

## An Imperialist Competitive Algorithm for Solving the Production Scheduling Problem in Open Pit Mine

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### Abstract

Production scheduling (planning) of an open-pit mine is the procedure during which the rock blocks are assigned to different production periods in a way that the highest net present value of the project achieved subject to operational constraints. This paper introduces a new and computationally less expensive meta-heuristic technique known as imperialist competitive algorithm (ICA) for long-term production planning of open pit mines. The proposed algorithm modifies the original rules of the assimilation process. The ICA performance for different levels of the control factors has been studied and the results are presented. The result showed that ICA could be efficiently applied on mine production planning problem.

**Keywords:** *Imperialist Competitive Algorithm; Integer linear programming; meta-heuristic methods; Open pit mine production scheduling.*

### 1. Introduction

Sustainable supplying of the raw minerals is the vital requirement of modern human societies. Mining in either surface or underground modes is still the major source of the raw mineral production; however, surface mining and in particular open pit mining plays a bigger role due to their higher production rates. Open pit Mining, in general concept, starts with a small excavation on the ground, called pit, and is extended to the larger and larger pits till the final outline of the mine, called ultimate pit limit or UPL, to be reached. Every pit comprises several benches on which the large-scale mining machinery operates as well as some inter-bench raps to provide the transportation paths for the equipment. In this

regard, having a sophisticated and optimized production plan is crucial for most of the current mining activities of marginal economic condition. Accordingly, it should be addressed that where the first access to deposit should be, how to expand the excavations towards the final configuration and how to differentiate between valuable ore and worthless waste material. This is a large scale and complicated optimization problem from mathematical point of view, which has been severely investigated by mining researchers and mathematicians.

In general, the studies are based on a geological block model, which is a set of imaginary regular rock cuboids that cover the ore deposit and surrounding rocks. Then a set of related characteristics such as metal grade, density and

existing impurities are considered for all blocks and their contemplated values are estimated based on the assayed sampling data coming from surface or drill-hole samples. The constructed block model together with the technical and economic parameters enter into production planning stage that in turn denotes to finding the extraction sequence of the blocks in a way that leads to the highest net present value, NPV, of the project cash flows, while meeting the technical constraints such as mining capacity, processing capacity, sequencing and pit slope constraints [1]. In other words, three distinct and very important decisions must be made about each block of the model during production planning process [2]:

- Whether a given block should be mined by the end of mine life or not?
- If yes, when should it be mined?
- Once the block is mined, to which destination should it be hauled?

This is usually executed in three levels of planning time frames:

- Long-term mine planning, which may cover a period of 5 to 10 years or even up to decades and generally addresses the overall developmental questions.
- Mid-term mine planning, which concerns to a shorter period of 3 to 5 years. It provides a forecast of the company's policies over the coming few years in terms of the feasibility, profitability and financing.
- Short-term mine plan focuses on a production period of some days to few years in order to increase the accuracy and reliability of the mining operation.

Studies on open pit production planning and optimization started from 1968 [3] and, conventionally, several methodologies have been applied such as integer programming [2, 4], mixed integer programming [5, 6], dynamic programming [7] and maximum flow algorithm [8]. The fundamental and significant drawback of these approaches refers to their high computational cost when applied to real size.

Investigation on higher class of heuristics, called meta-heuristic, has been of interest recently. A meta-heuristic is a set of algorithmic concepts that

can be used to enhance the applicability of heuristic methods on difficult problems. These concepts are usually inspired from the biology and the nature. The use of meta-heuristic methods has significantly increased the ability of finding very high quality solutions for hard combinatorial problems in a reasonable computation time [9].

Several of these methods used to solve open pit production planning problem such as Genetic Algorithms [10, 11], Simulated Annealing [12], Ant Colony Optimization [13, 14], Tabu Search [15] and Particle Swarm [16, 17].

This paper introduces the application of a relatively unknown meta-heuristic, called Imperialist Competitive Algorithm (ICA), on production planning of open pit mines and investigates its comparative advantages to the mathematical modelling.

### 1. Mathematical formulation of Production planning problem

The binary linear integer-programming (LIP) model has been quite frequently used to model the conventional production planning problem of open pit mines. Its net present value (NPV) maximization objective function could be stated as below [18]:

$$\text{Maximize } \sum_{t=1}^T \sum_{n=1}^N \frac{v_m}{(1+d)^{t-1}} x_m \quad (1)$$

Where,

$x_{tn}$ : Binary decision variables of the model ( $x_{tn} = 1$  if block  $n$  is mined in time period  $t$  and  $x_{tn} = 0$  otherwise).

$v_{tn}$ : Economic value of a block calculated from the net return from mining of block  $n$  in period  $t$ .

$d$ : Annual discount rate.

$T$ : Number of time periods.

$N$ : Number of mine blocks.

The economic value of a block equals to the difference between revenue earned from selling the commodity (mineral) content of that block and the total costs involved in its mining and processing. For example, for a copper mine, the economic value of block,  $n$ , is calculated as follows:

$$Return_n = \left( \begin{aligned} & [(Copper\ price - Sales\ costs) \times Recovery \times Tonnage_n \times grade_n] \\ & - [(Mining\ cost + Milling\ cost + Smelting\ cost \\ & + Refining\ cost + Transport\ cost)] \end{aligned} \right) \quad (1)$$

$$v_n = \max [Return_n, (-Mining\ cost_n)] \quad (2)$$

Described objective function is subject to the following constraints [18]:

Mining capacity constraints: Total tonnage of extracted material should be between a pre-determined upper and lower limit for each scheduling period.

$$MC_{min}^t \leq \sum_{n=1}^N W_n \cdot x_m \leq MC_{max}^t, \text{ for } \forall t \quad (3)$$

Where,

$W_n$  : Tonnage of block  $n$ .

$MC_{max}^t, MC_{min}^t$ : Maximum and minimum acceptable mining capacity for period  $t$ .

Processing capacity constraints: Quantity of ore production should satisfy processing capacity limitations:

$$PC_{min}^t \leq \sum_{n=1}^N O_n \cdot x_m \leq PC_{max}^t, \text{ for } \forall t \quad (4)$$

Where,

$O_n$  : Tonnage of ore block  $n$ . If the block economic value is greater than zero ( $v_n > 0$ ), it will be considered as ore.

$PC_{max}^{tm}, PC_{min}^{tm}$  : Maximum and minimum tolerable processing capacity for period  $t$ .

Reserve constraints: This constraint are mathematically necessary to ensure that a block is mined only once in the model.

$$\sum_{t=1}^T x_m \leq 1, \text{ for } \forall n \quad (5)$$

Sequencing constraints: The sequencing constraints ensure that a block can only be removed if all overlaying blocks have been removed in earlier periods or considered for the same  $t$  period.

$$\sum_{l=1}^t (x_{lj} - x_{li}) \geq 0, \quad t = 1, \dots, T, \text{ for } \forall (i, j) \in A \quad (6)$$

Where,  $A$  is the set of pairs  $(i, j)$  of blocks such that block  $j$  is a key block to block  $i$  and must be removed before block  $i$  can be mined.

## 2. Imperialist competitive algorithm (ICA)

Imperialist competitive algorithm (ICA) is a new population-based meta-heuristic algorithm proposed by Atashpaz-Gargari and Lucas (2007), inspired from the socio-political process of imperialism and imperialistic competition [19]. Algorithm's capability in dealing with different types of optimization problems has been proven by the authors [20]. Similar to any evolutionary algorithms, ICA also starts with an initial population of solutions, called countries, representing the concept of the nations. Reflecting the quality of objective function in each solution, some of the best countries in the population are chosen to be the 'imperialists' and the rest are assumed to be the 'colonies' of those imperialists. The set of one imperialist and its colonies is called an 'empire' [21]. Over the time, imperialists try to extend their own characteristics to the governing colonies; however, it is not totally a controlled procedure and revolutions might happen in each country. Countries can also leave from their empire to others if they see higher chance of promotion there. ICA has extensively been used to solve different kinds of optimization problems. For example, this method are used for stock market forecasting [22], digital filter design [23], traveling salesman problems [24], multi-objective optimization [25], integrated product mix-outsourcing problem [26] and scheduling problem [27, 28].

The methodology of ICA implementation in open pit mine production planning is described as follows. The flowchart of the proposed procedure has been illustrated in Figure 1.

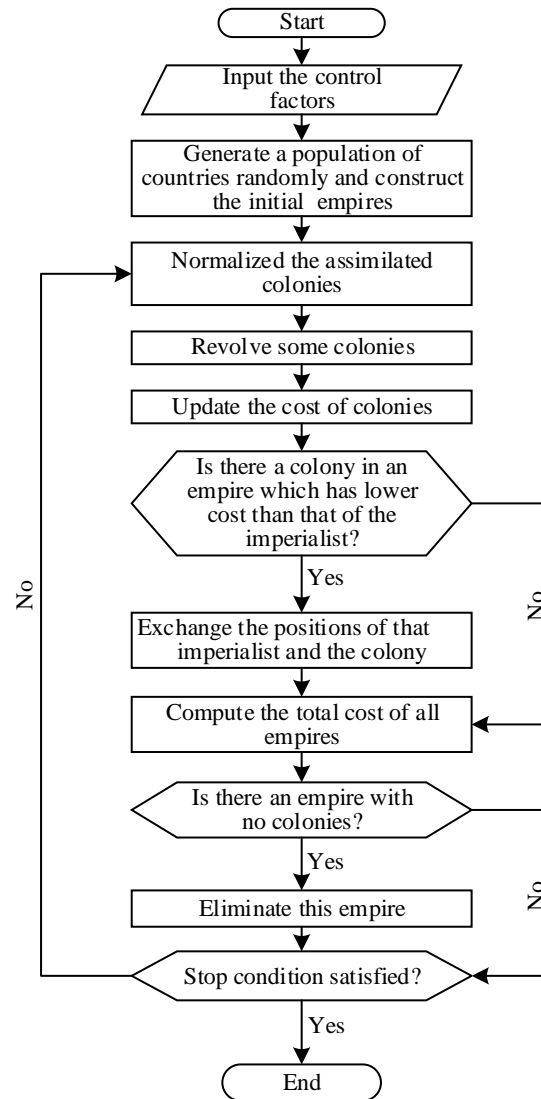


Figure 1. Flowchart of imperialist competitive algorithm

**2.1. Generating initial empires**

Solution of any optimization problem, herein called country, is an n-dimensional array as:

$$\text{country} = [p_1, p_2, p_3, \dots, p_n] \quad (7)$$

Where  $p_i$ s are decision variables that their values need to be determined in order to maximize or minimize the objective function. Decision variables of open pit mine production planning problem are the extraction time of the blocks in block model and the objective is to maximize the net present value (NPV) of the project. Processing capacity constraints is modelled by constant penalties into objective function for any unit of its exceeding from determined limits as below:

$$\text{Maximize} \sum_{t=1}^T \sum_{n=1}^N \frac{v_m}{(1+d)^{t-1}} x_m - P_t^- O_t^- - P_t^+ O_t^+ \quad (8)$$

Where,

$P_t^-, P_t^+$ : Represent the deducting unit costs (penalty) for shortage or surplus of ore produced in period  $t$ , respectively.

$O_t^-, O_t^+$ : Represents the shortage or excess amount of ore produced in period  $t$ , respectively.

To start the optimization algorithm, an initial population is generated containing  $N_{pop}$  solutions (mine schedules) among which  $N_{imp}$  of the most powerful solutions (according to their NPV) are selected as imperialists. The rest of the population members ( $N_{col} = N_{pop} - N_{imp}$ ) will be the colonies of the chosen empires. To form the primary empires, colonies are randomly divided among the imperialists based on their power as the higher the power of an empire, the more the number of colonies belong to that. To proportionally distribute the colonies among imperialists, normalized cost of  $n^{th}$  imperialist is defined as [26]:

$$C_n = \max \{c_i\} - c_n, \quad i = 1, 2, \dots, N_{imp} \quad (9)$$

Where  $c_n$  and  $C_n$  are the cost and the normalized cost of  $n^{th}$  imperialist, respectively. The objective of ICA is set to minimize the sum of the cost function value of all countries. To convert the production planning problem from maximization to minimization, minus of the net present value is considered as the cost function.

$$\text{cost}(\text{country}) = -NPV \tag{10}$$

Hence the normalized power of each imperialist can be determined as below [26]:

$$pow_n = \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \tag{11}$$

The normalized power of an imperialist indicates the number of colonies that should be probably controlled by that imperialist. Thus, the initial number of colonies of an empire will be as follows [26]:

$$ColEmp_n = \text{round}(pow_n \times N_{col}) \tag{12}$$

Where,  $ColEmp_n$  is the initial number of the colonies of  $n^{th}$  empire that are chosen randomly amongst whole colony population. Figure 2 illustrates the construction of the initial empires. As shown, the better (bigger) an empire, the more the number of its colonies and vice versa.

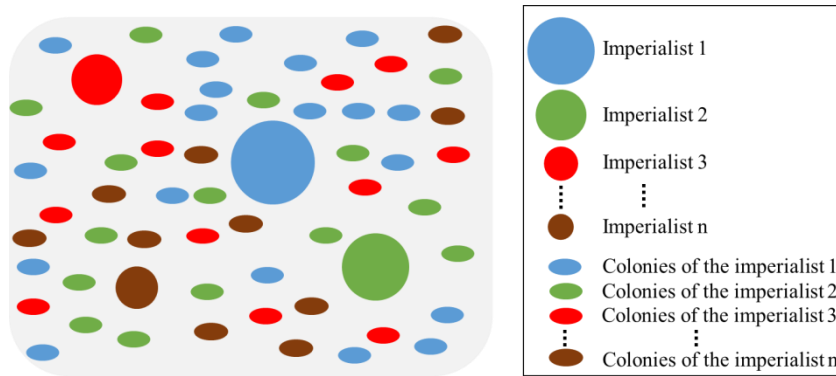


Figure 2. Generating the initial empires [26].

**2.2. Modified assimilation process**

The colony will approach to the imperialist along different socio-political axes such as culture, language, etc. Figure 3 sketches this movement. As shown in Figure 4, the assimilating operator for open pit mine production planning problem follows the below steps:

- Create the country array with the size equal to the number of blocks in the block

model of the mine. The value of the cell  $n$  in array is equal to 1 if block  $n$  is mined (the colored blocks in Figure 4) and equal to 0 otherwise.

- Select a sub-array randomly in imperialist array (for example: cells 5 to 11).
- Copy the imperialist sub-array to colony array.

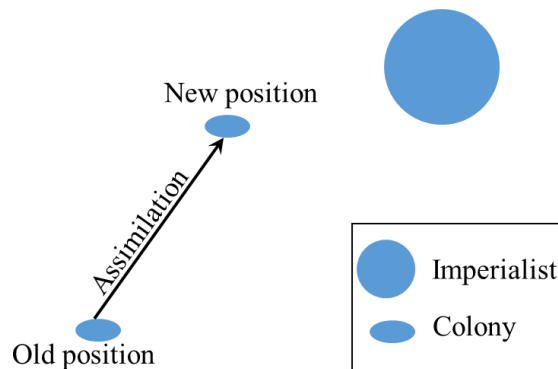


Figure 3. Movement of colonies toward their relevant imperialist [26].

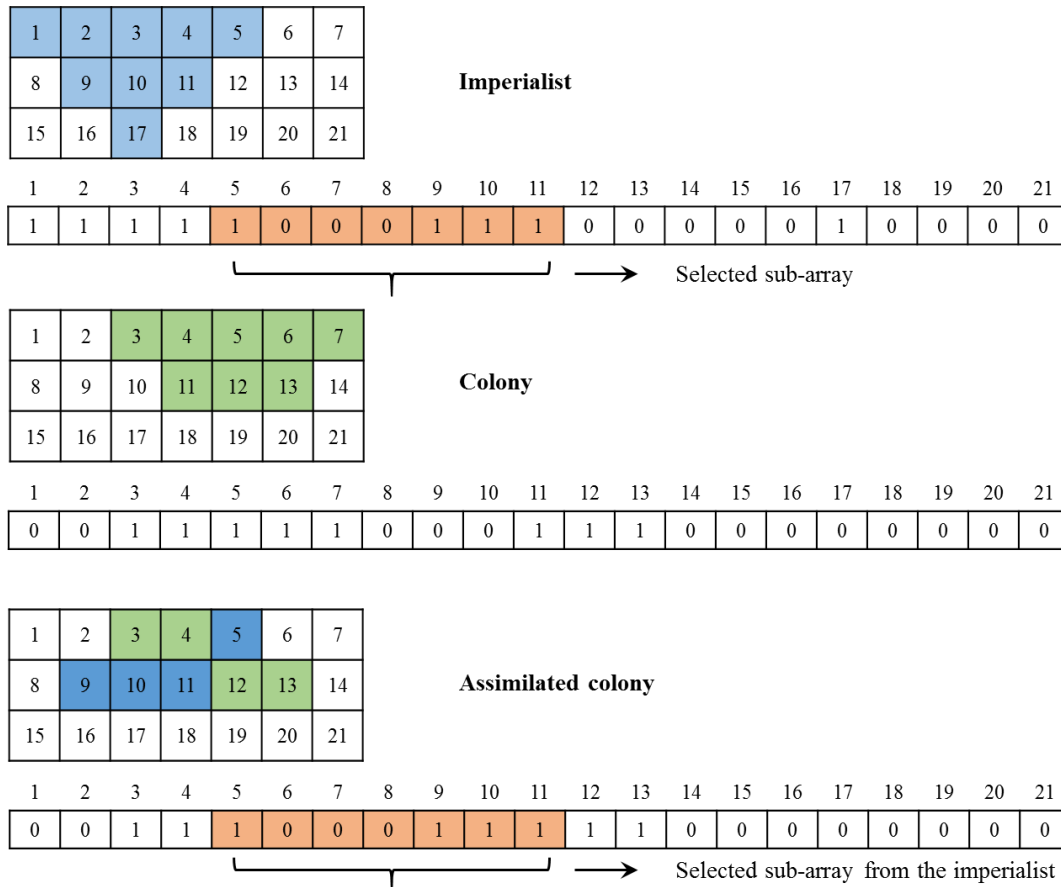


Figure 4. Assimilation operator for production planning problem

**2.3. Normalization**

Usually, the assimilated colony (Figure 4) do not result in a feasible pit shape respecting the slope angles and block precedence. Consequently, a

normalization procedure is required to fix the violations of the sequencing constraints (equation (7)), as shown in Figure 5.

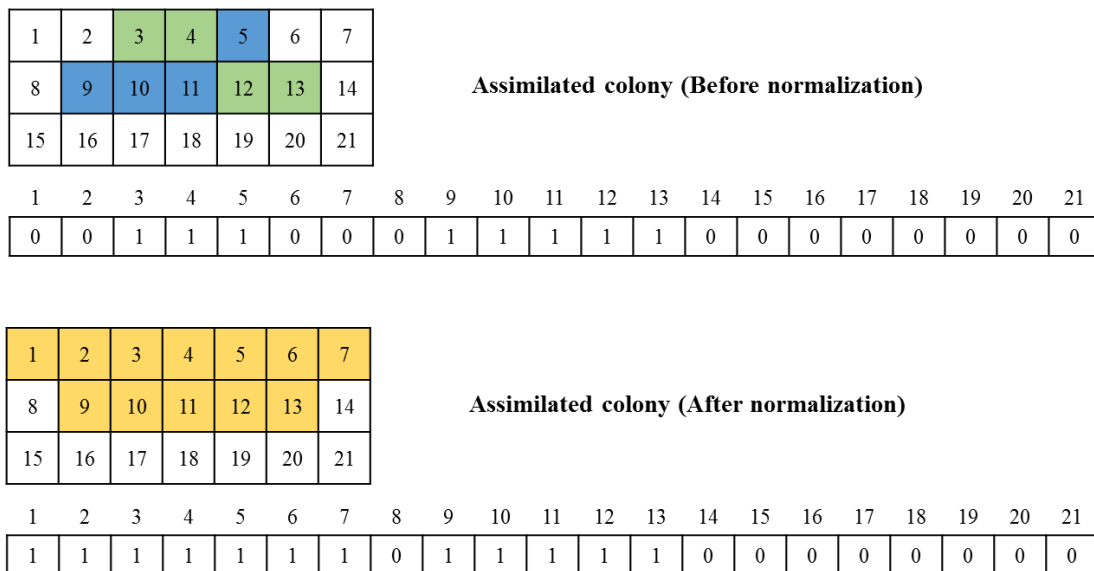


Figure 5. Normalization process for the assimilated colony

### 2.4. Revolution

Revolution operator diversifies ICA to explore more new regions. Revolution mechanism prevents the algorithm from being trapped in local optima [29]. For this purpose, the weakest colony (production plans with the lowest net present value) in each iteration is selected and replaced with a new one, randomly.

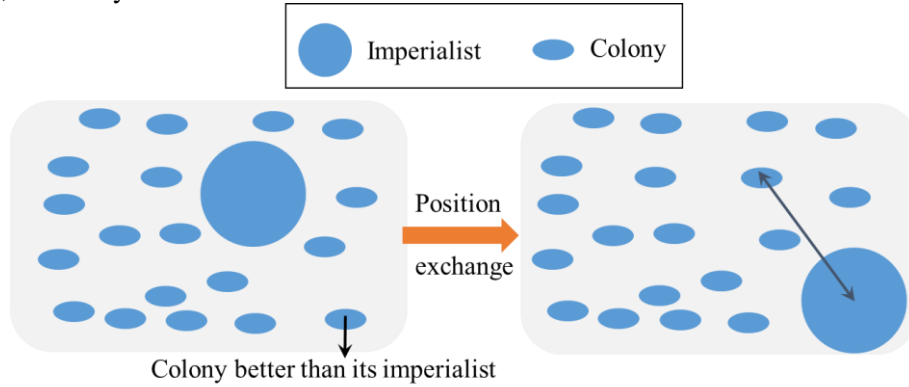


Figure 6. Exchanging the positions of a colony and the imperialist [26].

### 2.6. Total power of an empire

Total power of an empire is mainly based on the power of its imperialist country; however, the

$$TC = \text{cost}(\text{imperialist}_n) + \xi \text{ mean}(\text{cost}(\text{colonies of empire}_n)) \quad (13)$$

Where  $TC$  is the total cost of the  $n^{\text{th}}$  empire and  $\xi$  is a positive number which is considered to be less than 1.

### 2.7. Imperialistic competition

In general, empires try to take the possession of colonies of the other empires and control them. Through this process, the power of weaker empires will decrease and that of the more powerful ones will increase. This competition is

$$NTC_n = \max \{TC_i\} - TC_n$$

Where  $TC_n$  and  $NTC_n$  are respectively total cost and normalized total cost of  $n^{\text{th}}$  empire. Now the possession probability of each empire can be given by [20]:

$$p_n = \frac{NTC_n}{\sum_{i=1}^{N_{imp}} NTC_i} \quad (15)$$

$$r = [r_1, r_2, r_3, \dots, r_{imp}], \quad r_1, r_2, r_3, \dots, r_{imp} \sim U(0,1) \quad (17)$$

Then, vector  $D$  is formed by subtracting  $R$  from  $P$ .

$$D = P - R = [d_1, d_2, d_3, \dots, d_{N_{imp}}] = [p_1 - r_1, p_2 - r_2, p_3 - r_3, \dots, p_{N_{imp}} - r_{N_{imp}}] \quad (18)$$

Referring to vector  $D$ , the mentioned colonies in an empire whose corresponding index in  $D$  is

### 2.5. Exchanging positions of the imperialist and a colony

Whereas moving toward the imperialist, a colony might reach to a position with lower cost function than that of its imperialist. In this condition, the position of the imperialist and colony are swapped. Figure 6 depicts the position exchange between a colony and the imperialist.

powers of its individual colonies have also an effect, albeit relatively insignificant. Therefore, the total cost of an empire is as [20]:

modeled by choosing one of the weakest colonies of the weakest empire and making a competition among all empires to possess this colony. Figure 7 illustrates the modeling of the imperialistic competition. The possession probability of each empire is proportional to its total power. The normalized total cost of each empire is determined as [20]:

$$i = 1, 2, \dots, N_{imp} \quad (14)$$

To divide the mentioned colonies among empires based on the possession probability of them, vector  $P$  is formed as follows [20]:

$$P = [p_1, p_2, p_3, \dots, p_{N_{imp}}] \quad (16)$$

Then, the vector  $R$  with the same size as  $P$  whose elements are uniformly distributed random numbers between 0 and 1 is created.

maximum will be the winner of the possession competition [20].

2.8. Eliminating the powerless empires  
 Powerless empires will collapse in the imperialistic competition and their colonies will be distributed among other empires. In current

study, an empire collapses when it loses all of its colonies [20].

Stopping criteria

The algorithm continues until no iteration is remaining or just one empire exists in the world.

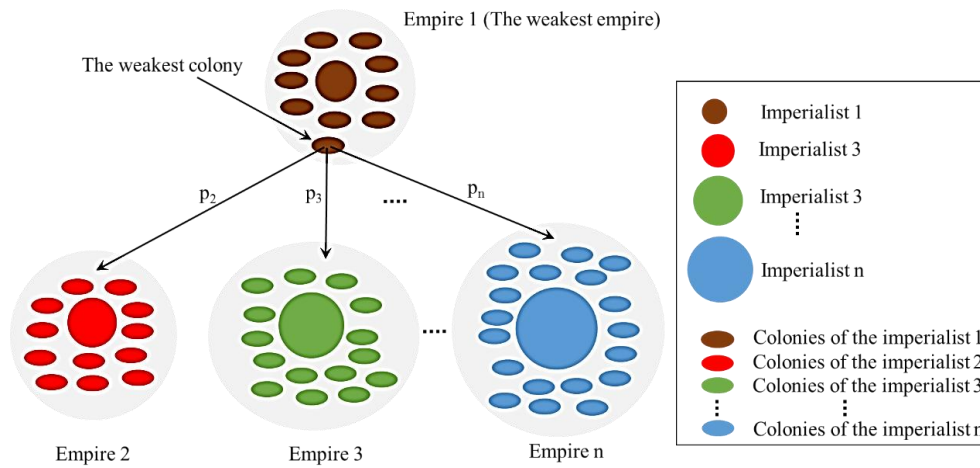


Figure 7. Imperialistic competition [20].

3. Numerical example

A hypothetical numerical examination was conducted to test the performance of the imperialist competitive algorithm in an open pit mine production planning problem. As shown in Figure 8, it consists of a copper deposit with geological block model containing 200 blocks. Table 1 displays the technical and economic

parameters assumed for the construction of the economic block model. The mine will be operated for 5 years and the maximum and minimum mining capacities are annually 23 and 17 blocks, respectively. The maximum and minimum processing capacities are considered to be 15 and 9 ore blocks per year, respectively, and the discount rate assumed to be 10%.

0.04	0.05	0.06	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
0.04	0.17	0.15	0.14	0.20	0.17	0.17	0.12	0.19	0.22	0.10	0.18	0.05	0.04	0.25	0.17	0.31	0.20	0.16	0.02
0.04	0.17	0.11	0.12	0.20	0.20	0.15	0.14	0.28	0.07	0.19	0.28	0.10	0.25	0.25	0.18	0.01	0.38	0.32	0.01
0.01	0.13	0.20	0.24	0.39	0.25	0.12	0.14	0.02	0.25	0.29	0.09	0.25	0.23	0.31	0.25	0.02	0.19	0.02	0.02
0.01	0.23	0.11	0.26	0.22	0.02	0.13	0.20	0.14	0.17	0.30	0.16	0.13	0.28	0.11	0.22	0.20	0.15	0.03	0.03
0.01	0.04	0.20	0.21	0.02	0.29	0.15	0.17	0.30	0.29	0.26	0.32	0.04	0.26	0.41	0.17	0.19	0.21	0.25	0.01
0.02	0.17	0.18	0.27	0.24	0.33	0.21	0.08	0.35	0.36	0.17	0.22	0.23	0.38	0.23	0.25	0.18	0.09	0.22	0.01
0.03	0.03	0.37	0.18	0.42	0.10	0.25	0.15	0.18	0.16	0.13	0.18	0.16	0.25	0.15	0.15	0.03	0.20	0.09	0.02
0.02	0.34	0.12	0.24	0.29	0.38	0.21	0.20	0.16	0.15	0.16	0.16	0.13	0.32	0.27	0.14	0.15	0.17	0.23	0.02
0.17	0.12	0.27	0.19	0.23	0.16	0.07	0.04	0.13	0.17	0.18	0.06	0.14	0.20	0.34	0.16	0.32	0.18	0.02	0.02

Figure 8. Geological block model assumed for the mine.

Table 1. the technical and economic parameters assumed for the construction of economic block model.

Parameters	Value
Mill recovery rate	80%
Mill concentrate grade	28%
Smelting loss (kg/ton)	10
Refining loss (kg/ton)	5
Mining cost (\$/ton)	1.5
Milling cost (\$/ton)	5.5
General and administration cost (\$/ton)	0.5
Amortization and depreciation cost (\$/ton)	0.8
Transport cost of mill concentrate to the smelter (\$/ton)	30
Smelting cost (\$/ton)	92
Transport cost of the blister copper to the refinery (\$/ton)	2
Refining cost (\$/ton)	184
Selling and delivery cost (\$/kg)	0.01
General plant cost (\$/kg)	0.01



Considering the mine life, fluctuation of the copper prices were collected in the last 5 years, i.e. from Aug 2010 to Aug 2015, Figure 9 [30]. The descriptive statistics parameters of the price

data are displayed in Table 2. The mean copper price was used for the construction of economic block model.



Figure 9. Fluctuation of the copper prices in the last 5 years [30].

Table 2. Descriptive statistics parameters of daily copper price changes.

Parameter	Value	Parameter	Value
Sample Size	1263	Min	5176.5
Mean	7551.6	25% (Q1)	6920
Variance	1.1114E+6	50% (Median)	7386
Standard deviation	1054.2	75% (Q3)	8221
Coefficient of variation	0.1396	Max	10148

The proposed procedure for applying ICA to production planning problem of open pit mines has been implemented using C++ programming environment. The most common performance measure of the algorithms used in the literature is the relative percentage deviation (RPD) which has been used in current study and is calculated as follow:

$$RPD = \frac{Math - ICA_{best}}{ICA_{best}} \times 100 \quad (19)$$

Where  $ICA_{best}$  and  $Math$  are respectively the best solutions generated by ICA and the optimal solution found by mathematical method i.e. equations (1) to (7).

The investigative runs of the program were conducted to test the performance of ICA. As mentioned earlier, the control factors in ICA are:  $\xi$ , the number of countries ( $N_{pop}$ ) and the number of imperialists ( $N_{imp}$ ). Different levels of these factors were used in optimization which is summarized in Table 3.

Table 3. Control factors of ICA

Factors	Value
Number of countries ( $N_{pop}$ )	15, 25 and 35
Number of imperialists ( $N_{imp}$ )	3, 4 and 5
$\xi$	0.03, 0.05 and 0.07

Due to the stochastic nature of the ICA, each problem was solved 10 times using ICA. The results of the investigations are shown in Figure 10. As it can be seen, the best solution has been

generated for  $N_{pop} = 35$ ,  $N_{imp} = 5$  and  $\xi=0.05$  that its RPD is equal to % 1.67. The convergence behavior of this solution in different iterations is shown in Figure 11.

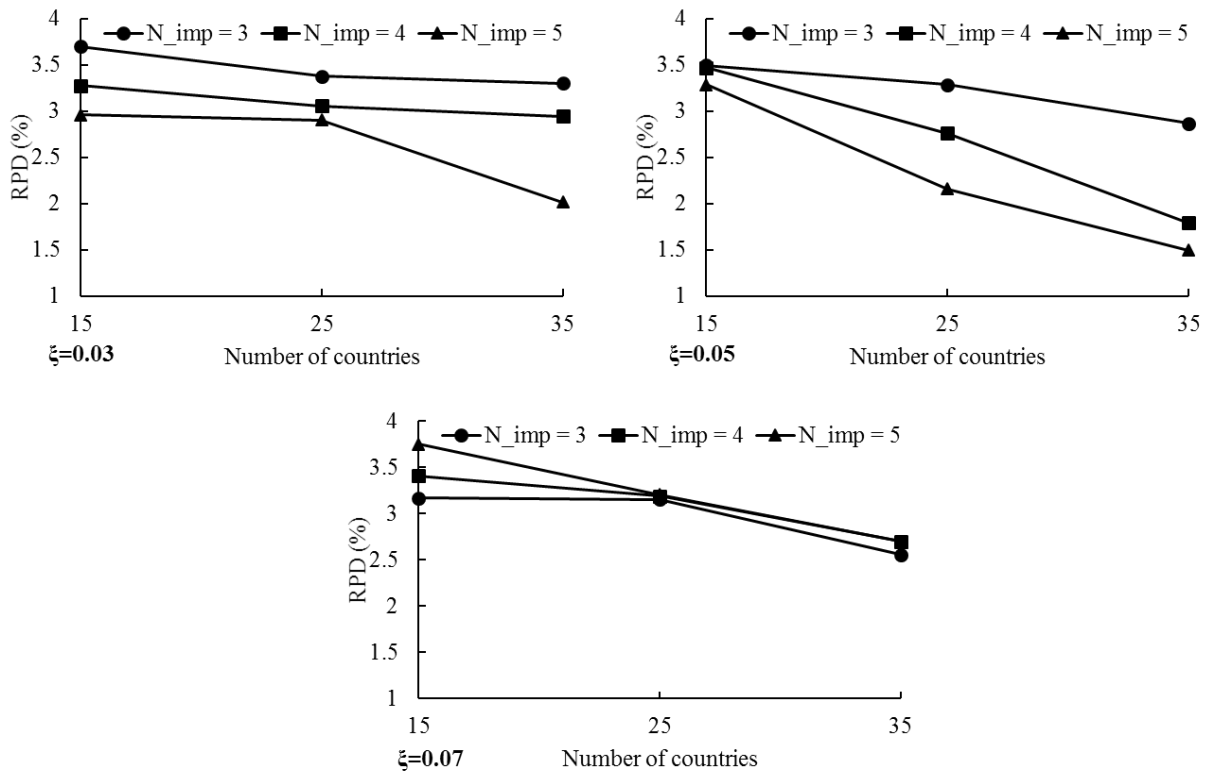


Figure 10. RPD for different values  $N_{pop}$  and  $N_{imp}$  in  $\xi=0.03$ ,  $\xi=0.05$  and  $\xi=0.07$

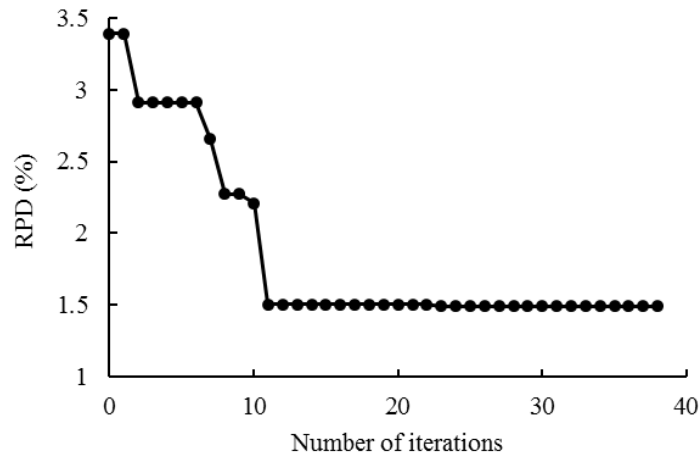


Figure 11. Convergence behaviour of the best solution.

To evaluate the effect of each control factor ( $N_{pop}$ ,  $N_{imp}$  and  $\xi$ ) on the ICA optimization procedure, mean value of RPD for different combinations of these factors was implemented that have been shown in Figure 12. It is obvious from Figure 12 that the performance of ICA becomes better while increasing the number of countries and imperialists, also ICA has good performance in  $\xi = 0.05$ .

Results of production planning for mathematical (LIP) programming method (the global optimum)

and the best solution of ICA are compared and shown in Figures 13 and 14. The detail of comparison is illustrated in Table 4. As shown in Table 4, the net present value of ICA approach is equal to \$ 199.032, which is almost equal to the LIP method and the small quantities of RPD show the effectiveness of ICA in production planning of mine. The number of blocks inside the ultimate pit limit (UPL) in both approaches is equal which represents ICA accuracy in determination of UPL.

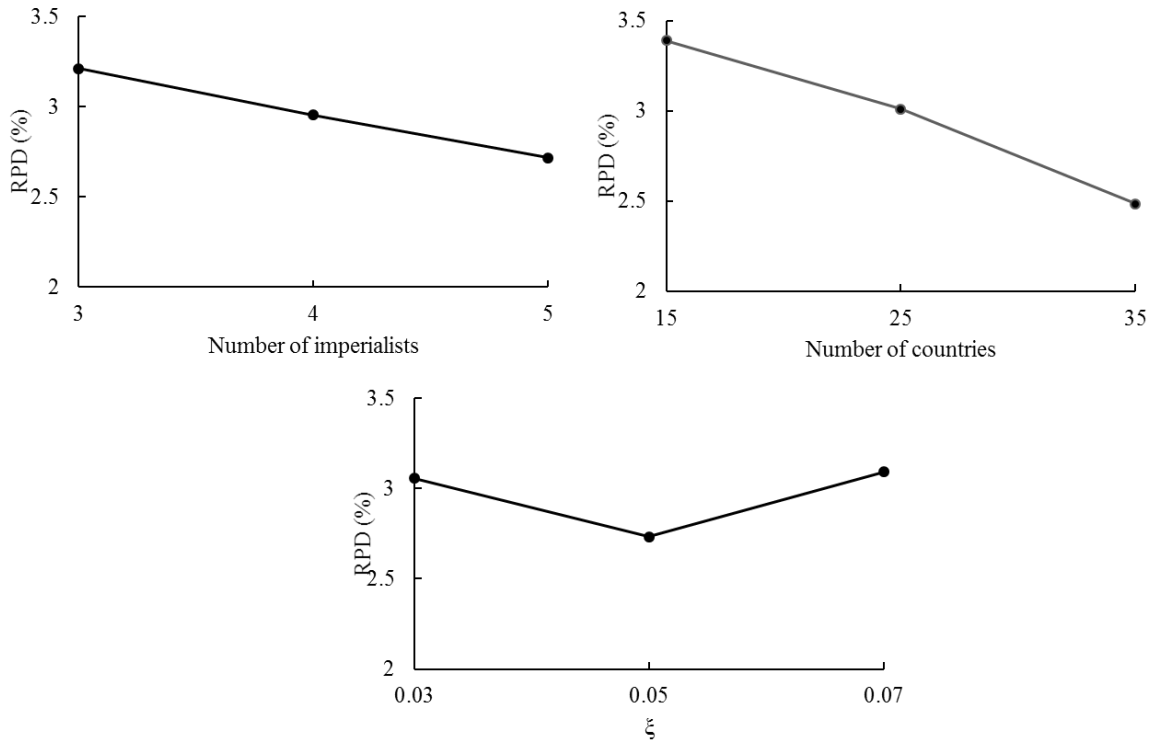


Figure 12. The mean RPD versus the number of imperialists, number of countries and  $\xi$ .



Figure 13. Mine production planning by mathematical method (The numbers are indicative of block extraction year).



Figure 14. Mine production planning by ICA method (The numbers are indicative of block extraction year).

**Table 4. Comparison of stochastic and conventional approaches.**

Factors	ICA	LIP
Net present value (1000 \$)	199.032	202.353
Relative percentage deviation (%)		1.67
Number of blocks inside the UPL	110	110
Number of blocks inside the first period	23	23
Number of blocks inside the second period	23	23
Number of blocks inside the third period	19	23
Number of blocks inside the fourth period	22	23
Number of blocks inside the fifth period	23	18

#### 4. Conclusions

The current paper presents a procedure for applying a population based meta-heuristic technique known as imperialist competitive algorithm (ICA) to solve the long term production planning problem of the open pit mines, and is able to solve UPL and long-term planning problems simultaneously. In order to encode the feasible solutions, an array was used. The original assimilation process of ACI was modified by selecting a sub-array randomly in imperialist array and copying the imperialist sub-array to colony array. Penalty and normalization methods were used for the handling of capacity and sequencing constraints, respectively. The algorithm can be implemented using different levels of the control factors. As the number of countries and imperialists increase, the performance of ICA becomes better which is best achieved in  $\xi = 0.05$ . Compared to the mathematical programming method result, it was revealed that the proposed procedure can produce good quality solutions with small RPD, showing the robustness of the procedure for solving the mine production planning problem.

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