
International Journal of Mining and Geo-Engineering

Application of truncated gaussian simulation on ore-waste boundary modeling of Golgohar iron deposit

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ARTICLE HISTORY

Received 18 Jan 2016, Received in revised form 19 Mar 2016, Accepted 21 Mar 2016

ABSTRACT

Truncated Gaussian Simulation (TGS) is a well-known method to generate realizations of the ore domains located in a spatial sequence. In geostatistical framework, the geological domains are normally utilized for stationary assumption. The ability to measure the uncertainty in the exact locations of the boundaries among different geological units is a common challenge for practitioners. As a simple and informative example of such a boundary, one can consider the boundary between ore and waste materials in an ore deposit. This boundary addresses the percentages of the ore and the waste and also affects the future economy of mine and all precedent mine designs and mine plans. Deterministic approaches, based on interpretation of geological phenomenon, provide just one scenario of ore-waste variation, and do not offer a model for uncertainty of boundaries. On the other hand, geostatistical simulations, based on stochastic models, can measure the uncertainty of such a boundary. Through different techniques for spatial simulation of the categorical data (geological domains), truncated gaussian simulation has been proved to be versatile when geological units have sequential geometries and/or there are few number of indicators (ore and waste). This study addresses the application of TGS for conditional simulation of ore and waste domains in Golgohar iron ore deposit. Separation of the ore and waste domains has affected the ore tonnage estimation and resource evaluation. Various simulations can be considered as the spatial realizations of ore and waste. TGS can generate realizations of the domains and measure the uncertainty of ore-waste boundary. The accuracy of results was checked through performance evaluation section and different scenarios (e.g. best, average and worst). The best scenario is the one with the most accuracy that is calculated from confusion matrix. The scenario No. 44 with 96 million cubic meters tonnage has an accuracy over 86 percent that is proposed as the best scenario for future planning and mine design.

Keywords: *Truncated Gaussian Simulation; Geological boundaries; Uncertainty modeling; Iron ore*

1. Introduction

Prior to grade and tonnage estimation, geological modeling has to be used to divide the deposit into stationary sub-domains called 'geological units'. This partitioning leads to a better characterization of the grade distribution within the deposits [1]. Stochastic simulations are among the best methods for modeling domains within ore deposits. Different geostatistical methods have been so far developed for modeling the

domains such as: sequential indicator simulation [2, 3], truncated Gaussian simulation [4], multiple point statistics and plurigaussian simulation [5].

SIS¹ is a simple and strong approach for simulating the categorical variables [6]. Also MPS² is another useful tool when nonlinear features exist, however it requires a training image containing complex features making the method hard to be applied [7]. Nevertheless, TGS and PGS³ methods have increased the ability to reproduce the complex features comparing to SIS, without requiring a training image, although they call for establishment of a truncation mask showing the geological orders in

¹ Sequential Indicator Simulation

² Multiple Point Simulation

³ Plurigaussian Simulation

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a flag [8].

In this paper, in order to model ore and waste domains in Golgohar iron deposit, TGS has been used. The purpose is to produce different realizations for ore-waste boundary, and choose the best and worst scenarios for tonnage estimation. Based on statistical and geological analyses, Golgohar iron deposit consists of six different rock types grouped into ore and waste domains. The best realization of boundary strongly affects the production and mine planning [9]. This study implements Truncated Gaussian Simulation to model the ore and waste domains within Golgohar deposit.

1.1. Truncated Gaussian Simulation

TGS method simulates one Gaussian variable at all data locations within the study area and then uses a flag to transform this Gaussian values into the premium geological domains. Figure 1 summaries this process for TGS method for the case of one Gaussian field. Figure 1.b shows that the grey domains can touch the other two domains in which the black and white never touch each other. Also, these domains have a fixed order which is defined by the sequence stratigraphy [10].

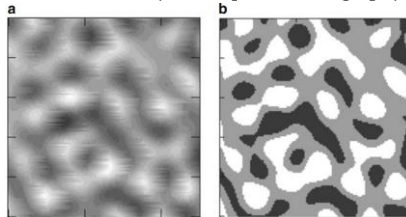


Figure 1. (a) Simulated greytone image. Values have an $N(0, 1)$ distribution. (b) Same image after being truncated at the cutoffs -0.6 and 0.5 . Values below -0.6 have been shaded dark grey, those between -0.6 and 0.5 are coloured light grey while values above 0.5 are shown in white [10].

Initially, TGS was used for simulation of an ordered category with locally changed proportions [5]. Suppose a vertically non-stationary sedimentary environment compared shale, shaly-sandstone and sandstone rock types. The expected order is that shaly-sandstone intercalates shale and sandstone. In Figure 2, every truncated zone integrates to the local rock type proportions. The continuity of rock types is determined by the variogram calculated for the simulation [8].

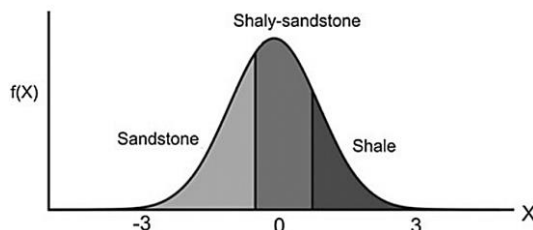


Figure 2. Sketch of truncation of Gaussian probability density function. Truncated probabilities integrate to the local proportion. Here, we expect sandstone and shale to be separated by shaly-sandstone [8].

A flag is a graphical sketch of the domains' orders and contacts that is drawn based on geological information. In Figure 3, the flag or rock type rule of this sketch is shown in which shale and sandstone have no contact based on geological knowledge.



Figure 3. The rock type rule.

The main steps in a truncated gaussian simulation are as follow [10, 11]:

Step 1: Estimation of two factors controlling the simulation results: (i) thresholds that truncate the gaussian random field into the domains which are determined by the flag (rocktype rule), the proportions of each domains, and (ii) the variogram model of this gaussian variable that

should reproduce the spatial relationship between the hard data.

Step 2: While the domains are known for each sample, the corresponding Gaussian values are unknown. After implementing the truncated rule, the domains reduce to rectangles. In this step, Gibbs sampler method can be used to generate Gaussian values using these intervals in respect to variogram model.

Step 3: Simulation of Gaussian values of variables at the grid nodes. In this case, any simulation approach, such as the sequential Gaussian or the turning band, can be implemented on the Gaussian variable in this step. In current study, the turning band algorithm is used.

Step 4: In the last step, the flag is used to convert the gaussian values at grid nodes back into the domains.

1.2. Gibbs Sampler

When all or part of the data values are interval constraints rather than single numbers, simulation of the Gaussian random vectors is subjected to inequality constraints that is created in the analysis of the spatial data. In mineral resource evaluation, this situation occurs when the depth of a geological horizon is greater than the depth at which drilling has stopped, or a measured grade is smaller than the detection limit, or when working with soft data defined by lower and upper bounds [12]. Interval constraints exist when simulating continuous indicators and variables are represented by chi-square random fields, as well as by truncated Gaussian or plurigaussian random fields [13]. For instance, let d be a positive integer and consider an indicator variable obtained by truncating a stationary Gaussian random field $Y = \{Y(x): x \in R^d\}$ at a given threshold $Y \in R$. The following procedure can be used to simulate the indicator [14]:

- (1) Simulate Y at the data locations, conditioned by the indicator data.
- (2) Simulate Y at the target locations of R_d , conditioned by the Y -vector obtained in step (1).
- (3) Truncate the simulated Gaussian random field to obtain a realization of the indicator.

Any multivariate Gaussian simulation algorithm can be considered in step (2). Regarding step (1), one can use an iterative algorithm known as the Gibbs sampler.

In the second step of TGS procedure, Gaussian random function is simulated with a specified covariance structure considering the observed lithotypes (domains) at sample points. Statisticians routinely, use iterative methods based on Markov chain Monte Carlo simulations (MCMC, for short) for sampling from complicated distributions and for estimating the parameter values. The best known algorithms are Hastings-Metropolis and the Gibbs sampler algorithms [10]. The Gibbs sampler is an iterative algorithm being used to simulate Gaussian random vectors subjected to inequality constraints [15].

2. Presentation of the deposit and the data

2.1. Geological description

The Golgohar iron deposit is located at about 55 km southwest of Sirjan and in the eastern edge of the Sanandaj-Sirjan structural zone of Iran (Figure 4). The host rocks of the ore deposit include metamorphosed sedimentary ± volcanic rocks of the greenschist facies, probably of Upper Proterozoic-Lower Paleozoic age. The most important host rocks include shale, sandstone, gabbroic-basaltic and diabasic sills, diamictite and cherty carbonatic sequences that are transformed to thick carbonate successions in the upper units. Magnetite banding, granular, banded and massive textures all represent deposition of iron as hydromagnetite [16].

2.2. Dataset

The study area has a dimension of 2750 m x 1000 m x 31921.9 m, respectively towards east-west, north-south and vertically. The drill holes were analyzed at every 3 m on average, comprising a total number of 11570 samples and rock type information, among which 4661 samples

were analyzed for Fe, S and P grades. An east-west cross section from the available data is shown in Figure 5.

Ore and waste domains are separated based on the rock types with similar statistical characterisations to be grouped into the domains. The statistical analysis is done for 3 meter composites. The length of composites is the mode of samples length in a drillhole. In Figure 6, ore

and waste domains are shown in different colors: red for waste and green for ore. The data show a clear boundary for ore and waste domains.

The summary statistics of the iron grade are shown in Table 1. Only high grades of the waste domain are reported, so the statistical analysis shows the similar parameters of ore domains.

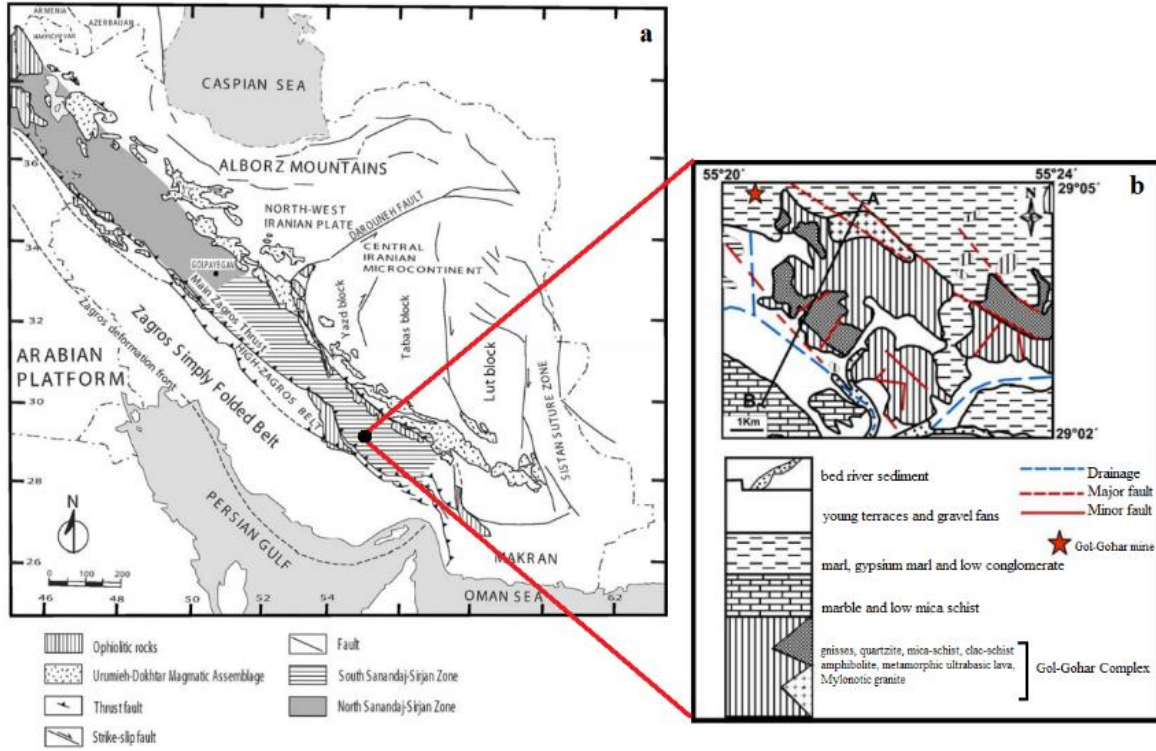


Figure 4. (a) Location of Golgohar deposit in Sanandaj-Sirjan zone [17], (b) a summarized geological map of Golgohar deposit [18].

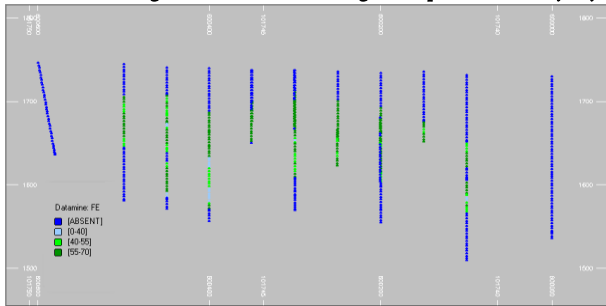


Figure 5. A cross section of available data (iron grade).

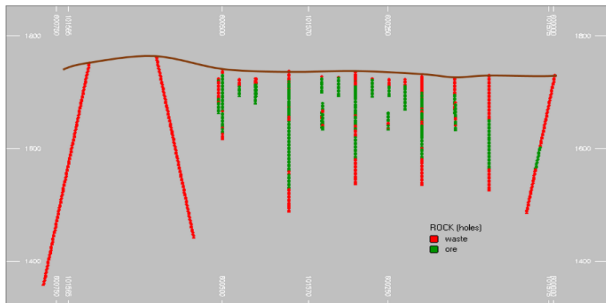


Figure 6. Distribution of available data over cross-section with north coordinate 101565.832 m.

3. Implementation of Truncated Gaussian Simulation

3.1. Truncation threshold

In TGS approach, one gaussian random field is used with each number of thresholds based on the number of the domains and their contacts. In this case, there is only one threshold (Figure 7). This gaussian field is truncated to create two domains:

$$\begin{cases} \text{if } Y < t & \text{waste} \\ \text{if } Y \geq t & \text{ore} \end{cases}$$

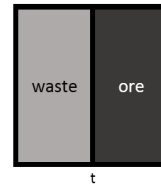


Figure 7. 2D flag of truncation rule for two domains.

The truncation threshold is calculated from experimental domains proportions: $t=0.2793$ (Figure 8).

Table 1. Main statistics of iron grade.

	Number of the data	Mean	Std. Deviation	Variance	Skewness	Kurtosis	Minimum	Maximum
Ore	4510	57.81	8.04	64.70	-1.83	4.51	8.70	68.00
Waste	7059	43.96	13.75	189.17	-0.52	-0.681	9.30	67.10
Total	11569	56.36	9.77	95.50	-1.80	3.59	8.70	68.00

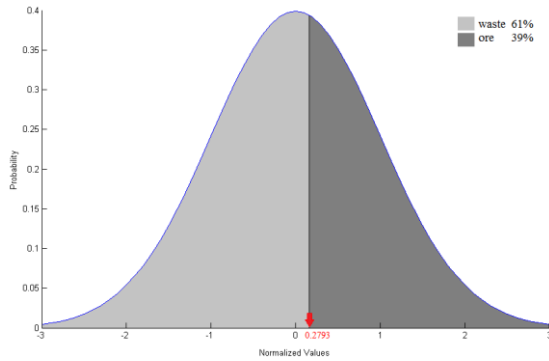


Figure 8. Implementation of rock type rule on the Gaussian variable.

3.2. Variogram analysis

The gaussian random field should reproduce the spatial relationship of the indicators. The variogram models of the gaussian random field are defined using the indicators experimental variograms. The domain variograms are calculated in different directions. The main direction of

the continuity is in dip 10° with azimuth 270°. Table 2 shows the parameters of the gaussian random field variogram, which has the best fit of the indicator variograms (Figure 9).

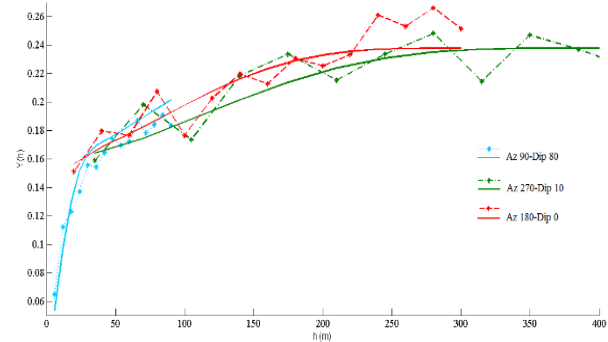


Figure 9. Sample (dots and dashed lines) and model (solid lines) indicator variograms along main anisotropy directions.

Table 2. Parameters of the variogram models of the Gaussian random field.

Gaussian random field	Nugget	Sill	Structure	Long range (m)	Short range (m)	Azimuth (°)	Dip (°)
Y	0	0.5	Cubic	60	10	90	80
	0	0.5	Cubic	400	250	270	0

3.3. Conditional simulation

The ore and the waste domains within the deposit can be simulated in a block grid with 10*10*10 meters by use of the obtained parameters from truncated gaussian model, including flag, truncation threshold and variograms of the underlying Gaussian random field. The simulation should be conditioned to the vertical proportion curve (VPC) of domains (Figure 10). VPC is a simple tool to quantify the variation in the amount of each domains existing as a function of depth that calculate the proportions from experimental data. They are computed along lines parallel to the selected reference level [10]. In this case, the references level is considered parallel to the horizon because the result of the simulations were consistent with the interpreted geology of the deposit and reproduce the spatial variations of the facies proportions.

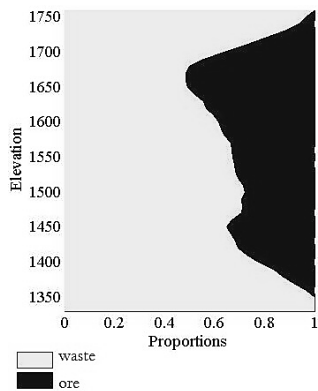


Figure 10. Vertical proportion curves.

3.4. Simulation results

Based on the hard data at drill hole locations, 100 realizations were calculated on a grid with block size of 10*10*10 m. displays one of these realizations which have a good match with the actual data. From the geological point of view, the boundaries between ore and waste domains are reproduced. Although, randomness in simulation procedure caused some simulated points as ore in which there are no ore data in the neighborhood of such points.

Figure 12 shows the probability map of 100 realization for ore domain. The map is obtained by calculating the frequency of ore over the 100 realizations for each location. In this map, the red and orange regions represent certain ore domains, while the blue regions correspond to locations where it is unlikely to be an ore domain. The intermediate regions indicate a greater uncertainty on whether the ore can be found or not.

3.5. Validation of the simulation

After producing different realization of the simulated domains, one can validate the simulation results by checking the hard data and the model parameters or comparing the histograms and variograms of the realizations with those of the experimental data [14, 19]. Figure 13 shows the histograms of the experimental and simulated values. It shows that the percentage of the waste has increased, because of adding a specific volume of the waste during the simulation procedure. Figure 14 indicates the directional variograms of the ore domain along the two principal directions (Dip=10 and Dip=80), calculated on the basis of 11,570 original and 726,331 simulated values. Again, the agreement is satisfactory; the simulated domains reproduce variability of the input data but the variograms are slightly different.

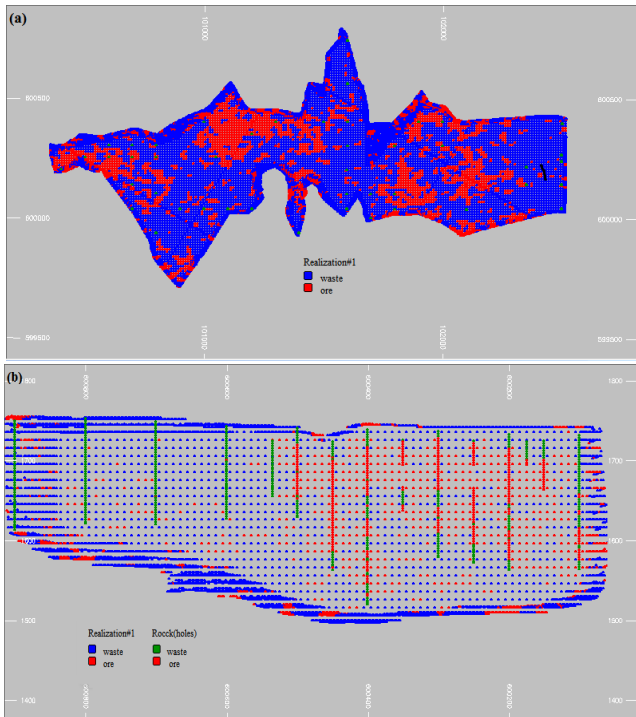


Figure 11. A conditional realization of domains in Golgohar deposit over the (a) plan with elevation=1530.6m and (b) cross-section with northing 101565.8 m.

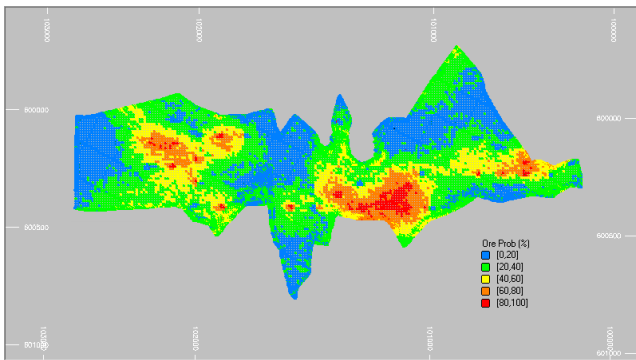


Figure 12. Probability map of ore domain in Golgohar deposit over the plan with elevation 1530.625 m.

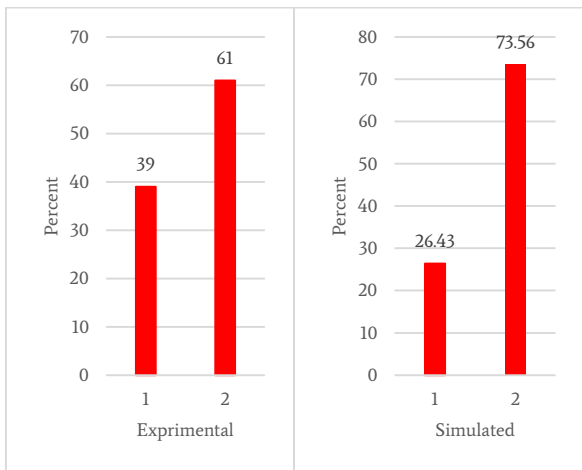


Figure 13. Histograms of experimental and simulated values (1=ore and 2=waste).

Finally, Figure 15 shows the convergence of the means and standard deviations of the facies proportions while increasing the number of realizations. The means of the facies proportions converge after about

18 realizations and their standard deviations converge after about 20 realizations.

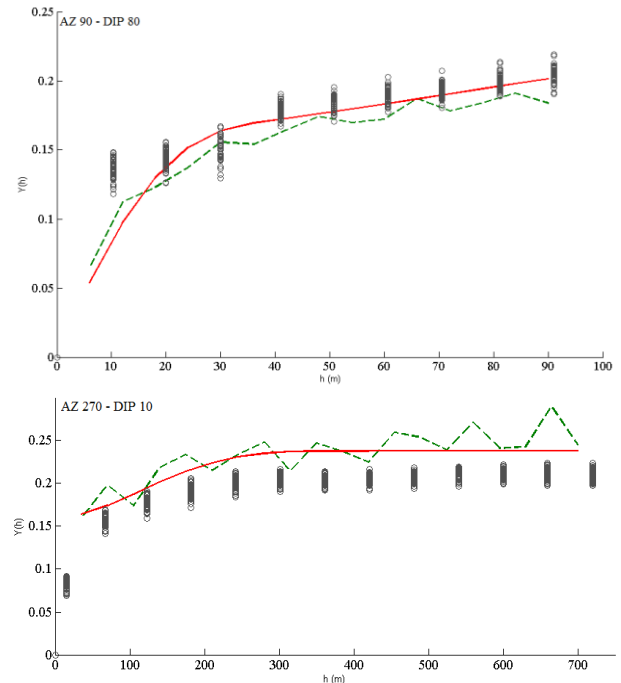


Figure 14. Experimental (green dashed lines), modeled (red solid lines) and post-simulation indicator variograms (dots) of the ore domain.

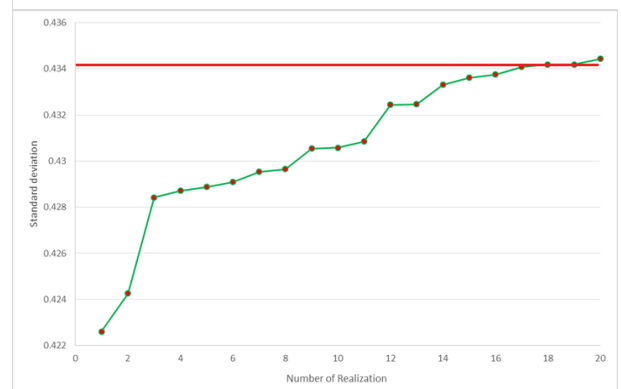
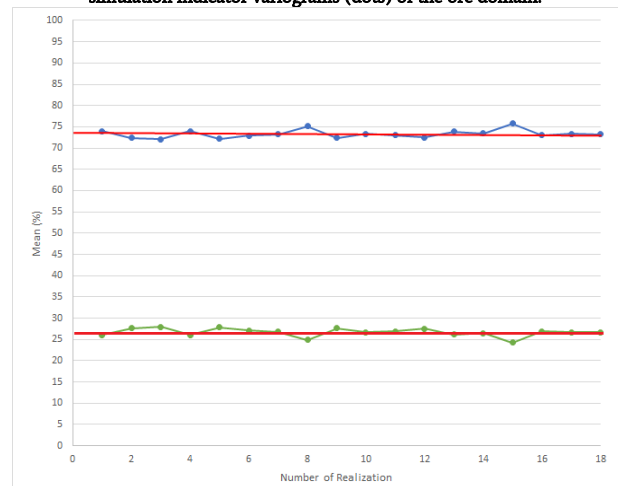


Figure 15. Dependence of the mean and standard deviations of the facies proportions to the number of the realizations.

4. Discussion

In this paper, a stochastic approach based on geostatistical simulation has been proposed for domain modeling. It considers the application of the truncated gaussian model for simulating the boundary between geological domains (in this case, ore and waste domains), their spatial continuity (indicator variograms), the sampling information, as well as the prior geological knowledge on spatial distribution of domains. The implementation of the truncated gaussian model is quite straightforward and leads to realistic realization of the rock type distribution in the deposit. The realizations are then used to calculate the probabilities of occurrence of the different lithotype over the region of interest, which in turn are combined with the grade estimates obtained by ordinary kriging for each lithotype. This approach accounts for the uncertainty in the geology of the ore.

Figure 16 displays the simulation results for one realization at a specific section at Golgohar iron deposit. Ore percentage at each data location within the deposit can be calculated through 100 realizations.

There are many tools to show the uncertainty of the boundary modeling. TGS provides many realization of the domains, each of which has a specific boundary between domains. The uncertainty of the boundary can be computed by means of the realizations. Figure 17 displays the ore probability and the chance of being ore at any data location. Within the areas with 100 % probability of ore, it can be concluded that the area is 100 % ore. Therefore, at this boundary, the uncertainty is almost 0 %. In Figure 17, domains with 80 % probability of ore reveal an area with almost 20 % uncertainty of ore-waste boundary.

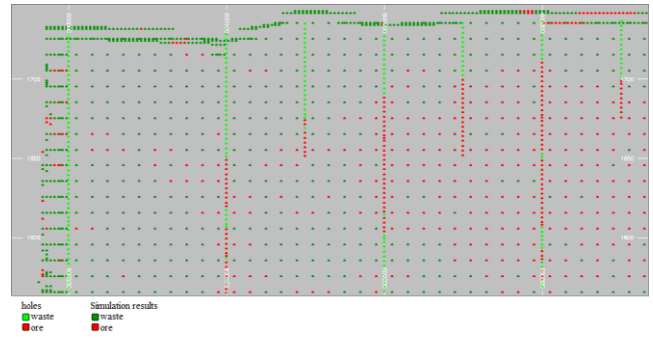


Figure 16. one realization of ore-waste simulation at a selected section.

Each realization is a scenario for ore tonnage estimation that affects future economy of mine and all preceding mine designs and mine plans. shows the most, least and moderate scenarios for ore tonnage estimation. As it shows, the realization number 27 has the most ore tonnage, but it does not mean that it is the best scenario for mine design. A good approach for this purpose is based on compositing matrix that can measure the accuracy of each realization with numbering the true simulated locations. All scenarios are ranked based on the calculated accuracies. The best scenario is the one with the most accuracy and 96 million cubic meters tonnage, and the worst one on the other hand has the least accuracy.

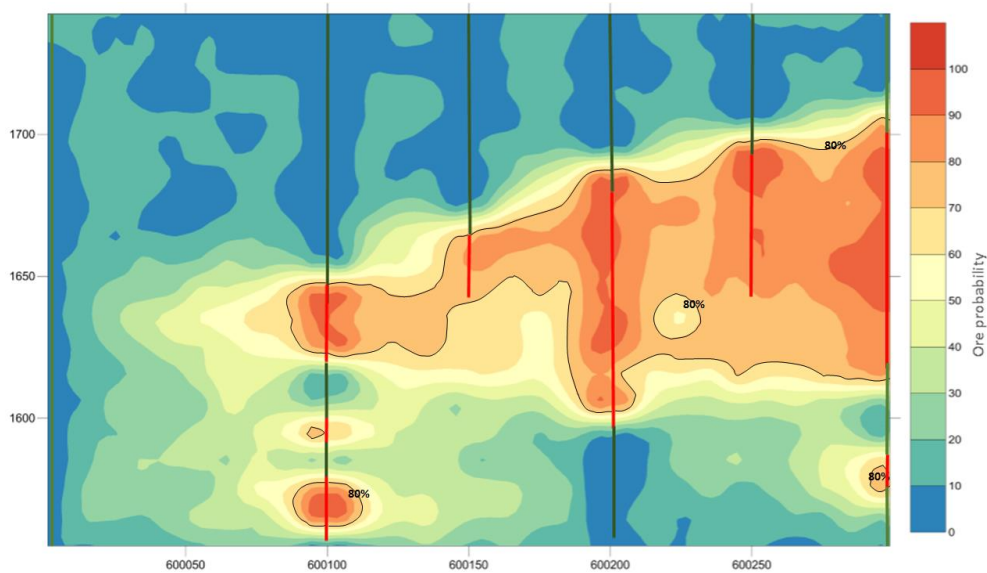


Figure 17. ore probabilities at a selected section.

Table 3. Ore tonnage estimation scenarios.

	Most	least	Moderate 1	Moderate 2	Mean of Tonnage
Number of realization	sim 27	sim 100	sim 92	sim 35	---
Ore tonnage (Mm ³)	104	86	94	94	94

Table 4. The accuracy of different realization.

	Best	Worst	Mean	Mean of accuracy
Number of realization	sim 44	sim 30	sim 100	---
Accuracy (%)	86.53	83.76	85.19	85.20
Tonnage (Mm ³)	96	87	86.5	---

5. Conclusion

In Golgohar iron deposit, separation of the ore and waste domains has affected the ore tonnage estimation and resource evaluation. Therefore, it is necessary to model the layout of the ore domain and its contact with waste domain, in order to create a better and more accurate reproduction of the deposit features and to plan the mining process. In this study, the focus was on stochastic approaches, based on conditional simulation, in order to reproduce the spatial variability of mineralization and to evaluate the uncertainty of the ore-waste boundary.

Modeling these domains boundaries is a challenge in mine planning and mine design. Deterministic approaches provide just one scenario of the domains but the stochastic methods, e.g. TGS, can generate

realizations of the domains and measure the uncertainty of such boundaries. Truncated Gaussian simulation has been proved to be versatile when geological units have a sequential geometry and/or there are few number of indicators (ore and waste). Using this method, one is able to propose the best scenario for future economy of mine and all precedent mine designs and mine plans. As a result, the scenario number 44 is chosen to be the best scenario for ore-waste layout.

Acknowledgments

The authors are grateful to Kusha-Madan consult Co. for providing the dataset used in this research. We also thank Hassan Talebi for his useful and constructive comments.

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