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Robustness price of open-pit mine production scheduling

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ABSTRACT

An Open-Pit Production Scheduling (OPPS) problem focuses on specifying block production scheduling to achieve the highest possible Net Present Value (NPV). This paper presents a new mathematical model for OPPS under uncertainty. To this end, a robust box and ellipsoidal counterpart approach was used. The proposed method was implemented in a hypothetical model. A Genetic Algorithm (GA) and an exact mathematical modeling approach were used to solve the model. It was shown that the scheduling of deterministic and robust models in various conditions is different. Considering the type of robust counterparts, different production plans under various conditions were scheduled. Furthermore, the price of robustness was determined for various levels of conservation.

Keywords : *Open-pit; Production scheduling; Robust counterpart; Uncertainty*

1. Introduction

One of the most serious problems in mine planning is the optimization of mine production scheduling. In an Open-Pit Production Scheduling (OPPS) framework, orebody blocks are assigned to different production periods to achieve the maximum Net Present Value (NPV) of the plan subjected to the limitations of the operation. In order to resolve an OPPS problem, an orebody is discretized into three-dimensional arrays, known as block models. This problem consists of deciding which blocks to be mined, when to be mined, and what to be done once they are mined.

Concerns about significant variation in the economic parameters (e.g., price, cost, discount rate, and block grade changes) of the problem have spurred an interest in designing production scheduling under uncertainty. Many researchers have studied this problem under uncertainty using stochastic programming. Open-pit mine stochastic production scheduling problems are commonly addressed under ore grade (geological) and price uncertainties.

For the purpose of dealing with OPPS uncertainties, convex set-based robust optimization can be used. Based on the box counterpart formulation, Robust Optimization (RO) was applied to solve the OPPS problem in [1]. In addition, the OPPS ellipsoidal robust counterpart in the Two-Dimensional (2D) format was studied in [2]. In the current paper, a 2D numerical study of a hypothetical open-pit mine is conducted to make a comparison of the solutions of the box counterpart and optimization model of the ellipsoidal counterpart. We considered a hypothetical copper deposit based on a geological block model with 200 blocks, and set-induced robust formulations were applied to the production scheduling of the hypothetical mine. Production scheduling is highly sensitive to ore grade, commodity price, cost, mining approach, processing capacity and block tonnage. Here, robust counterpart optimization was deployed according to the box and ellipsoidal counterpart, and violation sources were considered in block economic value, mining, processing capacity and block tonnages.

This article was prepared as follows. A summary of RO and as well as box and ellipsoidal sets of uncertainty with corresponding robust counterpart formulations are presented in Section 2. In Section 3, the mathematical model of the suggested uncertainty set is offered for an OPPS problem. The implemented results are discussed for the hypothetical open-pit mine as well. Finally, Section 4 presents the concluding and main remarks.

2. Robust Optimization

The solution of a linear programming problem sometimes shows high sensitivity to parameter violations, in which omitting the data uncertainty may produce non-optimal solutions that might be even infeasible for practical purposes. In order to deal with uncertainties, several methods (e.g., fuzzy programming, stochastic programming, chance-constrained programming and robust optimization programming) can be suggested. The RO concept offers a framework to handle the uncertainty of parameters in optimization problems that can protect an optimum solution for each case of uncertainty actualization in a specified set of uncertainty [3]. Table 1 provides an overview of different robust programming concepts from 1973 until now. Note that OR is different from the stochastic programming with recourse.

2.1. Robust optimization based on uncertainty set-induced

Linear programming meets the uncertainty in the coefficients of the objective function, left-hand side (LHS) and right-hand side (RHS) constraint coefficients [14]:

$$\begin{aligned} &\text{Max} \sum_j \tilde{c}_j x_j \\ &\text{s.t.} \\ &\sum_j \tilde{a}_{ij} x_j \le \tilde{b}_i, \forall i \end{aligned} \tag{1}$$

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where x_j is a continuous or an integer variable. It is possible to transform the objective function and RHS uncertainty into the LHS uncertainty using Eq. 2:

Table 1. Robust programming concepts from 1973 until now.

Reference	Event	Additional description
Soyster [4]	Used RO programming	Achieving feasible solutions with uncertainty under all possible conditions
Mulvey et al. [5] and Yu and Li [6]	Proposed a robust stochastic optimization model	Optimizing the objective function, finding a robust solution and making an insensitive solution to different realizations
Ben-Tal and Nemirovski [7] and [8]	Made a clear direction to RO for future work.	Proposing a non-linear model based on an ellipsoidal uncertainty set
Averbakh [3]	Proposed a method for generating min-max regret solutions for problems featuring coefficients of interval uncertain objective function.	
Ghaoui et al. [9]	Used worst-case probability distributions to widen the bounds of worst-case value-at-risk (VaR)	
Bertsimas and Sim [10] and [11]	Considered the degree of conservatism of the robust solution	Proposing a linear model using a set of mixed interval and polyhedral uncertainty
Verderame and Floudas [12]	Based on continuous and discrete uncertainty distributions to widen the RO framework	
Chen et al. [13]	Demonstrated equivalency of set-based RO formulations and conditional VaR bound-based approximations to chance constraints per individual.	
Li Zukui et al. [14]	Reviewed RO studies before 2011, and studied six uncertainty sets	Presenting MILP and RO models for the uncertainty on LHS, RHS, and objective function of the LP model
Li Zukui et al. [15]	Considered approximate probabilistic constraints.	Proposing a new method by using the advantage of priori and posteriori probability bounds, in which old RO and approximation frameworks were used before, respectively.
Li Zhangzhi and Li Zukui [16]	Applied the RO approximation to solve chance-constrained programming	

$$\begin{aligned}&\text{Max } z\\ &\text{s.t.}\\ &\quad z-\sum_j \tilde{c}_j x_j \le 0\\ &\quad \tilde{b}_i x_0+\sum_j \tilde{a}_{ij} x_j \le 0, \forall i\\ &\quad x_{0=-1}\end{aligned} \tag{2}$$

So, without loss of generality, the *i*-th constraint in the above linear programming model is assumed by considering only the LHS uncertainty:

$$\sum_j \tilde{a}_{ij} x_j \le b_i \tag{3}$$

where $\tilde{a}$ is the uncertainty factor presented by:

$$\tilde{a}_{ij}=a_{ij}+\xi_{ij}\hat{a}_{ij}, \forall j \in J_i, \tag{4}$$

where the nominal value of variables is denoted by a_{ij}, and $\hat{a}_{ij}$ indicates the violations of positive constant, ξ_{ij} represents independent random variables with uncertainty, and J_i indicates the index subset that covers all variables with uncertainty factors. To summarize constraint (3), it can be categorized into deterministic and uncertain parts for the LHS of (3) using Eq. 5:

$$\sum_j a_{ij} x_j+\sum_{j\in J_i}\xi_{ij}\hat{a}_{ij}x_j \le b_i \tag{5}$$

Using the set-induced RO technique and in order to avoid infeasibility, the goal is to find feasible solutions for any ξ_{ij} in the determined uncertainty set:

$$\begin{aligned}&\sum_j a_{ij}x_j+\Psi\sum_{j\in J_i}\hat{a}_{ij}u_j \le b_i\\ &\sum_j a_{ij}x_j+\max_{\xi\in U}\left\{\sum_{j\in J_i}\xi_{ij}\hat{a}_{ij}x_j\right\}\le b_i\end{aligned} \tag{6}$$

2.2. Box counterpart formulation

For Constraint (5), its corresponding box counterpart formulation (6) is similar to the following constraints:

$$\begin{aligned}&\sum_j a_{ij}x_j+\Psi\sum_{j\in J_i}\hat{a}_{ij}u_j \le b_i\\ &-u_j \le x_j \le u_j\end{aligned} \tag{7}$$

The $\infty-norm$ of the uncertain data vector defines the box uncertainty set as: $U_\infty=\left\{\xi \middle| \|\xi\|_\infty \le \Psi\right\}=\left\{\xi \middle| |\xi_j| \le \Psi, \forall j \in J_i\right\}$, where Ψ represents the adjustable parameter that determines the uncertainty size. Figure 1 shows the box uncertainty set for parameter $\tilde{a}_j$ introduced by $\tilde{a}=a_j+\xi_{j\hat{a}j}, j=1,2$, where $\tilde{a}_j$ signifies the correct value of the parameter, a_j indicates the nominal value for ξ_j, and $\hat{a}_j$ represents the uncertainty and constant violation, respectively, while the uncertain parameters are limited to specific intervals $\tilde{a}_{ij}\in\left[a_{ij}-\tilde{a}_{ij}, a_{ij}+\tilde{a}_{ij}\right]\forall j \in J_i$. Therefore, $\tilde{a}_{ij}=a_{ij}+\xi_{j\hat{a}ij}$ represents the uncertainty that is a form of box uncertainty set when $\Psi=1(i.e., U_\infty=\left\{\xi \middle| |\xi_j| \le 1, \forall j \in J_i\right\})$ [17]. Soyster [4] coined the "interval uncertainty set" to represent the box set with $\Psi=1$.

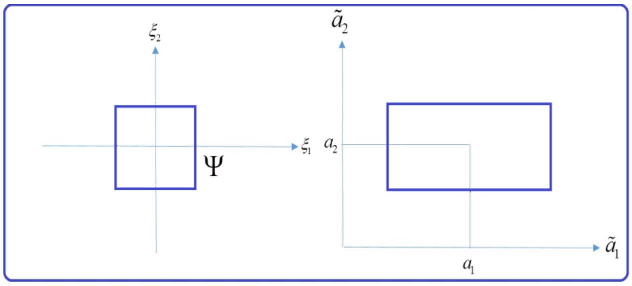

Figure 1. Box uncertainty set.

The formulation of box counterpart optimization for the *i*-th linear constraint with the LHS and RHS uncertainties is obtained as:

$$\sum_j a_{ij}x_j + \Psi\left(\sum_{j\in J_i}\hat{a}_{ij}\left|x_j\right| + \hat{b}_i\right) \le b_i \tag{8}$$

2.3. Ellipsoidal counterpart formulations

The ellipsoidal uncertainty set can be defined by the 2-norm uncertain data vector as:

$$U_2 = \left\{\xi \middle| \left\|\xi\right\|_2 \le \Omega\right\} = \left\{\xi \middle| \sqrt{\sum_{j\in J_i}\xi^2} \le \Omega, \forall j \in J_i\right\} \tag{9}$$

where Ω represents the adjustable variable, which controls the uncertainty set size. Based on the problem geometry, $\xi_j \in \left[-1,1\right]$ is true for the limited uncertainty and when $\Omega \ge \left(\left|J_i\right|\right)^{1/2}$ (where $\left|J_i\right|$ stands for the cardinality of the set J_i), the whole space of uncertainty is covered by the set of ellipsoid uncertainty. Figure 2 illustrates the ellipsoidal uncertainty set.

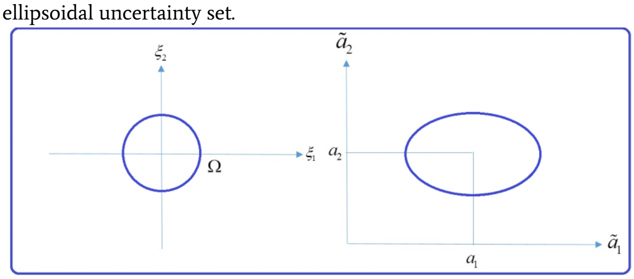

Figure 2. Ellipsoidal uncertainty set.

In the case of the ellipsoidal uncertainty set, represented by *U*, the following robust counterpart is obtained by incorporating substantiation reported in [14, 17]. Moreover, the formulations for the *i*-th linear constraint with the LHS and RHS uncertainties are as follows:

$$\sum_j a_{ij}x_j + \Omega\left(\sqrt{\sum_{j\in J_i}\hat{a}_{ij}^2 x^2{}_j + \hat{b}_i^2}\right) \le b_i \tag{10}$$

3. Computational study for robust mine production scheduling

The OPPS problem is designed to specify a block mining sequence to increase the NPV as much as possible based on the constraints of capacity and sequencing. In a deterministic form, this problem can be demonstrated as follows [18]:

Let x_{ij} be decision variables, and T be mining period counts, N refers to the number of blocks, V_{ij} is the current value of block j in period i, d_j is the mass of ore in block *j*, A, A' represent the highest and lowest processing capacities, respectively, v_j stands for the mass of waste in block j, and C, C' are mining capacity. The goal is to solve the following model:

$$\text{Max} \sum_{i=1}^{T}\sum_{j=1}^{N} V_{ij}x_{ij} \tag{11-1}$$

s.t.

$$\sum_{i=1}^{T} x_{ki} \ge \sum_{i=1}^{T} x_{ji},\ j \text{ blocks overlying block } k \tag{11-2}$$

$$C' \le \sum_{j=1}^{N}(d_j + v_j)x_{ij} \le C, \forall i = 1,\dots,T \text{ : Constraint of mining capacity} \tag{11-3}$$

$$A' \le \sum_{j=1}^{N} d_j x_{ij} \le A, \forall i = 1,\dots,T \text{ : Constraint of processing capacity} \tag{11-4}$$

$$\sum_{i=1}^{T} x_{ij} \le 1, \forall j = 1,\dots,N \tag{11-5}$$

Sequencing constraint (11.2): to access a block to be mined, mining the overlying must be done earlier or in the same period of that block.

Mining capacity (11.3): Mining capacity is selected based on the economic and operational limitations.

Processing capacity (11.4): Due to the sequencing constraint, ore and waste blocks are both mined under open-pit mining. Ore blocks are sent to the mineral processing plant along with waste blocks that are sent to the waste dumps. The volume of ore blocks must be proportional to the processing capacity.

Block conservation constraint (11.5): A block is mined only once.

where $\tilde{V}_{ij}$'s are subject to uncertainty and also show the true values of the parameters, V_{ij} 's represent the nominal values of the parameters.

3.1. Formulations of Box counterpart optimization for production scheduling

Here, the complete box uncertainty set containing robust counterpart formulation is introduced for the OPPS problem with constraints. It can be demonstrated by:

$$\begin{aligned}
&\text{Max} \sum_{i=1}^{T}\sum_{j=1}^{N} V_{ij}x_{ij} - \Psi\left(\sum_{i=1}^{T}\sum_{j=1}^{N}\hat{V}_{ij}x_{ij}\right)\\
&\text{s.t.}\\
&\sum_{i=1}^{T} x_{ki} \ge \sum_{i=1}^{T} x_{ji}\\
&\sum_{j=1}^{N} r_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{r}_j x_{ij} + \hat{C}\right) \le C, \forall i = 1,\dots,T, d_j + v_j = r_j \text{ : Mining capacity constraint}\\
&C' \le \sum_{j=1}^{N} r_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{r}_j x_{ij} + \hat{C}'\right), \forall i = 1,\dots,T, d_j + v_j = r_j \text{ : Mining capacity constraint}\\
&\sum_{j=1}^{N} d_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{d}_j x_{ij} + \hat{A}\right) \le A, \forall i = 1,\dots,T \text{ : Processing capacity constraint}\\
&A' \le \sum_{j=1}^{N} d_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{d}_j x_{ij} + \hat{A}'\right), \forall i = 1,\dots,T \text{ : Processing capacity constraint}\\
&\sum_{i=1}^{T} x_{ij} \le 1, \forall j = 1,\dots,N
\end{aligned} \tag{12}$$

where V_{ij} 's represent the nominal values of the parameters, Ψ stands for uncertainty and $\hat{V}_{ij}, \hat{r}_j, \hat{v}_j, \hat{C}, \hat{C}', \hat{A}, \hat{A}', \hat{d}$ represent constant violation of the objective function and constraint coefficients.

3.2. Ellipsoidal counterpart optimization formulations

Now, a generalized ellipsoidal uncertainty set is introduced, which incorporates robust counterpart formulation for the OPPS problem under constraints.

$$
\begin{aligned}
&\text{Max } \sum_{i=1}^{T}\sum_{j=1}^{N} V_{ij}x_{ij} - \Omega\left(\sqrt{\sum_{i=1}^{T}\sum_{j=1}^{N}\hat{V}_{ij}^{2}x_{ij}^{2}}\right)\\
&\text{s.t.}\\
&\sum_{i=1}^{T} x_{ki} \geq \sum_{i=1}^{T} x_{ji}\\
&\sum_{j=1}^{N} r_j x_{ij} + \Omega\left(\sqrt{\sum_{j=1}^{N}\hat{r}_j^{2}x_{ij}^{2} + \hat{C}^{2}}\right) \leq C, \forall i = 1,\dots,T, d_j + v_j = r_j : \text{ Mining capacity constraint}\\
&C' \leq \sum_{j=1}^{N} r_j x_{ij} + \Omega\left(\sqrt{\sum_{j=1}^{N}\hat{r}_j^{2}x_{ij}^{2} + \hat{C}'^{2}}\right), \forall i = 1,\dots,T, d_j + v_j = r_j : \text{Mining capacity constraint}\\
&\sum_{j=1}^{N} d_j x_{ij} + \Omega\left(\sqrt{\sum_{j=1}^{N}\hat{d}_j^{2}x_{ij}^{2} + \hat{A}^{2}}\right) \leq A, \forall i = 1,\dots,T \text{ : Processing capacity constraint}\\
&A' \leq \sum_{j=1}^{N} d_j x_{ij} + \Omega\left(\sqrt{\sum_{j=1}^{N}\hat{d}_j^{2}x_{ij}^{2} + \hat{A}'^{2}}\right), \forall i = 1,\dots,T \text{ : Processing capacity constraint}\\
&\sum_{i=1}^{T} x_{ij} \leq 1, \forall j = 1,\dots,N
\end{aligned}
\tag{13}
$$

where V_{ij} 's stand for nominal values, Ω denotes the size of uncertainty and $\hat{V}_{ij}, \hat{r}_j, \hat{v}_j, \hat{C}, \hat{C}', \hat{A}, \hat{A}', \hat{d}$ represent a constant violation of the constraints and objective function coefficients.

3.3. Implementation and evaluation

A hypothetical copper deposit economic block model is needed to elaborate on the details of implementing the production scheduling method. An open-pit mine was built using 2-D blocks. Figure 3 illustrates the economic block model of hypothetical copper as a case study. A mining operation needs to be run for at least four years. The highest and lowest mining capacities in a year are set equal to 24 and 18 blocks, respectively, and the highest and lowest processing capacities are considered to be 15 and 9 ore blocks, respectively. The discount rate is equal to 4%.

It is clear that the OPPS solution is affected by block economic value violation, ore volume, waste volume in each period, and operation capacities. Box counterpart and ellipsoidal counterpart robust optimization are used in a systematic manner. In addition, block economic value (coefficients of the objective function), block weights in each period (LHS), mining, and proceeding capacities in each period (RHS) are considered as uncertain parameters. It is not possible to determine the exact value of grade and weight (tonnage) of blocks. Therefore, the violations from the operational capacities are permitted. It is considered equal to 0.04 and 0.08 for the violation rates of the block weights and operation capacities, respectively. These violation rates are relatively acceptable because of the mine planer's consideration based on simulated grades and estimated block tonnages. In addition, the value 0.2 is considered as the violation rate of the block economic value.

-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227
-227	393	213	123	663	393	393	-56	573	843	-227	483	-227	-227	1112	393	1652	663	303	-227
-227	393	-146	-56	663	663	213	123	1382	-227	573	1382	-227	1112	1112	483	-227	2281	1742	-227
-227	34	663	1022	2371	1112	-56	123	-227	1112	1472	-227	1112	933	1652	1112	-227	573	-227	-227
-227	933	-146	1202	843	-227	34	663	123	393	1562	303	34	1382	-146	843	663	213	-227	-227
-227	-227	663	753	-227	1472	213	393	1562	1472	1202	1742	-227	1202	2551	393	573	753	1112	-227
-227	393	483	1292	1022	1831	753	-227	2011	2101	393	843	933	2281	933	1112	483	-227	843	-227
-227	1562	2191	483	2640	-227	1112	213	483	303	34	483	303	1112	213	213	-227	663	-227	-227
-227	1921	-56	1022	1472	2281	753	663	303	213	303	303	34	1742	1292	123	213	393	933	-227
393	-56	1292	573	933	303	-227	-227	34	393	483	-227	123	663	1921	303	1742	483	-227	-227

Figure 3. Economic block model of the hypothetical copper mine as a scenario.

OPPS is a complex mine planning problem. Inspired by operation research problems (e.g., knapsack), several authors have developed linear and integer programming models to solve this problem. This problem, to a large size (large open-pit mine), is an NP-hard problem. Meta-heuristic algorithms have been recognized to be particularly suitable for solving NP-hard problems.

Some of the meta-heuristics algorithms have been used in OPPS problems. GA is a well-known global search meta-heuristic algorithm, which uses population-based characteristics and improves multiple candidate solutions. It is used to generate near-optimal solutions with a small gap in an acceptable computational time. Alipour et al. [19] developed a new GA representation in OPPS of a hypothetical 2-dimensional block model. In this section, the same method is used to solve the OPPS problem. Since the OPPS ellipsoidal robust counterpart is non-linear and complex, it is not possible to solve the problem with known methods; thus, we use the GA to solve the problem approximately.

The results of production scheduling solutions in a deterministic state using the GA and CPLEX solvers are illustrated in Table 2. The NPVs measured using the CPLEX and GA solvers are 41373 and 39489 dollars for this case study, respectively. The computational gap between GA and optimal solutions are usually less than 5%. The number of the calculated blocks for each period are listed in Table 2.

The trade-off mechanism between the conservation level versus the uncertainty and objective function values is known as the price of robustness. The price of mine production robustness was computed for 2-dimensional block models of the hypothetical mine based on the CPLEX solver and GA solutions for quantitatively measuring the uncertainty and analyzing its effect on scheduling problems. The results of GA-based box counterpart optimization are presented in Table 3. In addition, similar results are presented in Table 4, according to the ellipsoidal counterpart.

Table 2. Results of production scheduling solutions used by GA and CPLEX solver.

Calculated Item	GA	CPLEX solver
NPV ($)	39489	41373
No. of blocks in ultimate pit limit	92	88
No. of blocks in the first period	24	23
No. of blocks in the second period	24	23
No. of blocks in the third period	21	22
No. of blocks in the fourth period	23	20

The computational gap between GA and optimal solutions is less than 5% for different box counterpart production scheduling. Therefore, according to the capability of GA in obtaining near-optimal scheduling with a small gap, it can be used in nonlinear ellipsoidal counterpart production scheduling.

The variation of conservation level (Ω, Ψ) versus NPV is represented in Figure 4. Obviously, the value of NPV attenuates by increasing the size of box and ellipsoidal sets Ω, Ψ.

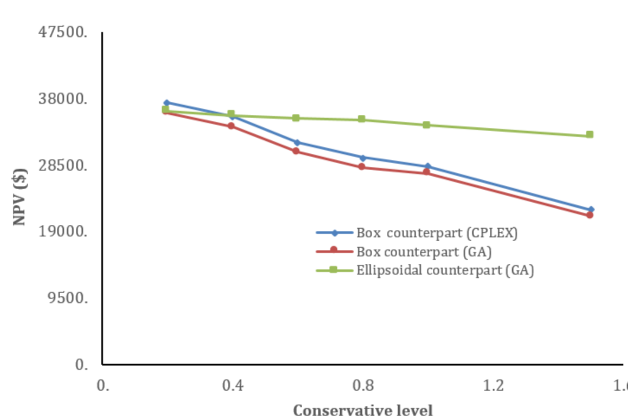

Figure 4. NPV, based on different counterpart optimization levels.

Table 3. Solutions summary for the box counterpart optimization model.

Conservative level (Ψ)	Source of violations	Violation rate	NPV, CPLEX ($)	NPV, GA ($)	Price of Robustness, CPLEX	Price of Robustness, GA
0, Deterministic	0	0	41373	39489	1	-
0.2	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	37492	35730	0.91	0.90
0.4	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	35394	33978	0.86	0.86
0.6	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	31692	30365	0.77	0.77
0.8	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	29579	28095	0.71	0.71
1, Soyster model	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	28263	27273	0.68	0.69
1.5	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	22081	21197	0.53	0.54

Table 4. Solutions summary for the ellipsoidal counterpart optimization model.

Conservative level (Ω)	Source of violations	Violation rate	NPV, GA ($)	Price of robustness
0, Deterministic	0	0	39489	1
0.2	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	36242.61	0.92
0.4	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	35624.52	0.90
0.6	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	35149.20	0.89
0.8	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	34922.76	0.88
1	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	34168.98	0.87
1.5	objective function, LHS and RHS uncertainty	$\hat{V}_{ij}$ =0.2, $\hat{r}$ =0.04, $\hat{C},\hat{C}',\hat{A},\hat{A}'=0.08$	32622.66	0.83

The final remarks of this research are presented below:

- There is considerable disagreement between the behavior of box and scheduling ellipsoidal counterpart production of the same size.
- The same sized box counterpart production scheduling is more conservative than the ellipsoidal counterpart.
- The size of uncertainty set, violation rate of block economic value, block weights and operation capacities in each period are decision-making factors in the scheduling plan election. In other words, mine designers should select the scheduling plan, according to the defined risk index.
- The prices of robustness are different in various cases of robust scheduling.
- The optimal solution cannot be obtained within a reasonable amount of running time using the exact CPLEX solver solution. However, the capability of GA in achieving near-optimal scheduling with small gap is acceptable. The CPU time for this GA to derive optimal solutions is relatively short compared to the complexity of the OPPS problem.

4. Conclusion

In this paper, the performance of the box and ellipsoidal robust counterpart set were evaluated for OPPS problems. Uncertainty appears in constraints and objective functions. Therefore, different production plans should be scheduled according to various conditions. Practically, the schedule plan can be selected based on the degree of conservation, the application of uncertainty in constraints (i.e., blocks weight, mining and processing capacity), and objective functions (i.e., economic values of blocks).

In large-scale robust mine production scheduling problems, an exponential increase in the problem size is encountered. Therefore, the use of approximation algorithms and heuristics are inevitable. In this research work, the GA was applied to complicated problems. By comparing the results of the exact linear mathematical solver using the CPLEX software, it was demonstrated that GA is capable of generating near-optimal solutions with a small error. Moreover, it could be used in non-linear integer programming and was an appropriate solver for ellipsoidal counterpart optimization formulations. The proposed framework can be more practically useful for solving uncertainty-based mine production scheduling problems in large-scale mines.

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