quad (26)$$

$$Q^{(t+1)} = Q^{(t)} + M_{13} \Delta Q^{(t)} \quad (27)$$

$$\begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \end{bmatrix} = \frac{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4.5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 80 \\ 0 \\ -100 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} = \begin{bmatrix} 10.5 \\ -10.38462 \end{bmatrix}$$

$$\begin{bmatrix} Q_{bc} \\ Q_{ab} \\ Q_{cd} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 10.5 \\ -10.38462 \end{bmatrix} = \begin{bmatrix} 12.5 \\ -16.88462 \\ -4.38462 \end{bmatrix}$$

3.2. Third-Order Newtonian corrected model (TONCM)

This new method is calculated based on equations (28-32). Therefore, for a better understanding of the third-order Newtonian corrected model of the Hardy Cross method pay attention to figure 3. According to figure 3, mathematical calculations of the first iteration in this method are as follows and its results are presented in figure 7. Also, the results of other stages are reported in table 4. If the accuracy of calculations is assumed 10^{-5} , therefore the iteration number of calculations in this method becomes 19. The final results of this method are completely the same as the results of Ventsim software. In this method, the airflow direction in each branch of the ventilation network is fixed for all iterations.

$$M_1 = |M_{31}| D_{11}^{(t)} M \quad (28)$$

$$M_2 = M_{31} A_{11}^{(t)} Q^{(t)} - M_{31} (P_{Fan} + P_{Natural}) \quad (29)$$

$$y^{(t)} = Q^{(t)} - M_{13} \frac{M_2}{M_1} \quad (30)$$

$$M_3 = M_{31} A_{11}^{n(t)} y^{(t)} - M_{31} (P_{Fan} + P_{Natural}) \quad (31)$$

$$Q^{(t+1)} = Q^{(t)} - M_{13} \frac{M_2 + M_3}{M_1} \quad (32)$$

$$M_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4.5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 80 \\ 0 \\ -100 \end{bmatrix} = \begin{bmatrix} -84 \\ 135 \end{bmatrix}$$

$$\begin{bmatrix} y_{bc} \\ y_{ab} \\ y_{cd} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -10.5 \\ 10.38462 \end{bmatrix} = \begin{bmatrix} 12.5 \\ -16.88462 \\ -4.38462 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 12.5 & 0 & 0 \\ 0 & 8.442308 & 0 \\ 0 & 3.288462 & 0 \end{bmatrix} \times \begin{bmatrix} 12.5 \\ -16.88462 \\ -4.38462 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 80 \\ 0 \\ -100 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 218.795118 \\ -56.963757 \end{bmatrix} \rightarrow \begin{bmatrix} Q_{bc} \\ Q_{ab} \\ Q_{cd} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 16.84939 \\ 6.002788 \end{bmatrix} = \begin{bmatrix} -14.84939 \\ 14.846602 \\ -0.002788 \end{bmatrix}$$

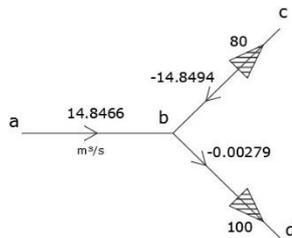


Figure 7. The calculation results for the first iteration of the corrected model of the third-order Newtonian model

Table 4. The results for the corrected model of the third-order Newtonian model

Description	Q_{bc}	Q_{ab}	Q_{bd}	δQ_1	δQ_2
Iteration 1	-14.849390	14.846602	-0.002788	16.849390	6.002788
Iteration 2	-3.938346	-4.381485	-8.319831	-10.911043	8.317043
Iteration 3	1.202404	-7.191987	-5.989583	-5.140750	-2.330248
Iteration 4	-2.321735	-1.56493	-3.886665	3.524139	-2.102918
Iteration 5	-54.690493	99.589293	44.898801	52.368757	-48.785466
Iteration 6	-13.477896	16.854734	3.376838	41.212597	41.521963
Iteration 7	-2.186521	-7.882689	-10.069209	-11.291375	13.446047
Iteration 8	2.498341	-10.391278	-7.892938	-4.684861	-2.176272
Iteration 10	4.000436	-11.090542	-7.090106	-0.353968	-0.134917
Iteration 15	4.153716	-11.20185	-7.048134	-0.000836	-0.000197
Iteration 19	4.154064	-11.202116	-7.048052	-0.000006	-0.000001

3.3. Sixteenth-Order Newtonian corrected model (SONCM)

This new method is calculated based on equations (33-42). Therefore, for a better understanding of the sixteenth-order Newtonian corrected model of the Hardy Cross method pay attention to figure 3. According to figure 3, mathematical calculations of the first iteration in this method are as follows and its results are presented in figure 8. Also, the results of other stages are reported in table 5. If the accuracy of calculations is assumed 10^{-5} , therefore the iteration number of calculations in this method becomes 8. The final results of this method are completely the same as the results of Ventsim software. In this method, the airflow direction in each branch of the ventilation network is fixed for all iterations.

$$M_1 = |M_{31} D_{11}^{(t)} M \quad (33)$$

$$M_2 = M_{31} A_{11}^{(t)} Q^{(t)} - M_{31} (P_{Fan} + P_{Natural}) \quad (34)$$

$$y^{(t)} = Q^{(t)} - M_{13} \frac{M_2}{M_1} \quad (35)$$

$$M_3 = M_{31} A_{11}^{n(t)} y^{(t)} - M_{31} (P_{Fan} + P_{Natural}) \quad (36)$$

$$z^{(t)} = y^{(t)} - M_{13} \left(\frac{M_3 (2M_2 - M_3) / (2M_2 - 5M_3)}{M_1} \right) \quad (37)$$

$$M_4 = |M_{31} D_{11}^{n(t)} M \quad (38)$$

$$M_5 = M_{31} A_{11}^{n(t)} z^{(t)} - M_{31} (P_{Fan} + P_{Natural}) \quad (39)$$

$$x^{(t)} = z^{(t)} - M_{13} \frac{M_5}{M_4} \quad (40)$$

$$M_6 = M_{31} A_{11}^{n(t)} x^{(t)} - M_{31} (P_{Fan} + P_{Natural}) \quad (41)$$

$$Q^{(t+1)} = x^{(t)} - M_{13} \left(\frac{M_6 (2M_5 - M_6) / (2M_5 - 5M_6)}{M_4} \right) \quad (42)$$

$$M_1 = \begin{bmatrix} 8 \\ 13 \end{bmatrix}, M_2 = \begin{bmatrix} -84 \\ 135 \end{bmatrix}, \begin{bmatrix} y_{bc} \\ y_{ab} \\ y_{cd} \end{bmatrix} = \begin{bmatrix} 12.5 \\ -16.88462 \\ -4.38462 \end{bmatrix}, M_3 = \begin{bmatrix} 218.795118 \\ -56.963757 \end{bmatrix}$$

$$\begin{bmatrix} z_{bc} \\ z_{ab} \\ z_{cd} \end{bmatrix} = \begin{bmatrix} 12.5 \\ -16.88462 \\ -4.38462 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 67.06063 \\ -33.56964 \\ 8 \\ 13 \end{bmatrix} = \begin{bmatrix} 4.117421 \\ -5.91975 \\ -1.80233 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 8.23484 & 0 & 0 \\ 0 & 5.91975 & 0 \\ 0 & 0 & 2.70350 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 14.154595 \\ 8.623253 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4.117421 & 0 & 0 \\ 0 & 2.959877 & 0 \\ 0 & 0 & 1.35175 \end{bmatrix} \times \begin{bmatrix} 4.117421 \\ -5.91975 \\ -1.80233 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 80 \\ 0 \\ -100 \end{bmatrix} \rightarrow M_5 = \begin{bmatrix} -45.525105 \\ 80.041955 \end{bmatrix}$$

$$\begin{bmatrix} x_{bc} \\ x_{ab} \\ x_{cd} \end{bmatrix} = \begin{bmatrix} 4.117421 \\ -5.91975 \\ -1.80233 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -45.525105 \\ 80.041955 \\ 14.154595 \\ 8.623253 \end{bmatrix} = \begin{bmatrix} 7.333698 \\ -18.418138 \\ -11.08444 \end{bmatrix}$$

$$M_6 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 7.333698 & 0 & 0 \\ 0 & 9.209069 & 0 \\ 0 & 0 & 8.31333 \end{bmatrix} \times \begin{bmatrix} 7.333698 \\ -18.418138 \\ -18.418138 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 80 \\ 0 \\ -100 \end{bmatrix} \rightarrow M_6 = \begin{bmatrix} 143.397026 \\ -161.7625 \end{bmatrix}$$

$$\begin{bmatrix} Q_{bc} \\ Q_{ab} \\ Q_{cd} \end{bmatrix} = \begin{bmatrix} 7.333698 \\ -18.418138 \\ -11.08444 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 41.605903 \\ -53.73399 \\ 14.154595 \\ 8.623253 \end{bmatrix} = \begin{bmatrix} 4.394307 \\ -9.247455 \\ -4.853149 \end{bmatrix}$$

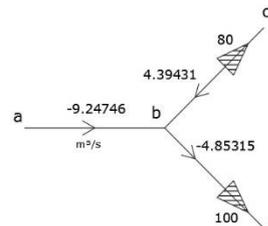


Figure 8. The calculation results for the first iteration of the corrected model of the sixteenth-order Newtonian model

Table 5. The results for the corrected model of the sixteenth-order Newtonian model

4. Second conflation model of Hardy Cross Method (SCM)

According to the presented results, the iteration number of mathematical calculations for the sixteenth-order Newtonian modified model and conflation model is 8 and 14, respectively. But sixteenth-order Newtonian modified model of the Hardy Cross method has

reached the final result by solving 10 mathematical equations. Increasing of equation number is caused to add volume and time of calculations in each stage. From the point of view of users, this function can cause public acceptance lack. But in conflation model is solved with a simple mathematical equation. Thus, this method from the point of view users will have more acceptance.

Description	Q_{bc}	Q_{ab}	Q_{bd}	δQ_1	δQ_2
Iteration 1	4.394307	-9.247455	-4.853149	2.939392	-6.231291
Iteration 2	4.311913	-11.012665	-6.700753	0.109723	-0.002345
Iteration 3	4.196317	-11.167392	-6.971075	0.015022	0.003001
Iteration 4	4.164607	-11.194732	-7.030126	0.002507	0.001211
Iteration 5	4.15664	-11.200457	-7.043817	0.000466	0.000362
Iteration 6	4.154689	-11.201734	-7.047045	0.000094	0.000097
Iteration 7	4.154217	-11.202028	-7.047812	0.000020	0.000025
Iteration 8	4.154103	-11.202097	-7.047994	0.000004	0.000006

Accordingly, a new method is presented based on the conflation model. The name of this new method is the second conflation model of the Hardy Cross method. The flowchart of this method is presented in figure 9. The problem-solving stages of this method are as follows:

Stage 1:

According to node law, for each branch of the ventilation network is assumed hypothetical flow intensity.

Stage 2:

Identify useful rings in the ventilation network under equation 2 and select hypothetical directions for them.

Stage 3:

Identify common and uncommon branches in between useful rings

Stage 4:

Production of an additional independent ring for uncommon branches in ventilation network, error quantity calculation of flow intensity of independent ring based on equations (43 or 44 or 45), and new flow intensity calculation of uncommon branches based on equations (46 or 47).

$$\Delta Q = -\frac{\sum \pm R_i Q_i^2 - (P_{i-Fan} + P_{i-Natural})}{2 \sum R_i Q_i} \quad (\text{for variable directions})$$

$$\Delta Q = -\frac{\sum \pm R_i |Q_i| |Q_i| - (P_{i-Fan} + P_{i-Natural})}{2 \sum R_i |Q_i|} \quad (\text{for fixed directions})$$

$$\Delta Q' = -\frac{M_{31} A_{11}' Q^{(i)} - M_{31} (P_{Fan} + P_{Natural})}{|M_{31}| D_{11}' \times M} \quad (\text{for fixed directions})$$

$$Q_i' = Q_i + \sum \pm \Delta Q_j$$

$$Q^{(i+1)} = Q^{(i)} + M_{13} \Delta Q^{(i)}$$

Stage 5:

Error quantity calculation of flow intensity of first useful ring based on equation (43 or 44 or 45), Investigation of this ring influence in ventilation network and new flow intensity calculation of ventilation network branches based on equations (46 or 47). Also, error quantity calculation of flow intensity of other useful rings and new flow intensity calculation of ventilation network branches.

Stage 6:

The above operations from the fourth stage to the fifth stage must be repeated several times until of calculation error of each ring is become less or equal to the calculation accuracy.

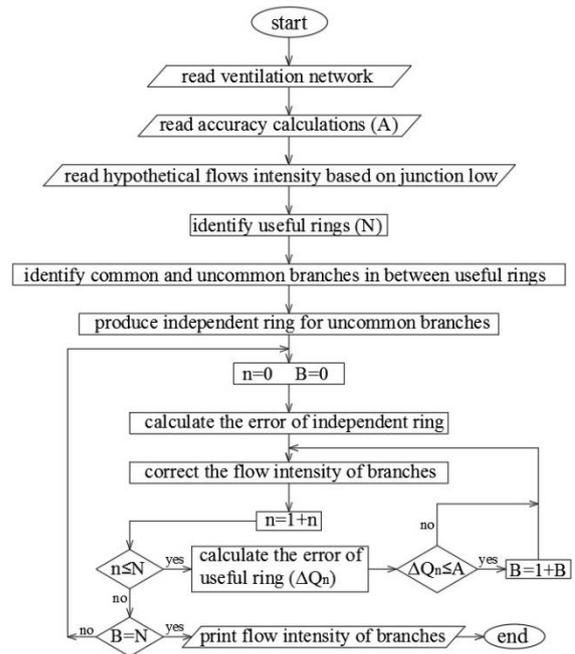


Figure 9. The flowchart of the second conflation model of the Hardy Cross method

For a better understanding of the second conflation model of the Hardy Cross method pay attention to figure 3. In figure 3 exist useful two rings. Accordingly, Branch (ab) is a common branch, and branches (bc, bd) are uncommon. According to figure 3, mathematical calculations of the first iteration in this model based on equation (43) are as follows and its results are presented in figure 10. Also, results of other stages are reported in table 6. If the accuracy of calculations is assumed 10^{-5} , therefore the iteration number of calculations in this method becomes 8. The final results of this method are completely the same as the results of Ventsim software. In this method, the airflow direction in each branch of the ventilation network is variable or fixed for all iterations.

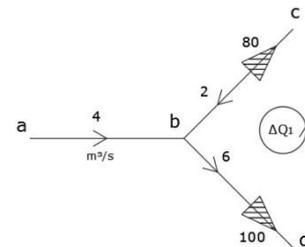


Figure 10.a. independent ring location in ventilation network

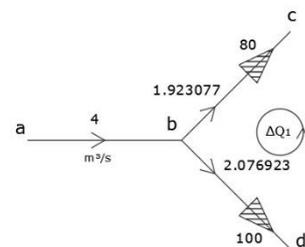


Figure 10.b. The calculation results for the first iteration of the independent ring in the second conflation model

$$\delta Q_1 = -\frac{1 \times 2^2 + 0.75 \times 6^2 + 100 - 80}{2(1 \times 2 + 0.75 \times 6)} = -3.923077$$

$$Q'_{bc} = Q_{bc} + \delta Q_1 = 2-3.923077 \rightarrow Q'_{bc} = -1.923077 \frac{m^3}{s}$$

$$Q'_{bd} = Q_{bd} + \delta Q_1 = 6-3.923077 \rightarrow Q'_{bd} = 2.076923 \frac{m^3}{s}$$

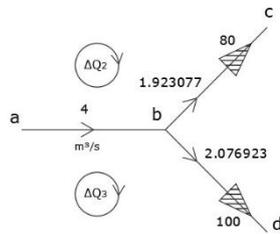


Figure 10.c. location of useful rings in ventilation network

$$\delta Q_2 = -\frac{-1 \times 1.923077^2 - 0.5 \times 4^2 - 80}{2 \times (1 \times 1.923077 + 0.5 \times 4)} = 11.687029$$

$$Q'_{bc} = Q_{bc} - \delta Q_2 = 1.923077 - 11.687029 \rightarrow Q'_{bc} = -9.763952 \frac{m^3}{s}$$

$$Q'_{ab} = Q_{ab} - \delta Q_2 = 4 - 11.687029 \rightarrow Q'_{ab} = -7.687029 \frac{m^3}{s}$$

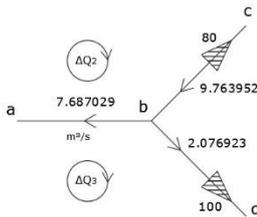


Figure 10.d. The calculation results for the first iteration of the first useful ring in the second conflation model

$$\delta Q_3 = -\frac{-0.5 \times 7.687029^2 + 0.75 \times 2.076923^2 + 100}{2 \times (0.5 \times 7.687029 + 0.75 \times 2.076923)} = -6.821624$$

$$Q'_{ab} = Q_{ab} - \delta Q_3 = 7.687029 + 6.821624 \rightarrow Q'_{ab} = 14.508653 \frac{m^3}{s}$$

$$Q'_{bd} = Q_{bd} + \delta Q_3 = 2.076923 - 6.821624 \rightarrow Q'_{bd} = -4.744701 \frac{m^3}{s}$$

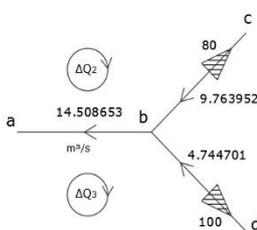


Figure 10.e. The calculation results for the first iteration of the second useful ring in the second conflation model

Table 6. The results of the second conflation model

Description	Q_{bc}	Q_{ab}	Q_{bd}	δQ_1	δQ_2	δQ_3
Iteration 1	9.763952	14.508653	-4.744701	-3.923077	11.687029	-6.821624
Iteration 2	3.739162	11.068324	7.329162	-3.694906	-2.329884	1.110445
Iteration 3	4.189508	11.219410	7.029902	0.341388	0.108959	-0.042127
Iteration 4	4.149715	11.200007	7.050292	-0.025753	-0.014040	0.005363
Iteration 5	4.154601	11.202378	7.047777	0.003169	0.001716	-0.000655
Iteration 6	4.154001	11.202087	7.048085	-0.000389	-0.000211	0.000080
Iteration 7	4.154075	11.202122	7.048047	0.000048	0.000026	-0.000010
Iteration 8	4.154066	11.202118	7.048052	-0.000006	-0.000003	0.000001

5. Other examples

5.1. Example 2

This example is based on figure 3. The accuracy of calculations in this example is assumed 10^{-15} . The final results of the Ventsim software simulation are the same as in figure 2. Accordingly, the results of Hardy Cross methods are reported in table 7. According to table 7, the second conflation model of the Hardy Cross method is considered the fastest method.

Table 7. The results of Hardy Cross methods for example 2

Description	Iteration
Hardy Cross method (HCM)	62
Wang model (WM)	62
Conflation model (CM)	32
Second conflation model (SCM)	19
First-order Newtonian corrected model (FONCM)	62
Third-order Newtonian corrected model (TONCM)	38
Sixteenth-order Newtonian corrected model (SONCM)	24

5.2. Example 3

This example is based on figure 11. This underground space has been formed from four shafts, four drifts, and three cross-cuts. Each branch resistance of this ventilation network is according to table 8. Two fans with productive pressure 80 (kg/m²) and 100 (kg/m²) exist in this model. The final results of the Ventsim software simulation are presented in figure 12. The intensity of hypothetical flows and network rings is presented in figure 13 and table 9. The mathematical calculation results of the first iteration of Hardy Cross methods are presented in table 10. If the accuracy of calculations is assumed 10^{-13} , therefore the iteration number of calculations in Hardy Cross methods will become according to table 11. The final results of these methods are completely the same as the results of Ventsim software. According to table 11, the second conflation model of the Hardy Cross method is considered the fastest method.

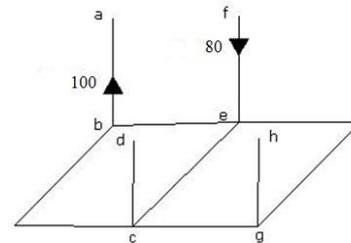


Figure 11. The second hypothetical ventilation network [39]

Table 8. Mine work resistance of second ventilation network [39]

Description	Resistance (kgs ² /m ⁸)
ab	0.10
bc	0.12
cd	0.20
Ce	0.15
be	0.18
Ef	0.12
cg	0.17
ge	0.22
hg	0.12

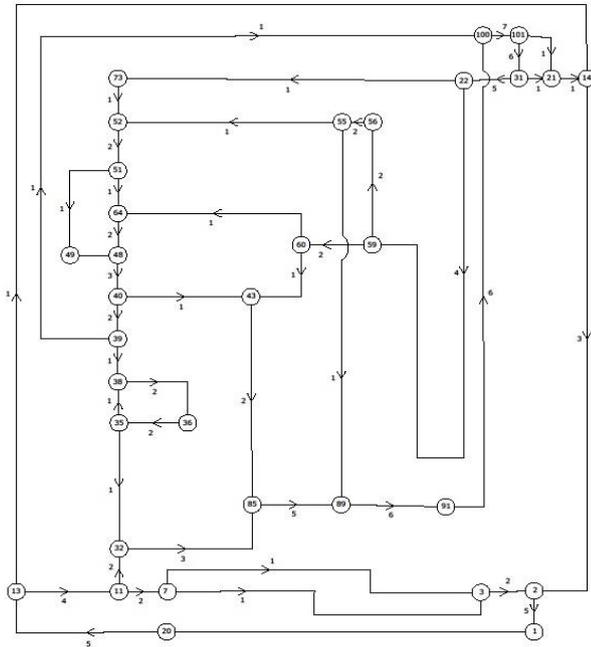


Figure 16. The intensity of hypothetical flows in example 4

Table 12. The rings of the third ventilation network

Num	Ring	Description
1	independent	1-20-13-14-2-1
2	useful	64-60-43-40-48-64
3	useful	48-49-51-64-48
4	useful	35-38-36-35
5	useful	7-5-3-7
6	useful	101-21-31-101
7	useful	56-59-60-64-51-52-55-56
8	useful	32-85-43-40-39-38-35-32
9	useful	52-73-22-59-56-55-52
10	useful	39-100-101-31-22-73-52-51-49-48-40-39
11	useful	11-13-14-21-101-100-39-38-35-32-11
12	useful	89-55-52-51-64-60-43-85-89
13	useful	11-32-85-89-91-100-101-31-21-14-3-2-3-5-7-11
14	useful	20-13-11-7-3-2-1-20
15	useful	89-91-100-39-40-48-49-51-52-55-89

Table 13. The results of the Hardy Cross methods for example 4

Description	Flow intensity	Iteration	Time (s)
Hardy Cross method (HCM)	21.572460	110	0.00
Wang model (WM)	21.572460	110	0.00
Conflation model (CM)	21.572460	60	0.00
Second conflation model (SCM)	21.572460	44	0.00
First-order Newtonian corrected model (FONCM)	21.572460	110	0.031
Third-order Newtonian corrected model (TONCM)	21.572460	73	0.032
Sixteenth-order Newtonian corrected model (SONCM)	21.572460	50	0.047

5.4. Example 5

This example is quite similar to example 4. But with this difference, this ventilation network has been used for three fans. Fans exist in branches 20-13, 39-38, and 59-60 respectively, the productive pressure of each fan is equal to 100 (kg/m²). If the accuracy of calculations is assumed 10⁻¹², therefore productive flow intensity in the ventilation channel is according to table 14 and the second conflation model of the Hardy Cross method is considered as the fastest method.

Table 14. The results of Hardy Cross methods for example 5

Description	Flow intensity	Iteration	Time (s)
Hardy Cross method (HCM)	divergent	∞	∞
Wang model (WM)	divergent	∞	∞
Conflation model (CM)	12.234138	227	0.016
Second conflation model (SCM)	12.234138	58	0.00
First-order Newtonian corrected model (FONCM)	divergent	∞	∞
Third-order Newtonian corrected model (TONCM)	12.262434	248	0.109
Sixteenth-order Newtonian corrected model (SONCM)	12.234138	270	0.25

6. Results and discussions

The ventilation network design is generally done using the computerized method according to section 1. The computerized method is designed based on mathematical approximate methods such as the Hardy Cross method and its corrected models.

In Section 2, the Hardy Cross method was presented for analyzing of water distribution networks but this method was later used for analyzing of ventilation networks. The Hardy Cross method calculates flow intensity error of each ring of ventilation network based on equation 1 and then flow intensity correction of each branch of ventilation network is done using equation 46 and the flowchart of this method has been presented in figure 17. Also, the principles of modified models of the Hardy Cross method such as Wang model, conflation model, first, third and sixteenth- order Newtonian models investigated. In normal conditions, the Hardy Cross method, Wang model, and conflation model are congruous with the principles of ventilation design. But Newtonian models of the Hardy Cross method are not perfectly congruous with the principles of ventilation design because those were designed for analyzing water networks. In these networks, the overall flow intensity is constant that this model is one of the mine conditions.

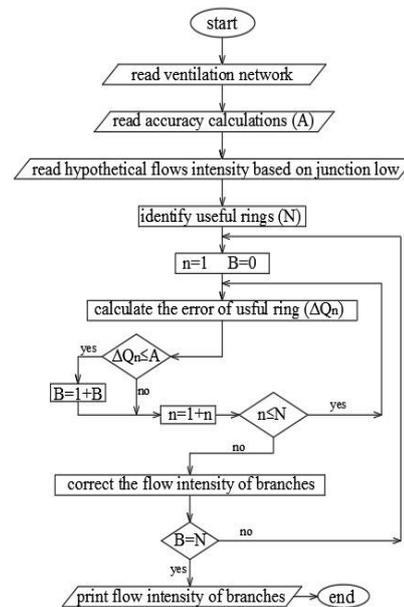


Figure 17. The flowchart of the Hardy Cross method

In Section 3 is observed that corrections must be done on first, third, and sixteenth-order Newtonian models because these models do not support all conditions of the mine. Therefore, this paper produced corrected models of Newtonian models for analyzing all ventilation networks.

In Section 4, a new method is presented for faster convergence of the Hardy Cross method to the final result. The name of this method is the second conflation model of the Hardy Cross method. This new method is full congruous with the principles of ventilation design.

In Section 5 is observed that various examples are presented to compare the Hardy Cross methods to better understand their performance. The results of this section indicate that the second conflation model of the Hardy Cross method has the best performance in terms of time and iteration number of calculations. Also, the ratio of calculation iteration number of Hardy Cross methods to second conflation model is reported in Table 15. According to table 15, the second conflation model is selected as the fastest method.

Table 15. The ratio of calculation iteration number of Hardy Cross methods to SCM

Description	Example 1	Example 2	Example 3	Example 4	Example 5
HCM	3.00	3.26	29.67	2.50	∞
WM	3.00	3.26	29.67	2.50	∞
CM	1.75	1.68	15.39	1.36	3.91
SCM	1.00	1.00	1.00	1.00	1.00
FONCM	3.00	3.26	29.67	2.50	∞
TONCM	2.38	2.00	16.78	1.66	4.28
SONCM	1.00	1.26	14.33	1.14	4.66

7. Conclusion

In the Hardy Cross method, Airflow direction in each branch of the ventilation network is variable for all iterations. The final results of the fifth example show that the Hardy Cross method is become divergent. Therefore, the correction of this method is necessary for the analysis of ventilation networks.

Wang's model was presented for using of easy of Hardy Cross method and the possibility of a better use of this model in the analysis of ventilation networks based on equation 4. Airflow direction in each branch of the ventilation network is fixed for all iterations in this method. The final results of the fifth example show that Wang's model is divergent. Therefore, the correction of this method is necessary for the analysis of ventilation networks.

The conflation model was designed based on simultaneous solving of the fourth and fifth stages of the Hardy Cross method. In other words, these two stages are interdependent and not independent. The findings results of this report indicate that this modified model is always convergent. According to the results of this paper, this method can reduce the iteration number and time performs of calculations in the Hardy Cross method. According to table 13, this method can reduce the iteration number of calculations in the Hardy Cross method to 45%. Therefore, the conflation model is a reliable model for reducing of iteration number of mathematical calculations in the Hardy Cross method.

The first-order Newtonian model was presented for analyzing water distribution networks but this method can be used for analyzing ventilation networks. For using this method in the analysis of ventilation networks is necessary to this method is corrected. The corrections of this method were done in section 3.1. The final results of the fifth example show that the first-order Newtonian corrected model has become divergent. Therefore, the correction of this method is necessary for the analysis of ventilation networks.

The third-order Newtonian model was presented for analyzing water distribution networks. For using this method in the analysis of ventilation networks are necessary to be corrected. The corrections of this method were done in section 3.2. The final results of this paper show that the third-order Newtonian corrected model can become convergent. This method is able to reduce the iteration number of calculations in the Hardy Cross method. According to table 13, this method can reduce the iteration number of calculations in the Hardy Cross method to 35% but the time of calculations has increased.

The Sixteenth-order Newtonian model was presented for analyzing water distribution networks. For using this method in the analysis of ventilation networks, it is necessary to be corrected. The corrections of this method were done in section 3.3. The final results of this paper show that the sixteenth-order Newtonian model can become convergent. This method is able to reduce the iteration number of calculations in the Hardy Cross method. According to table 13, this method can reduce the

iteration number of calculations in the Hardy Cross method to 55% but the time of calculations has increased. Also, the Maximum time of mathematical calculations is related to the sixteenth-order Newtonian corrected model.

The second conflation model of the Hardy Cross method has been designed based on three principles. The first principle is based on the conflation model. The second principle is based on the identification of common and uncommon branches. The third principle is based on the identification of independent and useful rings. These principles have been reported in section 4. According to the results of this paper, this method is able to reduce the iteration number and time performs of calculations in the Hardy Cross method. Also, the fastest method based on the iteration number and time performs of mathematical calculations is the second conflation model of the Hardy Cross method.

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