

# Recent developments in configuration design and optimization of mineral separation circuits; A Review

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Article History:

Received: 15 May 2018,

Accepted: 27 October 2018.

## ABSTRACT

The present research reviews two basic approaches for the separation circuit configuration analysis. The first approach is to optimize the circuit configuration. In this method, after a circuit modeling, a variety of search algorithms and mathematical optimization methods are used. Previous works show that this approach has more application in the flotation process. The second approach called the circuit analysis, evaluates the circuit configuration by considering a transfer function for each separation unit. This method provides great freedom for conducting a variety of investigations on the separation circuits.

**Keywords :** *Circuit configuration; Circuit analysis; Optimization techniques*

## 1. Introduction

Mineral processing involves two basic steps: comminution and separation. In the comminution section, valuable minerals are liberated from gangue and at the separation stage, two products named concentrate and tailings are obtained. Various equipment is used for comminution and separation operations. In the comminution operations, the crushing and grinding equipment is used to change the feed size to achieve the appropriate degree of liberation of particles. In the separation section, however, the equipment of size classification and gravity concentration such as heavy media, magnetic separators, flotation cells, and electrostatic separation is used [1]. The particle size remains unchanged in the separation section, and only the feed flow is divided into two different concentrate and tailings based on the physical and chemical differences between the different feed particles. If the flotation process is to use the regrinding, then the particle size changes [2]. The separation equipment divides the materials according to the properties such as density, magnetic susceptibility, conductivity, and the properties of surface to other parts. For example, a gravity concentration plant can contain a number of jigs and spirals that divide the feed into concentrate and tailings [3].

One of the most important issues in designing the separation circuits is how to connect the equipment to achieve the desired performance. In most cases, the arrangement of equipment in the processing circuits is based on a number of rules of the thumb and industrial experience. So far today, in most cases, two conventional rougher-cleaner and rougher-scavenger circuits are used for two-stage separations [4]. In these circuits, the rougher stage is used to achieve maximum recovery. The concentrate from the rougher stage is sent to the cleaner stage for further concentration. The tailing of the rougher stage, however, is sent to the scavenger section to ensure the recovery of the most valuable rougher stage materials. As the number of stages and the complexity of the circuit increase, its evaluation only based on empirical rules becomes

very difficult, and sometimes impossible [5]. Also, considering that there are different configurations for a separation circuit, finding the best combination through experiments is time-consuming and expensive [6]. In such situations, it is necessary to use a mathematical modeling and simulation for the evaluation of different circuit configurations. An optimal design of the separation circuits can eliminate the weaknesses associated with a single unit operation, while an improper design can lead to inefficient or even weaker circuits of single units [7]. On the other hand, due to the fact that millions of tons of ore are processed annually in the separation circuits, a slight improvement in the configuration of these circuits can have a significant economic impact [8]. In the literature, a variety of evaluation methods, and as a result, different circuit configurations have been proposed. Some only proposed counter-current circuits, in which just the number of stages were different [4, 9-19]; some others have tried to find the best configuration with circuit modeling and through the consideration of all possible layouts for a given feed and then the circuit optimization [20-27]. In such cases, a variety of circuit configurations were proposed that, according to researchers, would increase the grade, recovery, separation efficiency, and/or the economic indicators.

Examining the separation circuits used in chemical engineering, their circuits are often counter-current and this is the number of steps, the type of equipment and process used, the design and operational parameters that make the difference and not unconventional circuit configurations. For example, counter-current circuits can be mentioned for the distillation process [28]. However, it still seems difficult to answer some of the following questions:

- Which circuit configuration(s) [4] are more efficient for a given number of stages?
- Is there any configuration that always provides the best answer?
- Until now, what comparisons have been made between the proposed configurations through the optimization methods with a counter-current circuit?
- Are the proposed configurations able to prove their efficiency in

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practice through the optimization methods?

As a result, the present study reviews a variety of methods for evaluating and optimizing the configuration of mineral separation circuits along with the advantages and disadvantages of each method. After presenting an introduction (Section 1), the involved concepts such as separation systems and their models (Section 2), and then the circuit configuration evaluation methods (Section 3) are presented. In this section, various types of optimization methods are reviewed. Also, a circuit analysis is considered as a method for evaluating the configuration of the separation circuits without the need for accurate information from the unit operations. Section 4 attempts to critically examine the advantages and disadvantages of previous works. Section 5 presents the conclusions.

## 2. Concepts

### 2.1. Separation systems

A system can be defined as a group of objects that are interconnected in order to achieve a certain purpose within the framework of a relationship or interdependent correlation [29]. The system can be an arbitrary part of the whole process under consideration. In a mineral processing plant, grinding mill or flotation cell, part of a circuit or the entire processing plant can be a system. Materials or devices outside the system are the surroundings. The system is defined by drawing lines around that part of the process which is considered as the system boundary. To model the separation systems, understanding the concept of the system and the boundary of the system is essential. In mineral processing, in most cases, a single unit operation cannot achieve the desired quality (e.g., grade and recovery), and as a result, several devices need to be used to achieve the required performance [7]. The arrangement of the equipment in the separation section leads to the formation of separation systems or circuits. A separation system can be made of the same operational units, such as a flotation or a non-identical system, or a combination of the gravity, magnetic, and flotation equipment. Depending on the location of the unit operations, the inlet and outlet flows can be referred to as the feed, middling materials, or product streams. The layout design for separation circuits is often carried out based on past experiences as well as trial and error methods [2]. In addition to creating the dependence on a designer's experience, this approach can lead to inefficiencies in the separation circuits. As a result, the assessment of equipment configuration in the separation circuit is a way to properly understand and maximize the efficiency of separation processes.

### 2.2. Transfer function or mathematical model

A transfer function is a mathematical expression connecting the amount of a conserved quantity or component,  $j$ , entering to one step to the amount leaving that stage at a steady state [13]. For the  $j^{\text{th}}$  component that leaves stage  $k$  and enters stage  $l$ , the conservation relation is written as follows:

$$M_{jkl} = T_{jkl} M_{jl} \quad (1)$$

$T_{jkl}$  = fraction of the conservation component,  $j$ , which leaves the  $k^{\text{th}}$  stage and enters stage  $l$  (transfer function).

$M_{jl}$  = total amount of conserved quantity,  $j$ , which enters stage  $l$ .

$M_{jkl}$  = amount of conserved quantity,  $j$ , which leaves  $k^{\text{th}}$  stage and enters stage  $l$ .

The transfer function or model is very simpler to the phenomenon under the investigation, but from the mathematical point of view, it may still be complicated. The complexity of mathematical models may make their application difficult; therefore, there is a great desire to use simple models. In other words, it tries to make the model as simple as possible, since even if its result is not correct, it can be modified with the smallest and simplest change. For example, the first-order kinetic model has long been used for the flotation process, while in theoretical research, more complicated models are presented [30-37]. Another reason to keep a model simple is that if a complex model does not lead to an answer, the

correction will be as complex as it would be. While modifying a simple model (even if possible) is as simple as the model. In the following, the types of transfer functions or models presented for separation equipment are investigated.

#### 2-2-1. Empirical model

Curves that simply fit the shape of the recovery curves in a steady state are still widely used. These curves of the type of tromp do not attempt to establish logical relations or calculate mass transfer functions [38]. For example, empirical models for heavy media devices are provided by the modified Weibull function [39]:

$$f(x) = 100(f_0 + C \exp(-(x - x_0)^{\frac{a}{b}})) \quad (2)$$

In which:

$f(x)$  = coal fraction to clean coal

$x$  = reduced specific gravity and

$a$ ,  $b$ ,  $c$  and  $f_0$  are constant values.

The function  $f(x)$  is simply a good statistical fit to the washability data, as reflected in the correction coefficients. Other types of empirical models include trying to find the relationships between the response of a circuit, such as the recovery, and one or two other independent variables such as collector and frother concentrations [40]. Although these models are important, they do not have a theoretical basis.

#### 2-2-2. Probability model

Probabilistic models suggest that the flotation process is a sequence of events that must occur prior to collecting particles. These models are based on theoretical considerations of flotation sub-processes events and can describe the entire flotation recovery-time profile [41]. In this approach, the particle recovery rate in flotation is associated with the probability of flotation success,  $P_x$ . For a specific size:

$$P_x = P_c P_a F \quad (3)$$

In which:

$P_c$  = probability of collision

$P_a$  = probability of attachment

$F$  = froth stability factor

All kinetic models in a steady state are reduced to probability type models [42]. Also, according to [42], the bubble-particle collision zone around the agitators are very important. Nowadays, new cells are more likely to be different than the agitators. It was assumed that if the recovery or probability of flotation in a single cell is equal to  $p$ , in a bank, the values of  $p$  are equal in cells with probabilities of flotation  $p_1$ ,  $p_2$  to  $p_n$ . With this assumption, the bank recovery was proposed as follow:

$$R = R_{\infty} (1 - (1 - p)^n) \quad (4)$$

In the case of flotation cells in plants, it is better to use cells of the same size and only different in terms of number in different stages of the separation circuit (rougher, cleaner, scavenger, etc.) [19]. This can provide the possibility of generating the identical  $p$ 's.

#### 2-2-3. Kinetic model

Currently, a variety of kinetic models is used. Given the mass flow rate of the input and output of a cell, the total mass balance of component  $j$  for a single cell is as follows [40]:

$$\frac{dM_j(t)}{dt} = F_j(t) - T_j(t) - C_j(t) \quad (5)$$

In which:

$F_j(t)$  = feed rate

$T_j(t)$  = tailing rate

$C_j(t)$  = concentrate rate

$M_j(t)$  = mass of  $j$  in the cell at time  $t$

Equation 5 is the mass balance equation for a continuous flotation cell. This equation is often expressed kinetically [16]:

$$\frac{d(C(t)V(t))}{dt} = Q(t)C_f(t) - Q(t)C_j(t) - k_j(t)V(t)C_j(t)^{n_j(t)} \quad (6)$$

where:

$C_j(t)$  = concentration in the cell and tailings flow

$Q(t)$  = volumetric flow of feed and tailing

$V(t)$  = cell volume

$C_f(t)$  = concentration in feed flow

$k_f(t)$  = constant flotation rate

$n_j(t)$  = kinetic order

Perfect mixing assumption was applied to equation 6. Equation (6) can be simplified for a single species in a laboratory flotation cell ( $Q(t) = 0$ ) with first-order kinetics ( $n_j = 1$ ) and constant chemical and hydrodynamic environments in the cell [43]:

$$\frac{dc}{dt} = -kc \quad (7)$$

By integrating:

$$R(t) = R_0(1 - \exp(-kt)) \quad (8)$$

Where in:

$R(t)$  = floatable mass fraction after time  $t$

$R_0$  = floatable mass fraction at the infinity

Equation (6) at the steady state is simplified as follows [44]:

$$R = \frac{k}{k + Q/V} \quad (9)$$

### 2.3. Circuit modeling

The optimization of the flotation circuit configuration is based on the modeling of the flotation process and its development for the circuit. For example, all possible combinations for a circuit with 2 cells/banks, regardless of the physics of the flotation process, are shown in Figure 1 [45]. In this method, all possible connections between the flotation banks are considered, in which  $S$  is a splitter. The idea of this approach was initially proposed for heat exchanger systems [46] and then proposed for the optimization of flotation circuits [45]. In Figure 1,  $F$  is the feed entering the circuit,  $\delta_{Fi}$  is the contribution of the cell  $i$  to the input to the circuit,  $\beta_{ki}$  and  $\delta_{k0}$  are the mass proportion of the concentrate and the tailings of the  $j$ th cell that enters the cell  $i$ . In these flows, the subscript of zero means out of the circuit. A set of  $\delta_{Fi}$ ,  $\beta_{ki}$  and  $\delta_{ki}$  values, indicating a combination of the flotation circuit. For this reason, this set of variables is called structural parameters of the circuit. If  $C(K)$ ,  $T(K)$  and  $MF(K)$  represent the flow rate of solids with kinetic constant  $K$  in concentrate, tailings, and feed, respectively,

$$C(K) = M_{F,j}(K) \left( \frac{K\tau}{1 + K\tau} \right) \quad (10)$$

$$T(K) = M_{F,j}(K) \left( \frac{1}{1 + K\tau} \right) \quad (11)$$

In which,  $\tau$  is the mean residence time in the flotation cell. A simple mass balance, the total mass flow of  $j$  (for  $j=1,2, \dots, n$ ) input into the  $i$ th cell (for  $i=1,2, \dots, n$ ) is given as follows:

$$F_{j,i} = M_{F,j} \delta_{Fi} + \sum_{k=1}^m T_{j,k} \delta_{ki} + \sum_{i=1}^m C_{j,k} \beta_{ki} \quad (12)$$

If it is assumed that:

$$\alpha_{j,i} = \frac{1}{1 + K_j \tau_j} \quad (13)$$

The mass flow rate of the component  $j$  from the  $i$ th cell to the concentrate and tailing streams is as follows:

$$C_{j,i} = F_{j,i} (1 - \alpha_{j,i}) \quad (14)$$

$$T_{j,i} = F_{j,i} \alpha_{j,i} \quad (15)$$

$$C_{j,i} = T_{j,i} \left( \frac{1 - \alpha_{j,i}}{\alpha_{j,i}} \right) \quad (16)$$

Replacing  $C_{j,i}$  of Equation 16 in Equation 12:

$$F_{j,i} = M_{F,j} \delta_{Fi} + \sum_{k=1}^m T_{j,k} \left[ \left( \frac{1 - \alpha_{j,i}}{\alpha_{j,i}} \right) \beta_{ki} + \delta_{ki} \right] \quad (17)$$

By replacing this term in Equation 15, we get the following relation:

$$T_{j,i} = M_{F,j} \delta_{Fi} \alpha_{j,i} + \alpha_{j,i} \sum_{k=1}^m T_{j,k} \left[ \left( \frac{1 - \alpha_{j,i}}{\alpha_{j,i}} \right) \beta_{ki} + \delta_{ki} \right] \quad (18)$$

Equation 18 consists of a set of  $n \times m$  linear algebra equations which can be numerically solved for several  $T_{j,i}$  values according to the connection and design parameters. When the  $T_{j,i}$  values are known, we can obtain the mass flow rate of the component  $j$  in the concentrate stream of the  $i$ th cell using Equation 16. Now it is possible to calculate the total recovery for the flotation circuit based on the structural parameters:

$$R_c = \frac{\sum_{i=1}^N \sum_{j=1}^N C_{j,i} \beta_{i0}}{\sum_{j=1}^N M_{F,j}} \quad (19)$$

$$G = \frac{\sum_{i=1}^m \sum_{j=1}^n C_{j,i} \beta_{i0} w_j}{\sum_{i=1}^m \sum_{j=1}^n C_{j,i} \beta_{i0}} \quad (20)$$

In which  $w_j$  is the valuable mass fraction of the component  $j$  in the concentrate stream (assumed to be the same for all cells). This problem is completed by the following constraints:

$$\sum_{i=1}^m \delta_{Fi} = 1 \quad (21)$$

$$\sum_{i=1}^m \delta_{ki} + \delta_{k0} = 1 \quad (22)$$

$$\sum_{i=1}^m \beta_{ki} + \beta_{k0} = 1 \quad (23)$$

$$0 \leq (\delta, \beta) \leq 1 \quad (24)$$

$$0 \leq \alpha_{j,i} \leq 1 \quad (25)$$

$$H_i = \tau_i \sum_{j=1}^m T_{j,i} \quad (26)$$

$$V_i \geq H_i \left[ \frac{1}{d} + \frac{100 - \theta_d}{\theta_d} \right] \quad (27)$$

$$V = \sum_{j=1}^m V_j \quad (28)$$

In order to simplify the computations, the personal judgment enforces the arbitrary constraint  $\bar{\tau}$  on  $\tau_i$ :

$$0 \leq \tau_i \leq \bar{\tau} \quad i = 1, 2, \dots, m$$

To be acceptable, the concentrate grade must not be less than a minimum level of  $\bar{G}$ .

$$G \geq \bar{G} \quad (29)$$

As a result, the problem is to find the best possible values of the decision variables, including the structural parameters and residence time in stages, to optimize the objective function such as recovery or grade. Different methods have been used to solve the optimization problem of the flotation circuit arrangement according to the objective function, the constraints, and decision variables. The following section examines these methods.

### 3. Methods

Several methods have been considered for analyzing and optimizing the configuration of mineral processing circuits. In this section, two

basic approaches are reviewed. The first approach is to optimize the arrangement of separation circuits. In this approach, after modeling the circuit, various search methods are used to achieve the optimal circuit configuration. The second approach, called the circuit analysis, evaluates the circuit arrangement by considering a selection or transfer function for each unit operation. This method provides great freedom for conducting a variety of investigations on separation circuits.

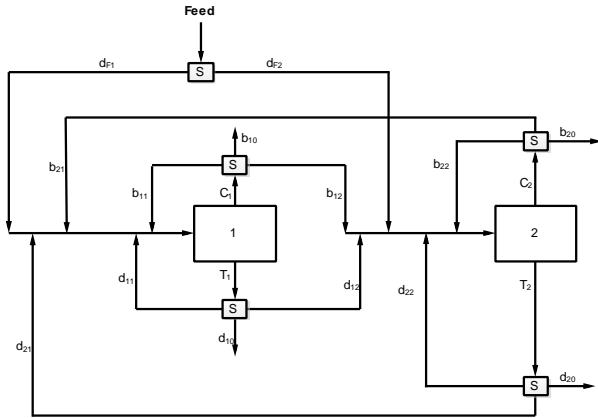


Figure 1. All possible configurations, regardless of the process physics, for circuits with two cells/banks.

3.1. Flotation circuit configuration optimization

Most of the optimization methods have been applied to the flotation process. Three basic parameters in each optimization process include the objective function, decision variables, and constraints. Considering the wide range of these three parameters in flotation circuits, numerous studies have been conducted in this field. A variety of objective functions such as recovery, separation efficiency, profit, cost, and variety of decision variables such as the connection of banks, residence time, number of cells, kinetic constants, as well as different constraints such as minimum grade, mass balances, unwanted elements, and environmental issues are considered. Basic steps for the optimization of the circuit configuration are shown in Figure 2. After modeling the circuit, one of the search algorithms is used to find optimal values. Since most of previous studies for circuit configuration optimization have used mathematical programming strategies and genetic algorithms; first, these two approaches are reviewed and then other less-used methods will be reviewed.

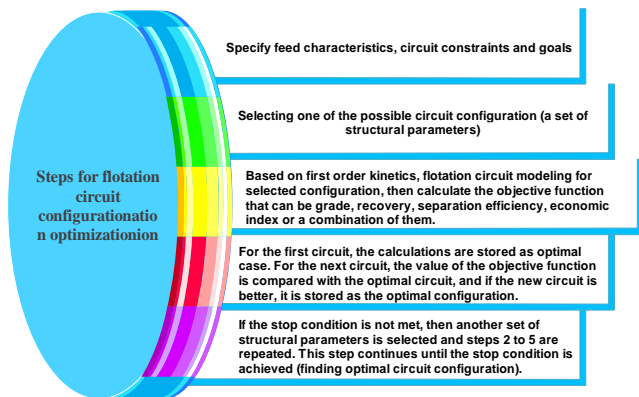


Figure 2. Details of steps to optimize the configuration of the flotation circuits [25].

3-1-1. Mathematical programming

To use a mathematical programming, the problem must first be converted into a mathematical model. To do this, we need an objective function for maximization or minimization. The mathematical model must also include decision variables affecting the objective functions and limiting constraints. By introducing a superstructure for all possible flotation circuit configuration such as Figure 1, it can formulate a circuit

optimization problem in a mathematical programming approach. Depending on the mathematical model, one of the linear programming (LP), mixed integer nonlinear programming (MINLP) or mixed integer linear programming (MILP) can be used to solve the optimization problem [6, 27, 47-50]. As shown in Figure 5, the steps to use the mathematical programming approach are to collect data and define the objective function, the mass balance, and the application of the mathematical programming model.

The resulted configuration will be analyzed for the simplest and most feasible circuits. Initial studies on the application of mathematical programming with consideration of structural parameters and enrichment factor used for flotation circuit modeling and optimal layout was obtained using linear programming [47]. Since the use of the economic objective function leads to the creation of a variety of nonlinearities in the circuit optimization problem [51, 52], the problem of nonlinearity was decomposed to a number of problems and then solved with MINLP [51]. In another study, two hierarchical superstructures were developed to represent a flotation circuit [6, 27]. This breakdown of the optimization problem facilitated the linearization of the economic objective function using the Taylor series so that the conditions for using the MILP were provided to optimize the flotation circuit configuration. Through the use of MINLP and MILP, various layouts and equipment, such as banks with a different number of cells, regrind units, and flotation columns were considered in circuit layouts.

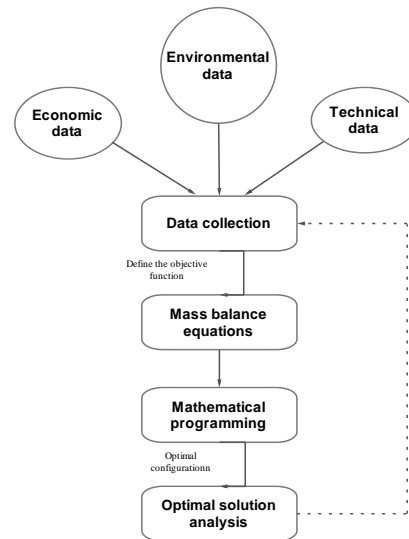


Figure 3. Optimization with mathematical programming [53].

3-1-2. Genetic algorithm

A genetic algorithm is an effective search method in vast and large spaces, which ultimately leads to the direction of finding an answer [54]. The use of GA to optimize the flotation circuit was initially proposed by Guria et al. [55]. These studies used the first rating approach developed by Mehrotra to model the flotation process [45]. GA was used to evaluate the layout with a primary population containing all the circuit design for the problem. The authors improved the efficiency of the algorithm by jumping gene adaptation through multi-objective problem-solving. Guria and colleagues obtained optimal solutions superior to the traditional techniques by solving the problem of optimization of two species, two cells, single objective [55]. The overall recovery of the concentrate flowrate was optimized for a desired concentrate and a total constant volume of the cell. Decision variables were the connection parameters between cells and the average residence time. In the following, the optimization of single-objective and multi-objective was used [56]. The functions objectives were: (1) maximizing the recovery of the ore under optimization and (2) maximizing the valuable minerals (grade) in concentrate ore. A fixed total volume restriction of the flotation cell was also used. Since these goals are focused on contradiction, the non-dominated Pareto solutions were

obtained. As a result, the designer can choose the right answer according to his experience. Fluorite flotation circuit was optimized using a multi-objective genetic algorithm elitist [26]. The phenomenological model of Mehrotra was used to model the plant in the absence of the regrinding section [45]. For a specific arrangement of the circuit and a set of operational data of the plant, the feed identification parameters were first determined using SGA-II. These parameters were then used to solve a two-objective optimization problem using NSGA-II-mjG to improve the plant efficiency. It was found that stream splitters in the proposed circuits could significantly improve the product recovery.

A genetic algorithm was used to simplify the circuit and optimize its efficiency [25]. Entrainment was considered in addition to the first order kinetics. The genetic algorithm was used for two optimization examples with the aim of achieving optimal concentrate grade for a given cell volume. Comparison of results with previous work showed that the proposed genetic algorithm significantly reduced the computational time for a two-stage flotation system, and a simpler circuit with similar efficiency was obtained. As the number of flotation stages increased, the number of circuit configuration increased significantly. It was found that many of the proposed layouts by GA were not logical due to the physical principles of the process and the experience of many years. Due to the random nature of the GA search, this limitation was not unexpected.

A robust GA with pulp and froth phase models was proposed to describe a steady state flotation system [24]. This algorithm and modeling system was used to produce the optimal flotation circuit for a given feed. The results showed that for three cells, the optimal circuit, the circuit with the highest profit from the combination of the grade and recovery, was a condition in which the cells were arranged in a row. For a circuit with 4 to 8 cells, adding cleaner cells resulted in higher incomes. In a recent study, a method for finding an optimal circuit arrangement for a coal washing plant was presented [23]. The objective function was to reach the highest mass recovery and at the same time produce a concentrate with a certain amount (11.2%) of ash. The input to the circuit was identified based on the size fractions and the corresponding rate constant. The results showed that, at a 95% confidence level, the difference between the modeled and measured values was 5.5 - 2.9% for mass recovery and 1.0-1.0% for the amount of the ash in the concentrate. When the proposed circuit configuration was implemented by GA at the plant, mass recovery from the initial value of 57.6% reached to 64.3%, while the amount of ash remained at an acceptable limit (10.9%). By adding one stage to the current three-stage circuit, it was predicted that mass recovery could increase by 3.8% while the quality of the concentrate was maintained in the desired range.

### 3-1-3. Other methods

Computational methods include direct and indirect search that have been used for the optimization of the circuit configuration [45]. Indirect search is trying to find local optimum by solving a set of nonlinear equations by assigning zero to the gradient of the objective function [57]. An optimal position can be found by limiting the search to zero gradient positions in all directions. Direct search methods (hill climbing) are local optimal search by following the objective function in the direction related to the local gradient [58]. However, both methods hunt only optimal points in the vicinity of the current situation, which depends on the initial guess in multiple optimal problems. As a result, there is no guarantee for global optimization, but local optimization can be used as a design guide. Enumerative method, search for any possible location in the search space (evaluates each possible arrangement for circuit optimization) and is therefore, inefficient [54]. This method was used for small-scale searches and guaranteed optimal global circuit design with an economic objective function [52, 59]. This method is limited to solving small problems. In two separate studies, a method for optimizing flotation circuits was introduced using the principles used in distillation columns [12, 60]. The author introduced a new structure for non-homogeneous flotation modeling based on the concept of enrichment functions. This function can be compared with the principles of flotation modeling, although the rate constant and residence time are not directly considered. In this method, the recovery

relations are similar to those of the chemical equilibrium based on the concentration. In the case of flotation, the concentration of mineral in the concentrate is shown as a function of feed concentration and mass recovery, which can be related to operational parameters, such as air flow for the amount of chemicals used. By evaluating the partial derivation of the enrichment function, an optimal analysis for the counter-current flotation system is obtained. McCabe Thiele Staircase Diagrams are used to interpret the optimization results.

### 3.2. Circuit analysis

Circuit analysis was introduced by Meloy in 1983 as a general technique for assessing the effect of different configurations of unit operations in the mineral processing and coal washing circuits [18]. The principles of circuit analysis were used to evaluate the dynamic models of flotation cells [16]. A dynamical model, lumped parameter for continuous flotation circuits was developed to study the transient responses and frequencies of co-current and counter-current flotation banks. Although both circuits were sensitive to low-frequency sinusoidal inputs, and both were stable, co-current banks were oscillating require large first cell and longer residence time to reach the steady state. In the next study, the general equations of steady state were developed for multi-feed separators with different transfer functions [15]. Using circuit analysis algebra, these general equations were solved in a closed form and used to select the optimal points of unit operations in the circuit. The optimization of the heavy media coal washing circuit including roughers, cleaners, and scavengers was the next research title in the circuit analysis series [61]. It was attempted to answer the question as to whether multi-stage coal washing circuits could be optimized to improve the overall efficiency. The authors state that the rougher-cleaner-scavenger circuits are not common in coal washing, especially in gravity separation circuits. Their investigations showed that the best condition was obtained at low separation sharpness values. In addition, by increasing the scavenger density and decreasing the cleaner density, better conditions can be achieved. The circuit analysis method continued with two alternative approaches for designing the circuit layout. Both methods origin from a similar theory of the circuit analysis. However, efforts were made to reduce the difficulties of the circuit analysis. The first method is to accurately define the separation functions in the circuit, namely rougher, scavenger, and cleaner [62]. According to their definition, a rougher is a unit whose feed is circuit feed, a cleaner is a unit that is fed by the concentrate flow, and a scavenger is being fed by tailings stream. In this study, the authors used the circuit analysis to represent four equivalent circuits, each of which represents a higher degree of detail consideration. The second circuit design approach defines three circuit design criteria: (1) the number of steps required; (2) the stage where the feed is introduced; and (3) the flow pattern of the product [13, 63]. This approach begins with the assumption of a generic cleaner of unspecified size, in which the concentrate is serially transferred to the next unit and the waste streams are returned to the previous point in the circuit so that the flow of the return stream is larger or is equal to the grade at the entry point, a law initially proposed by Tagarat [64]. By constructing this general structure, three design criteria can be solved algebraically on the condition that four design / operational parameters are identified: (1) a desired overall recovery, (2) the total ratio of the product to the tailings, (3) the ratio of the product to the waste that can be obtained per unit, and (4) the proportion of the feed component. These algebraic functions guide the initial design of the circuit because they naturally produce non-integers numbers. By rounding out and examining the different combinations of numbers, a design criterion that ends in desirable results can be determined. This configuration consists of valid designs that can be used in optimization and design process. In another study, with some changes, the circuit analysis method was used to find feasible designs [65]. In this study, the weaknesses of previous works, such as the assumption that the transfer function is uniform for each unit, changes the return flows to the waste stream at the time of the uncertainty of the return parameter and the lack of a standard method for the non-integers data were noted. In the new approach, the total

number of steps for the entire plant is first defined. The author assumed that each circuit had one rougher stage and an unspecified number of steps from scavenger and cleaner. The number of units in each stage is independently calculated by using the associated equation of tailings characteristics to the number of stages of the scavenger and the concentrate characteristics to the number of cleaner steps. Evaluation and correction of a gravity separation circuit were performed using circuit analysis method [4]. Using the principles of circuit analysis, the circuit configuration was simplified by reducing the number of spirals from 686 to 542. In addition, the new circuit was able to produce higher material grade at a higher recovery. In the original circuit, the materials were to be circulated 7 times in a 93% recovery to produce a certain grade. After the modification, the resulting circuit was able to achieve a desirable grade with one pass at the recovery of 94.74%. In other studies, the improvement of efficiency was achieved by implementing the principles of circuit analysis for spiral separation of coal [10] and column flotation [66]. Recently, a circuit analysis method has been used to the sensitivity analysis of the circuit [67]. In this study, the sensitivity analysis was introduced as a balance between experience and heuristic vision and numerical optimization strategies. Given that it is almost impossible to achieve global optimization through the experiment; a sensitivity analysis was used to determine the units with the highest sensitivity in the circuit. After identification of these units, empirical and laboratory analysis can be used to optimize or improve the efficiency of these units. Regarding the difficulties in circuit analysis calculations, a matrix-based algorithm was developed to obtain analytical and numerical solutions for circuit [7]. The main advantage of this technique is its numerical application that allows simple simulations in excel without performing iterative operations.

#### 4. Discussion

In this section, in addition to examining the questions raised in the introduction section, the advantages and disadvantages of the methods used in previous studies are also evaluated. In response to the question of whether there is a certain number of separation configuration(s) that is more efficient, it is better to consider the history of using mineral processing circuits. In the traditional perspective, separation circuits have been formed from two or three stages under the name of roughers, scavengers, cleaners, and other stages. Over the past years, the study of flowsheets used for various types of minerals including sulfide ore, coal, and even industrial minerals [68] have shown that counter-current separation circuit configuration can be used to process all kinds of minerals. It was found that for the separation of minerals, especially sulfide ores, usually 2 to 3 stages and maximum 4 separation stages is enough, due to the high separation factor of these minerals [19]. In these circuits, scavenger concentrate or cleaner tailing, the middle material, are returned one stage backward. The use of these circuits in the circuit analysis method that is based on the separation sharpness criterion has also been emphasized to assess the separation circuit configuration. The main advantage of the circuit analysis technique is that the resulting equations for describing process units are assumed linearly, which can be solved without using numerical methods. Circuit analysis evaluates the circuit arrangement by considering a selection or transfer function for each separation unit. This method provides great freedom for conducting a variety of investigations on the separation circuits. The main limitation of the circuit analysis method is the linear assumption. This assumption states that, regardless of the composition of feed or tonnage, the R value will remain constant [4]. In fact, this assumption is invalid because higher feed flow rate is effective on the operational behavior of separation. However, when designing a plant, it is possible to calculate the feed flow rate into a unit operation and then select a large enough equipment to meet the calculated tonnage. As a result, the designed circuit will show a linear behavior. Hence, this method is valid for designing new circuits. Previous researches have shown that almost all equipment is used in a linear range. Also, laboratory and industrial studies confirming the assumption of linearity in many examples [9, 10, 69, 70]. In the circuit analysis approach with the separation sharpness criterion for a given number of stages, the counter-current circuits

always have the highest separation sharpness. As a result, the optimal circuit is counter-current with return flow to the previous stage. One of the problems encountered in circuit analysis is that in some situations the circuit separation sharpness is the same, making it difficult to recognize the proper circuit [65]. For example, as shown in Table 1, the separation sharpness in the counter-current rougher-scavenger and rougher-cleaner circuits is 1.33.

However, other objectives such as recovery, grade or economic indicators can be as effective as the separation sharpness for assessing the separation circuit configuration. As a result, optimization methods of the flotation process configuration were introduced and have been used based on the objective function, decision variables, and various factors. Previous works used the structural parameter method to display the configuration options. One of the challenges of optimizing the circuit configuration with the structural parameter method is to create large streams and to search for many infeasible responses. In addition to generating computational costs, this often leads to complicated or even non-operational circuits. The common feature of the previous work is to use the flotation bank model and the structural parameter method to incorporate configuration options in the model. In this method, each output stream from a bank is connected to a splitter unit (Figure 1). The splitter divides the flow into the same parts and sends it to other banks or outgoing flows in the circuit. In this approach, mathematical formulation requires the use of complex equations and considerable effort to solve these equations. Also, optimum circuits may include a large number of stream splits or returns, and as a result, the circuit may have been complicated for application in operation.

Linear programming methods require an objective function and linear constraints to optimize the flotation circuit arrangement. While, in real-world situations, most industrial issues have a non-linear nature. As a result, the Taylor series was used to linearize the nonlinear objective function [6, 27]. This approach leads to more complex equations and optimization problems. The linear programming method can be used only to calculate a single goal, such as maximizing the profit or minimizing the cost. While, in today's dynamic environment, there is no single goal for all issues. In this case, genetic algorithms can be used to optimize the multi-objective problems [55, 56]. Genetic algorithms easily fit simulations and existing models. But specific optimization problems cannot be solved by a genetic algorithm. This is because of the poor recognition of fitness functions that produce bad chromosomes despite the fact that only good chromosomes do the crossover. There is no guarantee that genetic algorithms can be optimally found the global solution. This is often the case when the population has a lot of limitations. Like all other artificial intelligence techniques, the genetic algorithm cannot guarantee the constant optimization response time. Even more, the difference between the shortest and the longest optimization response times is much larger than the classical gradient methods. Table 1 presents recent studies conducted to optimize the arrangement of flotation circuits. The success of the configuration optimization approach requires the availability of an efficient optimization algorithm to work with a large number of variables and to handle multiple nonlinear constraints. Also, the correct definition of the objective function [71] plays an essential role in achieving an efficient configuration. One of the criticisms of the use of counter-current circuits [72] is the lack of consideration of the flow splitter into mineral separations circuits. A review of the published work has not provided any sign of the industrial use of the tailings or concentrates flow splitters in the plants. Similarly, studies [27] showed that stream split does not have any effect on improving the efficiency of plants. Over the past years, various circuit configurations have been proposed by optimization methods. The proposed optimal circuits are presented in Figure 4 using a mathematical programming approach ( $a_1$  to  $a_4$ ).

Counter-current circuit contains two stages of cleaner and one scavenger stage is shown in Figure 4a in order to compare with other proposed circuits with four separation stages. As seen, several types of circuit configuration are proposed in terms of the objective function and the desired constraints for the 4-stage circuit. In all circuits, except the  $a_1$ , two cleaner stages and one scavenger stage are used. In addition, the difference between the  $a_1$  and  $a_3$  circuits is at the point where the

scavenger concentrate and the first cleaner tailings are returned. The proposed circuits by the genetic algorithm search method are represented in Figure 5. Circuit b is a counter-current circuit consisting

of two stages of scavenger and one cleaner stage. Some of the GA suggested circuits are based on the designer's experience through eliminating the split streams.

**Table 1.** Recent research on optimization of flotation circuit configuration with mathematical programming and genetic algorithm.

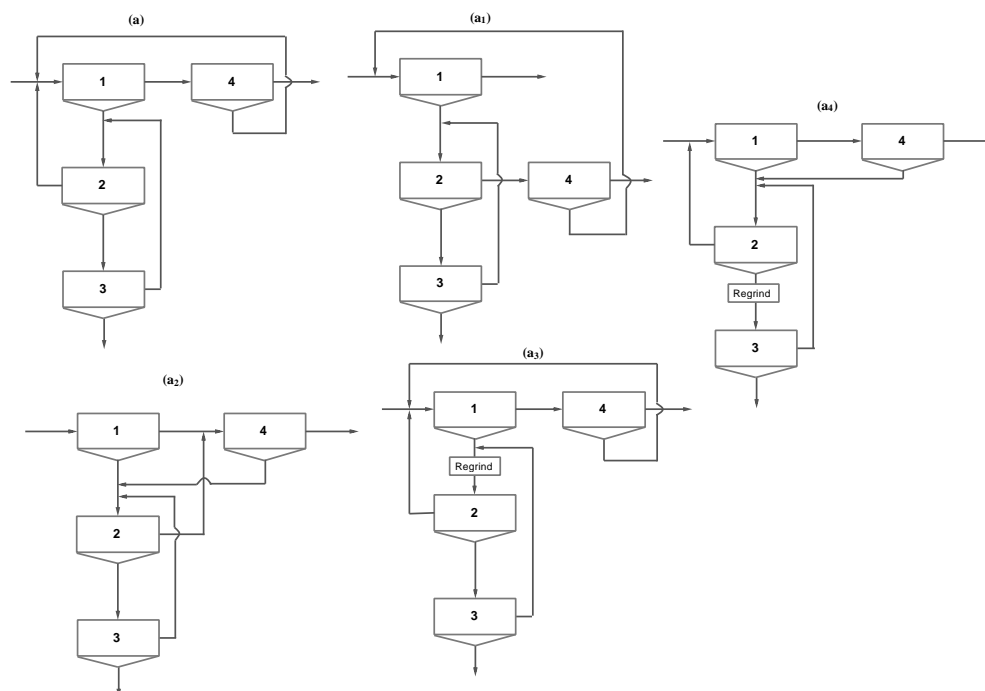
Technique	Year	Highlights	Weakness	Source
Mathematical programming	2004	Introducing the MILP model for flotation circuit design		[6]
	2006	MILP application for designing flotation circuit containing regrinding and column flotation	Extremely complex and many equations involved, difficulty in linearization, high dependence on flotation models, single objective	[27]
	2009	MILP Application for flotation and classification circuits		[73]
	2014	Investigating the objective function role in the design of the flotation circuit		[71]
	2016	Mathematical optimization based on laboratory data		[22]
Genetic Algorithm	2005	Introduction of a genetic algorithm for flotation circuit configuration Optimization		[55]
	2006	Multi-objective optimization flotation circuits	Require high memory and huge computations no guarantee for the optimal solution, weak mathematical support, different responses for different runs	[74]
	2009	Determining the optimal configuration of the circuit with many stages		[26]
	2011	Combination of genetic algorithm and circuit design rules		[25]
	2013	Optimization of flotation circuit with pulp and froth phase models		[24]
	2014	Pareto optimization using a genetic algorithm		[23]

As the number of separation stages increases, an optimization approach often faces the difficulty of finding the optimal configuration. For example, for the niobec flowsheet [75], with a large number of stages and a combination of different unit operations (Figure 6), the application of a superstructure approach and consideration of all possible configuration, in addition to the huge volume of computations, will have difficulties in interpreting the results of the problem.

A review of previous studies found that [26] and [4] evaluated the configuration of circuits with a large number of stages. In the first case, the arrangement of a circuit containing 15 separation units was evaluated using the genetic algorithm [26]. The resulting circuits are complex and have stream splits which were simplified according to the designer's experience. The second case was a 19-unit circuit, which was simplified by circuit analysis and separation sharpness criteria. The modified circuit has a lower number of streams and stages, while recovery and efficiency were higher.

## 5. Summary and conclusion

A review of previous work has shown that one of the challenges of optimizing the circuit configuration is the creation of a large number of streams and the search for many unnecessary points. In addition to generating computational costs, this often leads to complicated or even impractical circuits. In contrast, the circuit analysis approach and considering the transfer function for the unit operations in the separation circuits provide the ability to perform a variety of circuit evaluations. In order to find the suitable and applicable configuration for circuits with a large number of stages, it is possible to first calculate feasible designs based on counter-current circuits. Also, reliable results will be provided by reducing the number of search point in addition to saving the volume of calculations.



**Figure 4.** four stage counter-current circuit (a) and optimal circuits from mathematical programming (a<sub>1</sub> and a<sub>2</sub>) [6]; (a<sub>3</sub> and a<sub>4</sub>) [27].

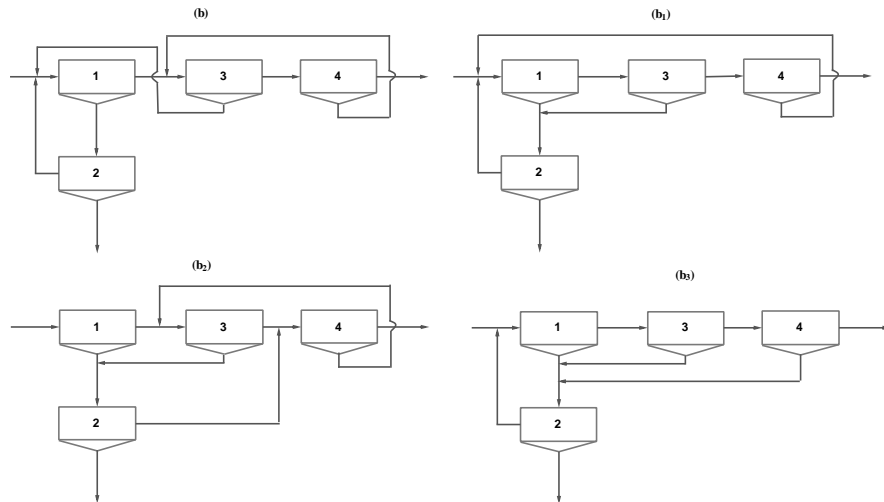


Figure 5. Counter-current 4 stage circuit (b) and optimal circuits proposed with genetic algorithm (b<sub>1</sub>) [74]; (b<sub>2</sub>) [25]; (b<sub>3</sub>) [24].

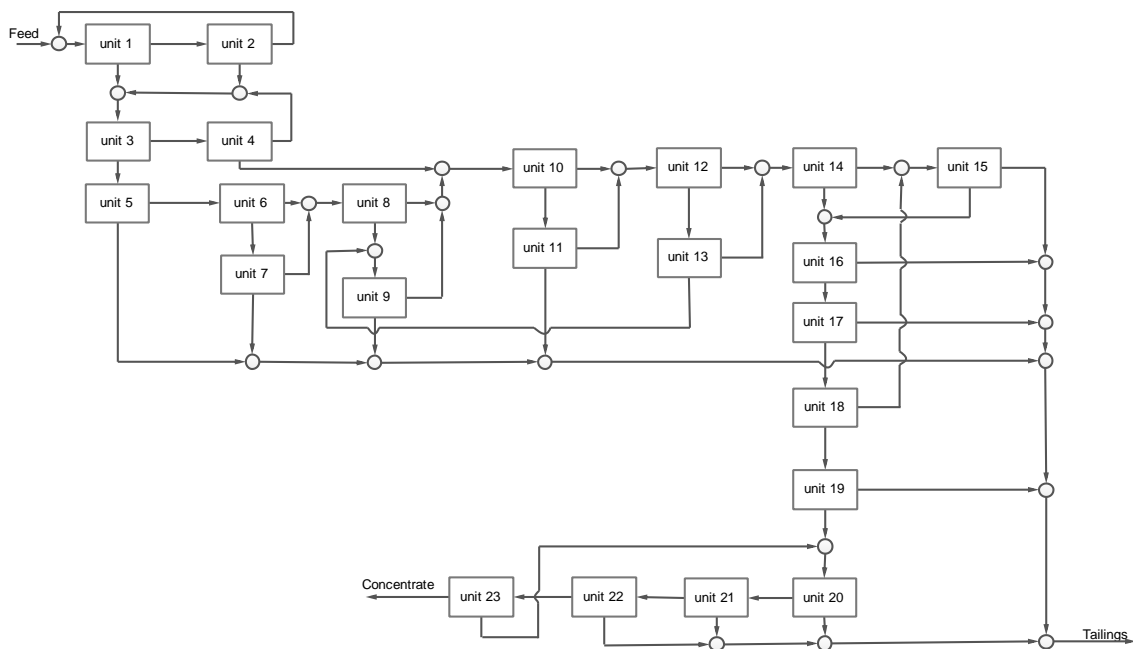


Figure 6. Schematic flowsheet of niobec processing plant.

### Acknowledgment

This research was funded by a Ph.D. grant of University of Tehran. We are grateful to the colleagues and professors at University of Tehran for sharing their pearls of wisdom with us during the course of this research.

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