A Statistical-Probabilistic Pattern for Determination of Tunnel Advance Step by Quantitative Risk Analysis

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ABSTRACT

One of the main challenges faced in designing and construction phases of tunneling projects is the determination of maximum allowable advance step to maximize excavation rate and reduce the project delivery time. Considering the complexity of determining this factor and unexpected risks associated with inappropriate determination of that, it is necessary to employ a method which is capable of accounting for interactions among uncertain geotechnical parameters and advance step. The main objective in the present research is to undertake optimization and risk management of advance step length in water diversion tunnel at Shahriar Dam based on uncertainty of geotechnical parameters following a statistic-probabilistic approach. In this research, in order to determine optimum advance step for excavation operation, two hybrid methods were used: strength reduction method-discrete element method- Monte Carlo simulation (SRM/DEM/MCS) and strength reduction method- discrete element method- point estimate method (SRM/DEM/PEM). Moreover, Taguchi analysis was used to investigate the sensitivity of advance step to changes in statistical distribution function of input parameters under three tunneling scenarios at sections of poor to good qualities (as per RMR classification system). Final results implied the optimality of the advance step defined in scenario 2 where 2m advance per excavation round was proposed, according to shear strain criterion and SRM/DEM/MCS, with minimum failure probability and risk of 8.05% and 75281.56 $, respectively, at a confidence level of 95%. Moreover, in either of normal, lognormal, and gamma distributions, as the advance step increased from Scenario 1 to 2, failure probability was observed to increase at lower rate than that observed when advance step in scenario 2 was increased to that in Scenario 3. In addition, Taguchi tests were subjected to signal-to-noise analysis and the results indicated that, considering the three statistical distributions of normal, lognormal, and gamma, under the scenario with poor section, effect of normal stiffness of joints on excavation advance step was larger than that of other parameters. Accordingly, as the RMR quality increased, normal stiffness of joints decreased, so that deformability modulus became more significant.

Keywords: Advance step, SRM/DEM/MCS, SRM/DEM/PEM, Risk analysis, Taguchi

1. Introduction

In last two decades, tunneling projects have had a considerable growth all over the world. Many of such structures are excavated into mediums of complex geological conditions with high degrees of uncertainty in rock mass parameters and poor quality of the rock under extensive stresses resulted by excavation operation [1]. For this reason, rock mass is generally heterogeneous and required information for designing underground excavations are mostly extracted from site investigations and laboratory tests; however, the amount of data to be obtained from such sources is very limited. Therefore, a large distribution of ambiguity has encompassed underground excavations [2]. Nevertheless, in order to analyze and design tunneling projects, statistical and probabilistic estimations can be useful as they can account for a wide spectrum of geotechnical parameters affecting the designs, specially advance step design. Accordingly, based on uncertainties associated with rock mass parameters, it is extremely important to determine appropriate values of the parameters at certain level of confidence when dealing with design and stability of structures excavated into rock masses, such as tunnels.

Accordingly, in underground space engineering, risk sources and accidents are generally resulted from either geotechnical uncertainties or errors [3]. As such, inappropriate designs may end up with unexpected risks such as reduced stability of tunnels. In order to prevent such problems, managers should implement safety risk management (risk identification, control and assessment) in underground projects [4]. Meanwhile, in recent years, geotechnical risk management has been promoted as an important issue in engineering geology and considering increased complexity and scale of geotechnical projects [5].

Excavation step length represents one of the important parameters in tunnel design which can lead to dangerous risks in cases where it is failed to be appropriately determined. In general, excavation step length is defined as the unsupported distance or length of span which is constructed in a single stage [6]. In preliminary tunnel design, excavation step should be determined as one of the key factors. Inappropriate excavation step can result in damages due to tunnel wall and roof displacements, which will continue to the back of tunnel face [7]. However, being difficult to determine, this factor is often determined based on expert judgments and designer’s experiences, and in cases where wrong decisions are made in this regard, unexpected risks will threaten the structure (e.g. reduced tunnel stability, economic losses due to tunnel failure, and over-maintenance). Advance step is closely related to the time and cost tunnel construction via stage excavation. Accordingly, optimally determining the excavation advance step represents one of the main challenges always faced in tunnels [2].

On this basis, design methods will be highly valuable where

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uncertainties and their resultant effect are taken into account. These methods are known as reliability assessment methods. In a general classification, reliability assessment methods can be classified into three classes: analytical methods, approximations, and simulations. Of these, Monte Carlo simulations (because of its vastness in undertaking different analyses) are more popularly used.

Numerous studies have been dedicated to the determination of excavation advance step in tunneling projects based on numerical modelling. Among others, one can refer to the study by Vermeer who utilized PFC software to validate input data into triaxial tests software to investigate face stability with different step lengths [8]. Within the same scope, Li and Shobert built physical models to determine the mechanism of the relationship between tunnel convergence and excavation step length. In this study, determination of the round length was investigated for tunnels in weak rock, where the behavior is not governed by discontinuities. This study focuses on shallow or medium depth tunnels so that squeezing or rock burst is not concerned. The behavior mode of the face and unsupported span was investigated by a series of small scale model tests and PFC3D analyses. Total five types of behavior modes are suggested for planning of excavation and support. Based on the results from PFC3D analyses, the equivalent models were analyzed by a FDM code, using elastic material behavior. The results are illustrated in the 'Conditional chart for excavation plan in weak rock tunneling' which shows the relationship between the safety factor and relevant behavior mode as the round length varies [9]. However, determination of tunnel advance step based on statistical analyses and risk probabilities has been rarely considered by researchers. In this regard, one may only refer to the study by Yu et al. where a solution is proposed to determine excavation advance step for designing a tunnel based on quantitative risk analysis with an emphasis on tunnel construction cost assessment from data obtained in similar projects to the one under consideration (which itself can introduce errors into risk calculation results) [2]. Moreover, statistical distribution of uncertain input parameters are solely taken as being normal; but other statistical distributions should also be further considered. Apart from that, so far, numerous studies have been conducted on sensitivity analysis of geotechnical parameters to deformation and stability of the rock mass surrounding the tunnel. However, in many of such studies, sensitivity analyses are not undertaken on multiple parameters at the same time, and the effect of distribution function of uncertain input parameters are not considered as well, so that just a few studies have accounted for such phenomena.

The results of a research by Hoy and Zhang showed that the modulus of elasticity served as the most effective parameter on subsidence and hence stability and displacements occurred around the tunnel [10]. A finite element model of variable parameters was established according to the geological conditions of the Galongla tunnel. Using deformation monitoring data, back analysis was performed for the mechanical parameters including elastic modulus, Poisson's ratio, cohesion, internal friction angle and lateral pressure coefficient in order to get the real values of surrounding rock. Then a sensitivity analysis was performed to determine the most effective geo-mechanical parameters. The results showed that, for vault settlement, the elastic modulus was more sensitive than the other parameters. For the surrounding convergence which was opposite to the vault settlement, the Poisson's ratio, cohesion and internal friction angle were more sensitive than the elastic modulus. Su and Sanky undertook sensitivity analysis of geo-mechanical parameters on tunnel stability considering excavation-damaged zone using partial factorial design (a design of experiment methodology) in FLAC 2D Software. They undertook the mechanical stability of an underground opening using FLAC, which is a two-dimensional modeling code, with a consideration of EDZ. A sensitivity analysis was also carried out with fractional factorial design. Modeling showed that the behavior and the stability of an underground tunnel were strongly dependent on the existence of the EDZ. The sensitivity analysis showed that the key parameters affecting the factor of safety around the tunnel were in-situ stress ratio, depth, cohesion, reduction ratio, internal friction angle, and height and width of the tunnel. It is necessary to consider the EDZ, which can significantly affect mechanical stability in tunnel design [11]. The stability analysis of Huocheling tunnel introduced the plastic-elastic mechanical analysis of surrounding rock stress-strain state, and explained the sensitivity of mechanical parameters (\(E, \sigma_t\)) influencing the surrounding rock stability according to FLAC3D numerical simulation software. According to the results of sensitivity analysis, modulus of deformability and friction angle were found to be relatively more effective than the other two parameters namely Poisson's ratio and cohesion force [12]. In all of aforementioned papers, only one parameter was changed at a time, keeping all other parameters constant. In cases the parameters interact, the results are inaccurate.

Following a quantitative approach to risk for underground designs, one can attain a better understanding of relative likelihood of risks arisen in the course of construction phase, so as to optimize excavation advance step in terms of both safety and economy. In the current research, a solution is proposed to determine appropriate excavation step in tunneling projects based upon risk analysis via two hybrid methods, namely strength reduction method- discrete element method-Monte Carlo simulations (SRM/DEM/MCS) and strength reduction method- discrete element method, point estimation method (SRM/DEM/PEM), where associated uncertainties are taken into account; the solution can be used for designing underground structures. In this probability-based approach, of all rock mass parameter, modulus of deformability, cohesion, internal friction angle and compressive strength, and of all discontinuity parameters, normal stiffness of joint sets were chosen as uncertain random variables and used in evaluation of the probability of failure at Shahriar water diversion tunnel in response to different advance step lengths and hence associated risks with each step length.

Further in the present research, effect of the most important geo-mechanical parameters of rock mass on advance step length at Shahriar water diversion tunnel were investigated across sections of good, fair and poor quality (as determined per RMR classification). Effects of changes in type of statistical distribution function assumed for each uncertain input parameter on Taguchi test index and thereby on advance step length were further analyzed. For this purpose, simultaneous analysis of the relationships among the parameters affecting rock mass behavior was undertaken; reportedly, this has been rarely addressed in previous studies. One of the methods which can be used to address this imperfection and simultaneously analyze effects of different geotechnical parameters is the design of experiment (DOE) methodology. Indeed, the main reason for using this methodology is that it allows user to vary multiple geotechnical parameters at the same time in a single test. For this purpose, both of under study sections were modeled in 3DEC Software into different classes. Then, DOE processes were carried out using Taguchi methodology. Finally, performances of Monte Carlo simulations and Rosenblueth's point estimation method in evaluating failure probability and risk assessment under the effect of different statistical distributions were compared in terms of both displacement and shear strain.

2. Analysis Methods

2.1. Quantifying uncertainty in geo-technical parameters

Developments of in-situ and laboratory tests, significant uncertainties in parameters of rock mass and intact rock are well understood. In order to present accurate regional geological models, these uncertainties should be quantified using different probabilistic techniques and then accounted for in design process [13]. Soil and rock represent heterogeneous and anisotropic media defined by a series of geometrical, mechanical and strength parameters. It may be the case that these parameters exhibited different values at two different points of the same soil or rock mass, making it impossible to express such parameters as exact values, unless measurements are carried out at every point across the soil or mass, which is practically impossible to undertake. These parameters will always have values which are impossible to exactly characterize. This suggests that these parameters need to be defined via a statistical process [14].
In the present research, multiple uncertain geotechnical parameters (cohesion, modulus of deformability, internal friction angle, uniaxial compressive strength, and normal stiffness of joints) are quantified and statistically analyzed using Hoek-Brown relationship and empirical correlations.

### 2.1.1. Hoek-Brown failure criterion

Hoek-Brown criterion was used to determine uncertain parameters in this research. In order to accurately analyze geotechnical designs such as underground excavations, one should begin with attaining a reasonable evaluation of mechanical behavior of rock mass which are expressed using strength and deformability parameters. In this regard, some empirical criteria are proposed for special cases. Of these criteria, Hoek-Brown failure criterion has been widely used in rock masses [15]. This empirical failure criterion is expressed as a nonlinear relationship between rock mass strength and principal stresses, with its general form given in Eq. (3) [16].

\[
\sigma_i = \sigma_3 + \sigma_2 \left(m \sigma_\tau + \sigma_3 \right)^a
\]

\[ m_b = m_h \exp \left( \frac{D}{150} \right) \]  

\[ S = \exp \left( \frac{-D}{150} \right) \]

\[ a = \frac{1 + \frac{1}{2} \left( 2 \mu - e^{-\frac{\gamma c}{k}} \right)}{6} \]

In above relationships, \( \sigma_i \) is the maximum effective principal stress, \( \sigma_3 \) is the minimum effective principal stress, \( \sigma_2 \) is uniaxial compressive strength of intact rock, GSI is geological strength index, D is turbidity index, and \( m_b \) is the intact rock constant. The value of \( m_h \) can be obtained from specific tables based on rock structure (lithology, mineralogy, and rock texture), \( m_h \) is a constant which depends on the state of rock mass, S and a are other rock mass parameters, with \( \gamma c \) being crushing index of the rock. The value of uniaxial compressive strength of the rock can be obtained from Eq. (5).

\[ \sigma_{cmass} = \sigma_{ci} \times S \]

Moreover, Eq. (6)-(8) are proposed for calculating internal friction angle, cohesion and strength, respectively [16].

\[ \phi' = \sin^{-1} \left( \frac{6 \sigma_3}{(1+a)(2+a)+6 \sigma_3} \right) \]

\[ \gamma' = \sigma_{ci}(1+a+2a) + \sigma_3 \left( \sigma_{ci}+\sigma_3 \right)^{a-1} \]

\[ \sigma_{cm} = \sigma_{cmass} \]

Where \( \sigma_{cmass} \) denotes upper bound of confining stress; in tunnels, it can be obtained from Eq. (9):

\[ \frac{\sigma_{cm}}{\sigma_{cmass}} = 0.47 \left( \frac{\sigma_{cm}}{H} \right)^{-0.94} \]

Where \( \sigma_{cm} \) represents total rock mass strength, \( \gamma \) is rock mass specific gravity, and H is the depth from ground surface. In cases where horizontal stress exceeds vertical stress, the horizontal stress replaces \( \gamma \). Finally, \( \sigma_{cm} \) can be calculated by Eq. (10).

\[ \sigma_{cm} = \sigma_3 \left( \frac{6 \sigma_3}{(1+a)(2+a) + 6 \sigma_3} \right)^{a-1} \]

Considering the importance and effect of statistical distribution of uncertain input parameters on determined advance step length, three popular statistical distributions, namely normal, lognormal and gamma distributions, are briefly introduced in the following sections.

### 2.2. Random variable distribution function

#### 2.2.1. Normal Distribution

Normal distribution is among the most important continuous distributions used in statistics. It is widely used as many natural phenomena follow this distribution.

Probability density function for normal distribution is expressed in Eq. (11) [17].

\[ f(x; \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

Where \( \mu \) is the position parameter with its value being equal to mean, and \( \sigma \) is the scale parameter with its value being equal to standard deviation.

#### 2.2.2. Lognormal Distribution

In some cases, continuous variable do not exhibit normal distribution and rather follows a lognormal distribution. Probability density function for this distribution is given below [17]:

\[ f(x; \mu, \sigma) = \frac{1}{x \sqrt{2\pi \sigma^2}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \]

Where \( \mu \) is the position parameter and \( \sigma \) is the scale parameter, with mean and standard deviation of this distribution being calculated by Eq. (13) and (14), respectively.

\[ E(x) = e^{\mu + \frac{\sigma^2}{2}} \]

\[ St.D. = \sqrt{e^{2\sigma^2} - 1} \]

#### 2.2.3. Gamma Distribution

If the variable X exhibits the following probability density function, it is said to be of gamma distribution [17]:

\[ f(x; a, b) = \frac{b}{x^a} e^{-bx} \]

Where a is the scale parameter and b is form parameter. Mean and standard deviation of this distribution can be calculated by Eq. (16) and (17), respectively.

\[ E(x) = ab \]

\[ St.D. = \sqrt{ab} \]

#### 2.3. Strength reduction method (SRM)

Commonly referred to as direct strain control, Sakurai’s method has been based upon critical strain parameter. Sakurai presented strain warnings at three levels for a rock, based on the rock’s modulus of elasticity (E). Of these three warning levels, the critical strain resulted by the warning level II is suggested as a basis for engineering designs. Allowable strain can be determined by Eq. (18) – (20) [18].

\[ \log c = -0.25 \log E - 0.85 \]

\[ \log c = -0.25 \log E - 1.25 \]

\[ \log c = -0.25 \log E - 1.50 \]

Where E is in kg/cm^2, and \( c_e \) is the critical shear strain under unconfined compression around the tunnel.

Sakurai et al. proposed Eq. (21) and (22) for calculating three-dimensional critical shear stress (\( \gamma_c \)) and critical displacement, respectively [19].

\[ \gamma_c = (1 + u)\sigma c \]

\[ \epsilon_c = \frac{c}{\gamma_c} \]

Where \( u \) is critical displacement and a is tunnel radius.

In strength reduction technique, cohesion and internal friction angle at given safety factor are calculated via Eq. (23). Then, a new model is developed using newly calculated values of \( C \) and \( \phi \). Once the model was balanced, excavation is initiated at a specific advance step length. Afterward, shear strain of the considered model is compared against Sakurai’s allowed shear strength. If the shear strength is lower than the allowed value, the procedure will be repeated with a longer step length; otherwise the procedure will be stopped.

\[ FOS = \frac{tan \phi}{tan \psi} = \frac{C}{\gamma_c} \]
In fact it could be stated that the reliability analysis methods, transfer uncertainties associated with the input parameters to the function’s output value. In other words, the impact of input parameters' uncertainty is considered in the output value. This method has two advantages: first, it allows the designers to manage ambiguities present in the design and computations in a logical way and therefore accurately design or determine the sensitivity associated with different variables of the model. Second, decision making is more desirable and these methods provide a more logical basis for decision making in an analysis which is completely certain.

2.6. Design of experiments methodology

The main objective of the present research is to investigate the most effective geotechnical parameters on advance step length at water diversion tunnel at Shahriar Dam. Reportedly, simultaneous analysis of the relationships among the parameters affecting rock mass has been rarely addressed in previous literatures. For this purpose, here we use DOE.

In general, DOE is composed of 7 main steps [35]: (A) selection of independent variables, (B) selection of test levels for each independent variable, (C) selection of orthogonal array, (D) determination of independent variables across each column, (E) testing, (F) data analysis, and (G) inference.

2.6.1. Taguchi method

Generally, DOE follows either of single factor, full factorial, or partial factorial approach. In single-factor approach, only one factor is changed at a time, with all other factors kept constant. If the factors interact, results of this approach will be inaccurate. In full factorial method, all possible combination of factors will be considered. Although this method can provide extremely accurate results, it requires a great deal of time and money. In partial factorial approach, a limited number of all possible combinations of factors are selected, and once the required tests were undertaken in this selected set of combinations, data analysis is undertaken [36]. As one of the most popular partial factorial methods, Taguchi technique was employed in the present research. In other words, this technique was used as it allows the user to vary multiple geomechanical parameters of the same numerical model at the same time before undertaking data analysis.

Therefore, based on the methods presented in this research, a quantitative approach to risk was developed to assess the effect of associated uncertainty with geological conditions on excavation advance step length in tunneling projects, ground response and tunnel stability performance based upon risk analysis and reliability assessment methods. Implementation process of the newly proposed approach is demonstrated in Fig.1.

3. Project background

Shahriar double curvature storage dam is constructed on Gezel Ozan River, 40 km downstream of Miyaneh City, at North West of Iran (Fig. 2.a). To protect the dam site against floods, a water diversion tunnel was constructed at the left abutment of the dam site with the horse shaped section and the diameter of 14.7 m and length of 514 m (Fig. 2.b).
3.1 Geological Environment

Shahriar Dam site is comprised of volcanic rocks belonging to the Karaj formation of Eocene age, dioritic masses and quaternary sediments. The constituent components of the Karaj formation at the site include Tuff, Agglomerate and Andesite. The dioritic unit which probably has influenced the Karaj formation during Oligocene period comprises the major portion of the diversion tunnel. The dioritic rocks of this formation are green to grey colored and have a fully crystallized porphyritic texture. At the margins of the unit, especially at the river upstream where the entrance of diversion tunnel is located, signs of alteration and formation of hornfels are observed. Also, at the contact surface of this mass and the Karaj formation, parts of the Karaj formation are surrounded by the dioritic masses which have been outcropped by erosion. Totally, less than 60 Km of the tunnel length at the entrance and exit are located in this area. The remaining parts of the tunnel are excavated in the dioritic unit. The geological map together with the rock units’ details and the topography along the longitudinal axis are shown in Fig. 3.

3.2 Study Area

Prior to construction of the tunnel it was predicted that the 350 – 360 m section (scenario 2) would be the most problematic part along the water length of diversion tunnel due to the presence of uncertainties in the tunnel’s geo-mechanical parameters. In this section, five parameters including $C$, $E$, $\phi$, $K_n$ and $\sigma_{c,max}$ have uncertainties which would result into high risks in the design of tunnel support system. Based on the high instability potential of the tunnel, this section of the tunnel was selected to be analyzed as the study area. This section is an ideal one for the risk analysis due to a) variety in the rock mass behavior, b) uncertainties in the parameters and c) this fact that the consequences due to improper support system could considerably affect the project’s budget.

Although the accurate estimation of the rock mass parameters is a way for reliable design and stability of the excavated space, estimation of these parameters is difficult due to the presence of weak discontinuous plates which produce anisotropy. In the stability analysis and design of the support system, measurement of the geo-mechanical quantities of the rock mass is of vital importance, so that accuracy and validity of the obtained results from the analyses are in close connection with the precision employed in measuring these quantities.

In cases the rock mass contains important faults and joints, due to intersection of these discontinuities, some segments of the rock are separated from the main rock in form of blocks or wedges and there is possibility of collapse or sliding. These collapses are the result of intersection of some discontinuity systems after excavation and creation of free faces.

Therefore, prior to any excavation at these areas, first the probable wedges shall be identified, and then, to prevent fall of the lose wedges, the support system shall be designed based on the rock type, weight of the wedge and other geological conditions. Hence the importance of accurate and optimal determination of the support system is understood, so that in case of its precise determination the risks associated with the tunnel collapse and probable damages could be avoided. In Table 1, the rock mass and joint sets parameters of three scenarios of tunnel are shown. All original data of parameters presented in this research are obtained through field investigation and experiment.
4. Application of risk analysis

In this section, utilizing the following three stages which are based on the randomness of rock parameters surrounding the tunnel and the reliability analysis, the failure probability for each scenario of the advance step is determined.

Step 1: Quantifying uncertainty
A) Determination of C, E, Q:

One of the common methods for determining the geo-mechanical parameters of rock mass, like cohesion and internal friction angle, is application of the Hoek - Brown failure criterion. Hoek and Brown believed that a failure criterion is accurate and successful when the following three conditions are satisfied by it: a) A full description of the rock reaction and response against all stress situations. b) Prediction of the randomness of rock parameters surrounding the tunnel and the number of joint sets in full scale. The analytical results corresponding to the rock masses failure criteria for the investigated tunnel sections are obtained using RocLab, 2007 software. The input parameters of RocLab software are [37]: $\sigma_{ci}$, the uniaxial compressive strength of intact rock, $m_i$: The intact rock parameter, D: the disturbance factor. GSI: geological strength index. The rock mass input parameters are presented in Table 2.

<table>
<thead>
<tr>
<th>Equation no.</th>
<th>Equation</th>
<th>Limitation</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>$E_i = \frac{2RMR - 100}{10}$</td>
<td>$RMR &gt; 50$</td>
<td>26</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>41</td>
<td>$E_i = \frac{10^{0.016Q}}{10}$</td>
<td>$RMR \leq 50$</td>
<td>27</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>42</td>
<td>$E_i = 0.1(RMR/19)^3$</td>
<td>$RMR &gt; 30$</td>
<td>28</td>
<td>23.83</td>
<td>18.52</td>
</tr>
<tr>
<td>43</td>
<td>$E_i = \left(1 - \frac{Q}{Q_{ci}}\right)^{10} \frac{Q_{ci}}{Q}$</td>
<td>$Q_{ci} &gt; 100$ Mpa</td>
<td>29</td>
<td>12.68</td>
<td>10.67</td>
</tr>
<tr>
<td>44</td>
<td>$E_i = 10 Q_i^{1.5}$</td>
<td>$Q = Q_{ci}/100$</td>
<td>30</td>
<td>21.79</td>
<td>17.43</td>
</tr>
<tr>
<td>45</td>
<td>$E_i = E_i(10^{0.018(QD-1.9)})$</td>
<td>$QD &gt; 2$</td>
<td>31</td>
<td>12.33</td>
<td>20.66</td>
</tr>
<tr>
<td>46</td>
<td>$E_i = E_i(0.02 + \frac{1}{1+\frac{100}{E_i}})$</td>
<td>$E_i &gt; 100$</td>
<td>32</td>
<td>19.35</td>
<td>14.53</td>
</tr>
<tr>
<td>47</td>
<td>$E_i = E_i(0.02 + \frac{1}{1+\frac{100}{E_i}})$</td>
<td>$E_i &gt; 100$</td>
<td>33</td>
<td>10.7</td>
<td>7.18</td>
</tr>
<tr>
<td>48</td>
<td>$E_i = E_i(0.025\log(Q - 2.875))$</td>
<td>$Q &lt; 10$</td>
<td>34</td>
<td>11.34</td>
<td>8.24</td>
</tr>
<tr>
<td>49</td>
<td>$\sigma_{cmax} = \sigma_{ci} e^{E_i(QD-Q_{ci})}$</td>
<td>$QD &gt; 2$</td>
<td>(35)</td>
<td>16.95</td>
<td>11.47</td>
</tr>
<tr>
<td>50</td>
<td>$\sigma_{cmax} = \sigma_{ci} e^{E_i(QD-Q_{ci})}$</td>
<td>$QD &gt; 2$</td>
<td>(36)</td>
<td>18.45</td>
<td>12.62</td>
</tr>
<tr>
<td>51</td>
<td>$\sigma_{cmax} = \sigma_{ci} e^{E_i(QD-Q_{ci})}$</td>
<td>$QD &gt; 2$</td>
<td>(37)</td>
<td>28.74</td>
<td>20.83</td>
</tr>
<tr>
<td>52</td>
<td>$\sigma_{cmax} = \sigma_{ci} e^{E_i(QD-Q_{ci})}$</td>
<td>$QD &gt; 2$</td>
<td>(38)</td>
<td>14.21</td>
<td>9.91</td>
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<tr>
<td>53</td>
<td>$\sigma_{cmax} = \frac{RMR}{100}$</td>
<td>$RMR &gt; 50$</td>
<td>(39)</td>
<td>20.63</td>
<td>15.28</td>
</tr>
<tr>
<td>54</td>
<td>$\sigma_{cmax} = \beta \frac{RMR}{100}$</td>
<td>$RMR &gt; 50$</td>
<td>(40)</td>
<td>29.93</td>
<td>22.62</td>
</tr>
<tr>
<td>55</td>
<td>$\sigma_{cmax} = \frac{5\gamma(Q_{ci}/100)^{1.5}}{10}$</td>
<td>$QD &gt; 2$</td>
<td>(41)</td>
<td>30.50</td>
<td>24.41</td>
</tr>
</tbody>
</table>

Table 2. The proposed empirical equations for calculation of $E_i$ (GPa).

Table 3. The proposed empirical equations for calculation of $\sigma_{cmax}$ (MPa).
Moreover, being jointed, the medium into which diversion tunnel at Shahriar Dam is excavated has uncertainties associated with normal stiffness. Normal stiffness can be calculated by Eq. (42). As can be observed, this parameters depends on $E_{\text{mass}}$ and since $E_{\text{mass}}$ itself is uncertain, the parameter $K_n$ will be of uncertainty as well. So, it can be stipulated that, across Sections 1 and 3, six different values were obtained for the two uncertain parameters of $C$ and $\Theta$, and 14 different values were calculated for the three parameters of $E_{\text{mass}}$, $\sigma_{\text{mass}}$ and $K_n$. However, the corresponding figures to Section 2 were 11 and 19, respectively.

$$K_n = \frac{E_{\text{mass}}}{S(D-E_m)}$$  \hspace{1cm} (42)

$S$ represents average spacing of discontinuities.

Since many of natural phenomena exhibit normal distribution, in many probabilistic investigations on geo-mechanical problems, input data are assumed to be distributed normally. This is while there are cases where available data exhibit non-normal distributions, or no specific distribution can be well fitted to the data. In such cases, the point is that how does the form of probability distribution function of input data affect outputs of a probabilistic method? For this purpose, once finished with finding all possible values of parameters, two other important and popularly used distribution functions (lognormal and gamma) were further used to fit to the five parameters considered in the present study.

Characteristics and parameters of these functions are reported in Table 5. Subsequently, 1000 random data points were generated for each random variable under each scenario and numerical modelling was conducted. Further, according to the analysis using point estimation method, with 5 random variables under each scenario, we ended up with $2^5 = 32$ combinations of parameters based on two estimated points (lower and upper bounds of each variable).

### Table 5. Parameters of normal, lognormal and gamma distribution for each scenario.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Scenario</th>
<th>Normal</th>
<th>Log normal</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td></td>
<td>1.5073</td>
<td>0.108</td>
<td>0.608</td>
</tr>
<tr>
<td>$\Theta$</td>
<td></td>
<td>55.442</td>
<td>0.713</td>
<td>0.018</td>
</tr>
<tr>
<td>$E$</td>
<td>1</td>
<td>14.633</td>
<td>4.445</td>
<td>2.647</td>
</tr>
<tr>
<td>$\sigma_{\text{mass}}$</td>
<td>26.712</td>
<td>4.086</td>
<td>3.256</td>
<td>0.256</td>
</tr>
<tr>
<td>$K_n$</td>
<td>40.601</td>
<td>16.550</td>
<td>3.638</td>
<td>0.339</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>0.944</td>
<td>0.145</td>
<td>0.068</td>
<td>0.146</td>
</tr>
<tr>
<td>$E$</td>
<td>2</td>
<td>12.194</td>
<td>3.898</td>
<td>2.055</td>
</tr>
<tr>
<td>$\sigma_{\text{mass}}$</td>
<td>21.422</td>
<td>5.750</td>
<td>3.021</td>
<td>0.132</td>
</tr>
<tr>
<td>$K_n$</td>
<td>26.247</td>
<td>9.489</td>
<td>3.207</td>
<td>0.343</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>0.271</td>
<td>0.018</td>
<td>1.306</td>
<td>0.060</td>
</tr>
<tr>
<td>$E$</td>
<td>3</td>
<td>41.762</td>
<td>1.238</td>
<td>3.732</td>
</tr>
<tr>
<td>$\sigma_{\text{mass}}$</td>
<td>6.379</td>
<td>2.296</td>
<td>1.796</td>
<td>0.339</td>
</tr>
<tr>
<td>$K_n$</td>
<td>5.462</td>
<td>3.906</td>
<td>1.511</td>
<td>0.578</td>
</tr>
</tbody>
</table>

Step 2: Numerical modelling of the statistical analysis results

The selected geometry of built models was 70 m width, 70 m height and 10 m length and then the discontinuities including the joint sets were added to them. In this case all boundaries are taken fixed and only the upper boundary is taken free. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance.

The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance. The proposed excavation method for Shahriar Dam tunnel is by step method. First, the curved section of the model was solved for reaching the initial balance. After applying the boundary conditions, the built model was solved for reaching the initial balance.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario</th>
<th>Normal</th>
<th>Log normal</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1</td>
<td>1.440</td>
<td>0.318</td>
<td>0.220</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>2</td>
<td>1.448</td>
<td>0.325</td>
<td>0.224</td>
</tr>
<tr>
<td>$E$</td>
<td>3</td>
<td>1.437</td>
<td>0.331</td>
<td>0.230</td>
</tr>
<tr>
<td>$\sigma_{\text{mass}}$</td>
<td>1.356</td>
<td>0.601</td>
<td>0.265</td>
<td>0.531</td>
</tr>
<tr>
<td>$K_n$</td>
<td>1</td>
<td>1.445</td>
<td>0.363</td>
<td>0.251</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>1</td>
<td>1.444</td>
<td>0.354</td>
<td>0.244</td>
</tr>
<tr>
<td>$E$</td>
<td>2</td>
<td>1.429</td>
<td>0.332</td>
<td>0.226</td>
</tr>
<tr>
<td>$\sigma_{\text{mass}}$</td>
<td>1.423</td>
<td>0.339</td>
<td>0.238</td>
<td>0.414</td>
</tr>
<tr>
<td>$K_n$</td>
<td>3</td>
<td>1.422</td>
<td>0.323</td>
<td>0.226</td>
</tr>
<tr>
<td>(a) Shear strain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SRM/DEM/MCS</td>
<td>1</td>
<td>1.440</td>
<td>0.318</td>
<td>0.220</td>
</tr>
<tr>
<td>SRM/DEM/MCS</td>
<td>2</td>
<td>1.448</td>
<td>0.325</td>
<td>0.224</td>
</tr>
<tr>
<td>SRM/DEM/PEM</td>
<td>3</td>
<td>1.437</td>
<td>0.331</td>
<td>0.230</td>
</tr>
<tr>
<td>SRM/DEM/PEM</td>
<td>1</td>
<td>1.444</td>
<td>0.323</td>
<td>0.224</td>
</tr>
<tr>
<td>(b) Displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SRM/DEM/MCS</td>
<td>2</td>
<td>1.429</td>
<td>0.323</td>
<td>0.226</td>
</tr>
<tr>
<td>SRM/DEM/PEM</td>
<td>3</td>
<td>1.423</td>
<td>0.339</td>
<td>0.238</td>
</tr>
</tbody>
</table>

Step 3: Estimation of $P_f$ and risk for each scenario

In order to calculate failure probability under either of the scenarios, number of the samples given by Monte Carlo analysis wherein failure occurred (advance step lengths smaller than 1 m) was divided by total number of samples (1000 samples from Monte Carlo simulations plus 32 samples given by point estimation method). In fact, from
mathematical point of view, a cumulative distribution is defined as the integral of normalized probability density function, while technically, a point on cumulative distribution gives that probability; such a point will refer to a random variable with its value being either smaller or larger than a given value. A step length of 1 m was chosen because advance steps of smaller than 1 m were economically unviable. Cost of constructing 10 m of tunnel was obtained by summing up the excavation cost and the cost incurred by tunnel maintenance system. Once finished with calculating maintenance system and excavation costs for each section with different qualities (as per RMR classification system) and accounting for failure probability via both methods, expected cost of damage was evaluated for different advance step lengths. Figs. 4 and 5 compare the results of failure probability calculation for different lengths of advance step according to the two proposed methods considering either of normal, lognormal, or gamma distributions. Finally, Table 7 presents associated risks with each scenario based on both shear strain and displacement.

![Fig. 4](image1.png) Probability of failure for each scenario in SRM/DEM/MCS method based on (a) shear strain (b) displacement.

![Fig. 5](image2.png) Probability of failure for each scenario in SRM/DEM/PEM method based on (a) shear strain (b) displacement.

<table>
<thead>
<tr>
<th>Table 7. Risk computed for 3 different distribution types by SRM/DEM/MCS and SRM/DEM/PEM methods.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Criterion</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>shear strain</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>displacement</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>shear strain</td>
</tr>
<tr>
<td>displacement</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
According to the results, it is obvious that a change in probability distribution of input data, different values of failure probability risk for each advance step length are obtained with either of the proposed methods. This is while no changes is imposed to the final solution in terms of optimum advance step length. In other words, an advance step length of 2 m was suggested as the optimal advance step length in terms of both economic factors and safety considerations as it provided lowest risk level according to either of the two proposed hybrid methods, considering either of the three probability distribution functions for input data, and using both criteria of shear strain and displacement.

As can be observed from the results of Monte Carlo simulation and point estimation methods for optimization of advance step length, in spite of the optimality of the 2 m advance step length, final risks associated with different scenarios are just slightly different. Such a slight difference can be explained by the number of data points modelled in terms of C, E, and Ø values. Indeed, only two points (upper and lower bounds of the parameters of the variable) were accounted for in the point estimation method while Monte Carlo simulations were fed by 1000 data points when it came to reliability calculations. In other words, it can be concluded that, one can end up with more reliable results by taking into account all possible values of uncertain parameters in risk assessment and reliability calculations. Hence, generally speaking, it can be said that, one can attain higher confidence levels in determining excavation advance step length in tunneling projects by increasing the number of tests or more generally the deal of available random data for risk assessment and reliability analysis. Moreover, it is seen on Figs. 6 and 7 that, under different scenarios, an increase in advance step length can boost failure probability, such that as the step length is increased from 1.5 m to 2 m, corresponding probability of failure increased at lower rate, while the probability increased at considerably higher rate when the step length was further increased to 3 m. Moreover, it should be noted that, failure probabilities obtained with shear strain, rather than displacement, as failure criterion were more in agreement with experimental data regardless of the statistical distribution assumed.

5. Sensitivity analysis

In the present research, in order to investigate the effect of type of probability density function of uncertain parameters on optimum length of advance step in tunneling excavations, three types of probability distribution functions (normal, lognormal, gamma) were chosen to be fitted on quantified data. Characteristics of required parameters for each of the distribution functions are reported in Table 8. Once required parameters for each probability density function were obtained, mean and standard deviation of each uncertain input parameter was calculated for either of the three distribution functions.

5.1. Determination of test levels, orthogonal arrays and test index relationships

In order to apply DOE methodology, three levels were assumed for each factor: good, fair, poor. The three test levels for each variable with each distribution function were determined according to the following approaches: (A) unbiased point estimator, and (B) the two points returned by Rosenblueth’s point estimation method [54-55]. In other words, the parameter \( \mu \) was recognized as having three estimators: \( \bar{x} \), \( M_d \) and \( M_p \). However, it has been shown that fluctuations in mean are less than those of median and mode, i.e. mean value is more stable than median or mode, and the most important characteristic of mean is its unbiasedness. So, the three levels considered for each parameter at three sections in the present research were \((\mu + \sigma, \mu, \mu - \sigma)\). Quantitative values of the four levels determined for the considered factors are presented in Table 9.
According to DOE methodology, a parameter called test index can be used to help the user make proper judgments. An appropriate test index should be 1) sensitive to input parameters, and 2) measurable [56]. In the present research, the test index is defined in Eq. (43) and (44).

\[
\text{TI} = \frac{\delta_{\text{max}}}{\delta_{\text{Sakurai}}} \quad (43)
\]

\[
\text{TI} = \frac{\sigma_{\text{max}}}{\sigma_{\text{Sakurai}}} \quad (44)
\]

Where TI is the test index, \(\delta_{\text{max}}\) represents maximum displacement around the tunnel, \(\sigma_{\text{max}}\) is maximum shear strain (as obtained from 3DEC software), and \(\delta_{\text{Sakurai}}\) and \(\sigma_{\text{Sakurai}}\) are allowable displacement and shear strain from Sakurai’s relationships.

In order to calculate test index, once finished with numerical modelling and extracting maximum displacement and strain for the 27 Taguchi tests assuming different probability distribution functions, values of displacement and shear strain were further calculated by Sakurai’s relationships. Finally, dividing maximum displacement and shear strain by corresponding values obtained by Sakurai’s relationships, test index was calculated.

When designing experiments according to Taguchi, special tables called orthogonal arrays are used. Application of these tables largely facilitate the DOE process. For this purpose, in order to investigate the effect of the factors under test, one should consider appropriate orthogonal arrays. Next, appropriate orthogonal arrays are selected for each section in each probability distribution. Taguchi orthogonal arrays are expressed as \(L_c(b^n)\) where a represents the number of times the test has been conducted (numerical modeling), \(b\) denotes the number of levels considered, and \(c\) is the number of factors [57]. As such, in the present research, considering five uncertain parameters and three test levels for each, we ended up with orthogonal arrays of the following form \(L_{27}(3^5)\).

### 5.2 Numerical modelling and test index

After statistically analyzing the uncertain parameters at each section and determining test levels and Taguchi’s orthogonal arrays, 27 numerical simulations were run for each section and each probability distribution \((L_{27}(3^5))\).

Considering discontinuity of the medium into which water diversion tunnel at Shahriar Dam is excavated, numerical simulations were undertaken utilizing 3DEC finite element software. Considering the order and sequence of excavations, the tunnel is excavated in stages, i.e. four stages.

### 5.3. Taguchi sensitivity analysis

In DOE via Taguchi’s methodology, targeted and simultaneous changes are made in factors affecting a system, and the resulting changes in outputs are investigated to attain an extensive understanding of how such factors affect the outputs. Identification of the factors of high signal-to-noise ratios will attenuate the effect of adverse factors [58]. In this section, once finished with determining test levels, orthogonal arrays, and test index, the orthogonal arrays were analyzed on the basis of the statement “the larger, the better” as defined in Eq. (45) [59].

\[
n_{ij} = -10 \log_{10} \left( \sum_{k=1}^{3} \frac{1}{C_k} \delta_{ij} \right)  \quad (45)
\]

Where \(n_{ij}\) denotes signal-to-noise ratio of the ith test and nth replicate of the tests.

Design process of underground structures are often performed with some approximate knowledge of geo-mechanical parameters of rock mass. This is while an accurate knowledge of such parameters is necessary for determining excavation methods, designing maintenance systems, and finally analyzing stability state of the structure. Among these four parameters, Young's modulus is the most highlighted parameter indicating mechanical behavior of rock mass; it is necessary for determining excavation methods, designing maintenance systems, and finally analyzing stability state of the structure. Among these four parameters, Young's modulus is the most highlighted parameter indicating mechanical behavior of rock mass; it is necessary for determining excavation methods, designing maintenance systems, and finally analyzing stability state of the structure. 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for stress distribution and displacement analyses, or more generally, describing rock mass behavior in rock engineering projects [60]. Moreover, discontinuities in rock masses surrounding a tunnel (e.g. faults, joints, fractures) play a critical role in rock mass stability around the tunnel, particularly tunnels excavated into hard rocks [61]. Many rock structures are designed to be constructed into jointed rock masses at large scale; this makes the rock structure difficult to design and maintain. Stability of these structures is mainly affected by the distribution and behavior of discontinuities [62]. As such, in discontinuous rock medium, instability conditions are further controlled by strength coefficients of discontinuities. Among these parameters, normal stiffness of joints are of the most important parameters affecting the behavior exhibited by discontinuities and hence stability of rock structures. According to the aforementioned discussion, one can conclude that modulus of deformability and normal stiffness of joints significantly affect stability and behavior of rock mass. Results of the analyses undertaken by taking displacement and strain as test index in Minitab Software are presented in Figs. 8 and 9, respectively.

Fig. 8. The effect of the uncertainty variables on the advance step based on displacement test index.
As the results show, for both criteria (displacement and shear strain), different levels of sensitivity were found depending on the type of used probability function. Therefore, as rock mass quality (as measured per RMR classification system) decreases, effect of normal stiffness on tunnel stability increases. This is while a reduction in rock mass quality was observed to increase the effect of deformability modulus on the tunnel stability. At scenario 1, for both test indexes, $E_{\text{mass}}$ and $\theta$ in the normal distribution, $E_{\text{mass}}$ and $C$ in the lognormal and gamma distribution were found as the most effective parameters on tunnel stability. Moreover, at scenarios 2 and 3, one can see that normal stiffness of joints represent the most effective parameter on the tunnel stability. At scenario 2, according to displacement criterion, three parameters of $E_{\text{mass}}$, $\theta$, and $C$ were found to be of large contributions into tunnel stability, while the other four parameters were found to be of virtually the same deals effective when shear strain was taken as test index. Finally, at scenario 3, following normal stiffness, the parameter $\theta$, in normal distribution, and the parameters $\theta$ and $C$, in lognormal and gamma distributions, were found to be the most effective parameters. According to these results, one can say that, failing to consider type of probability distribution function of uncertain input parameters when calculating tunnel stability is likely to introduce significant deals of error into calculations. Accordingly, achieving more accurate estimations is easy by prioritizing the distributions based on their effectiveness.

Table 10 reports statistics of test index based on both criteria: displacement and shear strain. Moreover, by simultaneously changing probability density distribution functions of all five uncertain variables, values of Taguchi test index were seen to change. Considering median of the confidence interval over the three confidence intervals calculated for test index of each section, and determining maximum difference between the three medians, one can determine maximum difference in values of test index as types of probability density distribution functions.
of five uncertain factors change. It is worth noting that, within the mentioned confidence intervals, the median of confidence interval is the same as mean of data (Table 11). Maximum difference in the values of test index across the three sections is presented in Table 12.

Table 10. Statistical information obtained from Taguchi experimental index in each scenario of tunnel.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Scenario</th>
<th>Distribution</th>
<th>Mean</th>
<th>STD</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>1</td>
<td>Normal</td>
<td>0.644</td>
<td>0.249</td>
<td>0.64</td>
<td>1</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Lognormal</td>
<td>0.608</td>
<td>0.232</td>
<td>0.64</td>
<td>1</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Gamma</td>
<td>0.628</td>
<td>0.251</td>
<td>0.64</td>
<td>1</td>
<td>0.16</td>
</tr>
<tr>
<td>Shear strain</td>
<td>1</td>
<td>Normal</td>
<td>0.604</td>
<td>0.237</td>
<td>0.64</td>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Lognormal</td>
<td>0.602</td>
<td>0.227</td>
<td>0.64</td>
<td>0.94</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Gamma</td>
<td>0.606</td>
<td>0.203</td>
<td>0.59</td>
<td>0.94</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 11. Confidence intervals of Taguchi experimental index.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Scenario</th>
<th>Normal Distribution</th>
<th>Lognormal Distribution</th>
<th>Gamma Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>1</td>
<td>0.520-0.767</td>
<td>0.493-0.723</td>
<td>0.504-0.753</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.487-0.722</td>
<td>0.490-0.714</td>
<td>0.505-0.707</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.481-0.677</td>
<td>0.491-0.660</td>
<td>0.500-0.678</td>
</tr>
<tr>
<td>Shear strain</td>
<td>1</td>
<td>0.532-0.735</td>
<td>0.507-0.716</td>
<td>0.529-0.722</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.537-0.753</td>
<td>0.472-0.702</td>
<td>0.515-0.729</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.502-0.680</td>
<td>0.300-0.679</td>
<td>0.534-0.718</td>
</tr>
</tbody>
</table>

Table 12. Maximum difference in the values of test index.

<table>
<thead>
<tr>
<th>experimental index</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>3.59</td>
<td>3.37</td>
<td>1.37</td>
</tr>
<tr>
<td>Shear strain</td>
<td>3.92</td>
<td>5.78</td>
<td>3.66</td>
</tr>
</tbody>
</table>

6. Results and discussion

Uncertainty is inherent in geological engineering problems and can have a significant impact on design performance if it is not properly taken into account. Currently, the reliability-based design approach is the only methodology that quantifies uncertainty and provides a consistent measure of safety by determining the probability of failure for a system. In this paper, a new quantitative risk method based on reliability approach was presented that utilizes discrete element method, point estimate method, Monte Carlo simulation and strength reduction method to assess advance step length. To demonstrate the value of this methodology, a case study was performed for the diversion tunnel of Shahriar Dam. The benefits of such an approach were demonstrated through the assessment of different advance step categories along a section of the tunnels.

The combinations of SRM/DEM/MCS and SRM/DEM/PEM methods in selection of optimal advance step for each scenario can be seen in Fig. 1. It is clear due to the Figs. 4 and 5 that scenario 2 (2m) shows the lowest P<sub>f</sub> based on shear strain criterion when compared to other scenarios. While advance step of scenario 3 in SRM/DEM/PEM method has the highest risk, the high probability of failure leads to the financial damage. The results for all scenarios are shown in Table 7. The benefit of presented new approach in this paper is decreasing of negative impact of simplifications made by the numerical modelling and uncertainty of geotechnical parameters on the results.

7. Conclusion

In stage excavation, excavation step length serves as the most principal factor form both economic and operational points of view. This factor is closely related to the time and cost to construct tunneling projects via stage excavation. As such, optimal determination of this factor is among the main challenges in tunneling projects. When determining tunnel excavation advance step length using deterministic method, uncertainties within the rock mass are neglected, so that the final result is neither accurate nor reliable. As such, more reliable results can be achieved by statistical and probabilistic risk analysis. In the present research, a novel approach was proposed based on two hybrid methods (SRM/DEM/MCS and SRM/DEM/PEM) to undertake risk analysis on the tunnel excavation advance step length at Shahriar Dam following a statistic and probabilistic approach. Further, sensitivity of the excavation advance step length in water diversion tunnel at Shahriar Dam to each of the uncertain geo-mechanical parameters of rock mass was addressed according to Taguchi’s DOE methodology based on two main criteria (displacement and shear strain) playing fundamental roles in stability of underground spaces.

Therefore, according to investigations and considering the results of geotechnical-statistical researches, the following conclusions can be drawn:

- Optimum excavation advance step length under scenario 2 with fair quality as per RMR classification system based on both hybrid methods (SRM/DEM/MCS, SRM/DEM/PEM) considering the three probability distribution functions for input variables was found to be 2 m. This step length was analyzed by comparing the shear strain and displacement obtained from Sakurai’s relationships to those of numerical modelling in 3DEC discrete element software. Shear strain and displacement are...
two important parameters in the field of tunnel and underground space stability.

- In the present research, effect of the type of probability distribution function of input parameters on failure probability and associated risk with different lengths of advance step at different scenarios was investigated. According to Figs. 4 and 5 and Table 8, different values of failure probability and risk were obtained with different methods, criteria, and types of probability distribution function assumed for input parameters. However, in all cases, the advance step length of 2 m was found to result in the lowest failure probability and risk.

- Under scenario 2 with an advance step length of 2 m, minimum failure probability (8.05%) was that of shear strain criterion in SRM/DEM/MCS method with lognormal probability distribution function, while the maximum failure probability (11.52%) was found based on displacement criterion in SRM/DEM/PEM method with lognormal probability distribution function.

- Also, at optimum scenario 2, minimum risk (75281.56 $ & ) was that in SRM/DEM/MCS method with lognormal probability distribution function, while the maximum risk (77699.20 $) was found in SRM/DEM/PEM method with lognormal probability distribution function.

- As advance step length was increased from 1.5 m to 3 m, failure probability increased accordingly; however, the increase in failure probability followed a slower rate when step length was increased from 1.5 m to 2 m, rather than when it was further increased to 3 m. Moreover, as advance step length was increased from 1.5 m to 3 m (incurring increased failure probability), expected cost of damages incurred by tunnel failure increased rapidly.

According to Taguchi’s sensitivity analysis undertaken in the present research, the following important conclusions were drawn:

- At the scenario of good quality, considering either of displacement or shear strain as the judgment criterion, advance step length exhibited highest sensitivity to the three main parameters of \( E_{\text{mass}} \), \( \sigma_{\text{mass}} \), and \( C \). Indeed, when input parameters were considered to be distributed normally, \( \Theta \) and \( E_{\text{mass}} \) where of the largest contributions into advance step length, while \( C \) and \( \sigma_{\text{mass}} \) played the same role when input parameters were assumed to follow lognormal distribution.

- At the section of fair quality, \( K_w \) was found to be of more effective role, while other parameters had virtually the same decisions on the excavation advance step length. Indeed, according to both displacement and shear strain criteria, \( K_w \) was recognized as effective with normal and gamma distributions of input parameters, while \( K_w \) and \( \sigma_{\text{mass}} \) were found as the most effective parameters on the excavation advance step length when input parameters were assumed to follow normal distribution. Further, with normal distribution of input parameters and based on shear strain, \( K_w \) and \( E_{\text{mass}} \) were identified as the most effective parameters.

- At the scenario of poor quality, \( K_w \) was found to be of primary and fundamental role, followed by \( C \) and \( \Theta \), on the excavation advance step length. Results of sensitivity analysis at this section were different from those at the section of fair quality in that \( K_w \) was by far more sensitive at this section rather than the fair section. So that, according to displacement criterion, in normal distribution, \( K_w \) followed by \( \Theta \), in lognormal distribution, \( K_w \) followed by \( \Theta \) and \( E_{\text{mass}} \) in gamma distribution, \( K_w \) followed by \( C \) were found to be of the largest contributions into the excavation advance step length. Further according to shear strain criterion, in lognormal distribution, \( K_w \) and \( C \) exhibited similar effects on the excavation advance step length, implying the role played by cohesion of rock masses in the excavation advance step length at poor sections.

- In general, in all three probability distribution functions (normal, lognormal, and gamma), with increasing the RMR value of rock mass, three parameters of \( E_{\text{mass}} \), \( \sigma_{\text{mass}} \), and \( C \) (specially \( E_{\text{mass}} \) ) exhibited further effectiveness on the excavation advance step length. This was while, at sections of poor RMR, contributions from \( K_w \) were by far larger than those of other rock mass parameters (\( E_{\text{mass}} \), \( \sigma_{\text{mass}} \), and \( C \)).

- Calculated confidence intervals based on 95% confidence level (Table 10) for Taguchi test index for both criteria at all three scenarios were of finite ranges. This properly demonstrates enhanced confidence within the data once Taguchi sensitivity analysis was undertaken. This is because application of most of existing empirical correlations and Hoek-Brown failure criterion and also the relationship for determining normal stiffness of joints (\( K_w \)), virtually all viable combinations of uncertain parameters are practically identified for probabilistic analyses and incorporated into computations, so that the number of combinations omitted from computations is very limited.

- The changes in Taguchi test index in response to simultaneous change of types of probability distribution functions assumed for all of the five uncertain variables (\( C \), \( \Theta \), \( E_{\text{mass}} \), \( \sigma_{\text{mass}} \), and \( K_w \)) were different at different sections along the tunnel. In fact, according to displacement criterion, in the fair scenario, a change in probability distribution functions resulted in minimum change in test index (0.37%), while the same change at the good scenario (as per RMR) resulted in largest change in test index (3.59%). Moreover, when Taguchi shear strain criterion was concerned, the minimum (3.66%) and maximum (5.78%) changes in test index were observed in the section of firm quality.

Totally, the rock mass properties affected by the random nature of discontinuity characteristics and intact rock properties, which are widely scattered and variable, cannot be sufficiently represented by a single value for each input characteristic and a single output value. Therefore, it is highly recommended that other probabilistic- reliability analyses should be applied, such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and First Order Second Moment (FOSM) methods. Moreover, statistical distribution of uncertain input parameters are taken as being normal, log normal and gamma; but other statistical distributions such as beta, F, exponential and binomial should also be further considered.

REFERENCES


