

Considering the Epistemic Uncertainties of the Variogram Model in Locating Additional Exploratory Drillholes

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Abstract

To enhance the certainty of the grade block model, it is necessary to increase the number of exploratory drillholes and collect more data from the deposit. The inputs of the process of locating these additional drillholes include the variogram model parameters, locations of the samples taken from the initial drillholes, and the geological block model. The uncertainties of these inputs will lead to uncertainties in the optimal locations of additional drillholes. Meanwhile, the locations of the initial data are crisp, but the variogram model parameters and the geological model have uncertainties due to the limitation of the number of initial data. In this paper, effort has been made to consider the effects of variogram uncertainties on the optimal location of additional drillholes using the fuzzy kriging and solve the locating problem with the genetic algorithm (GA) optimization method. A bauxite deposit case study has shown the efficiency of the proposed model.

Keywords: *additional drillholes, extension principle, fuzzy variogram model, geostatistics, kriging.*

1. Introduction

After Kim et al.'s [1] and Walton and Kaufman's [2] studies regarding the use of geostatistical methods in optimally locating additional samples and drillholes, some relatively vast and comprehensive researches have been performed to complement their results by putting together geostatistical and optimization methods. Two general tendencies can be observed in these researches: 1) the sampling plan is optimized and its objective is to optimally estimate the spatial structure [3-8] and; 2) a specified spatial structure for the known variable is assumed and its objective is

to reduce the block model's uncertainties. The focus of this paper is on the second tendency as our objective is optimally locating additional drillholes, whereas the first tendency is more related to initial drilling pattern. The main developments in the second tendency can be summarized as those related to the objective function [9, 10], extension of the studies on 3D space [11, 12], optimization of the number of additional drillholes [12, 13], and the optimization method used [7, 11, 15, 16]. The inputs of all the above methods include the locations of the initial drillholes, the variogram model parameters and the 3D geological model

of the deposit. Since the initial drillholes are limited, fitting a crisp model to the experimental semivariogram is usually difficult and the fitted model generally faces high epistemic uncertainties. Epistemic uncertainty arises from a lack of knowledge regarding the true value of variogram model parameters and is typically specified using an interval. Since the outputs of the locating procedures highly depend on the variogram model parameters, the optimum locations of additional drillholes are usually tainted with uncertainty. This uncertainty originates from epistemic uncertainty of variogram model parameters. Common kriging methods (ordinary, simple, universal, indicator, cokriging, and disjunctive) are not made for considering the effects of such uncertainties and therefore, these effects are not considered in the additional drillholes' optimal locating algorithm either. Although the effects of epistemic uncertainties are usually ignored in geostatistical studies, two fuzzy [17-19] and Bayesian Kriging [20] methods, respectively, have been proposed to study the effects of such uncertainties. Effort has been made in this paper to consider the effects of epistemic uncertainties in the objective functions of additional drillholes' optimization studies and investigate the consequences of the improvements in the results.

2. Materials and Methods

2.1. Problem statement

Consider deposit $D \subset R^3$ where in K drillholes have been drilled (in positions $x_i \in D, i=1, \dots, K$) and M samples taken and assayed. The sample grade $Z(x_i) \in D, i=1, \dots, M$ follows an intrinsically stationary process. If the deposit is divided into N blocks $v_i \in D, i=1, \dots, N$ (with specified shapes and sizes), each block's estimated grade $Z^*(v)$ and kriging variance $\sigma_k^2(v)$ can be calculated as follows:

$$z^*(v) = f(z(x_1), \dots, z(x_n), a, v) = \sum_{i=1}^n \lambda_i(x_1, \dots, x_n, a, v) Z(x_i) \quad (1)$$

$$\sigma_k^2(v) = g(x_1, \dots, x_n, a, v) = \sum_{i=1}^n \lambda_i(x_1, \dots, x_n, a, v) \bar{\gamma}(x_i, v) + \mu - \bar{\gamma}(v, v) \quad (2)$$

where $\bar{\gamma}(v, x_i)$ is the average variogram value between block v and location x_i , μ is the Lagrange multiplier, $\bar{\gamma}(v, v)$ is the average of semivariogram values of all possible paired points within the block being estimated, $a = \{a_j, j = 1, \dots, p\}$ are the variogram model parameters (nugget effect, sill, and range), p is the number of variogram model parameters (commonly $p=3$), and $\lambda_i, i=1, \dots, n$, are weights obtained by solving the kriging system [21]. The problem is to optimally locate T additional holes.

2.2. Fitness function

The common objective in studies of optimally locating additional drillholes is to minimize the average kriging variance (AKV):

$$AKV = \frac{1}{N} \sum_{\alpha=1}^N \sigma_k^2(v_\alpha) \quad (3)$$

The reason for selecting the estimation variance as a criterion for locating the additional samples is that it is known to be homoscedastic as it does not depend on data values but on semivariogram model [22]. Therefore, AKV has been used as an optimality criterion in locating additional samples by many authors [1-2,7,13]. However, it gives just the spatial configuration of neighbor data used to estimate an unsampled location (Journel and Rossi 1). It means that if the same configuration is used for distinct locations, the kriging variance will be exactly the same. Each candidate location does not have the same importance because the impact of the data obtained from drilling in different points is different [9]. Soltani et al. [23] changed the traditional objective function of AKV according to the hints of Burger and Birgenhake [24] who proposed the minimization of weighted average kriging variance (WAKV) with respect to the estimated grades of the blocks as follows:

$$WAKV = \frac{1}{N} \sum_{i=1}^N Z^*(v_i) \sigma_k^2(v_i) \quad (4)$$

The result of this change in the objective function was more tendency towards additional drill holes in the high-grade zones.

Sample locating depends not only on the initial sampling pattern, but also on the

variogram model parameters. Since this is generally impossible due to insufficient data or the experimental semi-variogram behavior (difficult fit), to exactly fit a variogram model with no epistemic uncertainties necessitates considering the effects of uncertainties in the sample locating procedure using the fuzzy kriging method. Bardossy et al. [17] model the uncertain variogram parameters with fuzzy numbers $\hat{a} = \{\hat{a}_j, j = 1 \dots p\}$ and calculated the membership function of $z^*(v)$ as follows:

$$\mu_{z^0}(z^*(v)) = \sup_{z^0, z^*(x_0)=f_0(z, a, v)} (\mu_{\hat{a}_i}(a_i)) \quad (5)$$

They defined the “width of the fuzzy number” parameter as follows to show the effects of the parameters’ uncertainties on the estimated value at any point [19]:

$$W(v) = \sup(x, \mu_{\hat{a}_i}(x) > 0) - \inf(x, \mu_{\hat{a}_i}(x) > 0) \quad (6)$$

Since the fuzzy width depends on both the arrangement as well as the grade of the samples, it is an appropriate parameter to be used in locating additional samples and therefore the objective function of the studies of locating the additional drillholes’ can be rewritten as follows:

$$WAKV = \frac{1}{N} \sum_{i=1}^N Z^*(v_i)W(v_i)\sigma_K^2(v_i) \quad (7)$$

2.3. Developing a genetic algorithm (GA) for the optimally locating the additional drill holes under the quadratic model

A genetic algorithm is used to optimally locate additional drillholes. The optimization objective is so defined as to find the optimal decision variable vector $DV = \{(x_i, y_i); i=1, \dots, T\}$ in such a way that it maximizes the objective function (Eq.7). The objective is to find the optimal set "S" containing the collars of additional drillholes, but optimal set DV contains the horizontal projection (x_i, y_i) of additional drill holes. The grade distributions along the additional drill holes could be determined based on the DV, block model boundary, composite length and Azimuth and dip of additional drill holes [11].

GA is a general-purpose search strategy for generating useful solutions to optimization

problems; it uses recombination and selection strategies, improves the solutions and produces better results [27]. Problem variables in GA are represented as genes in chromosomes which are evaluated according to their fitness values using a fitness function. This algorithm starts with a set of randomly selected chromosomes as the initial population that encodes a set of possible solutions. The optimization chromosomes consist of genes twice as many as the number (T) and corresponding to two easting/ northing directions of the additional drillholes; each gene is represented by 20 bits of binary codes, hence each chromosome is represented by 40T bits. Two genetic operators, crossover and mutation, alter the composition of genes to create a new chromosome called the offspring. The selection operator is an artificial version of the natural selection to create populations from generation to generation and chromosomes with better fitness values have higher probabilities of being selected in the next generation; after several generations, GA can converge to the best solution. A GA pseudo code is shown in Figure 1.

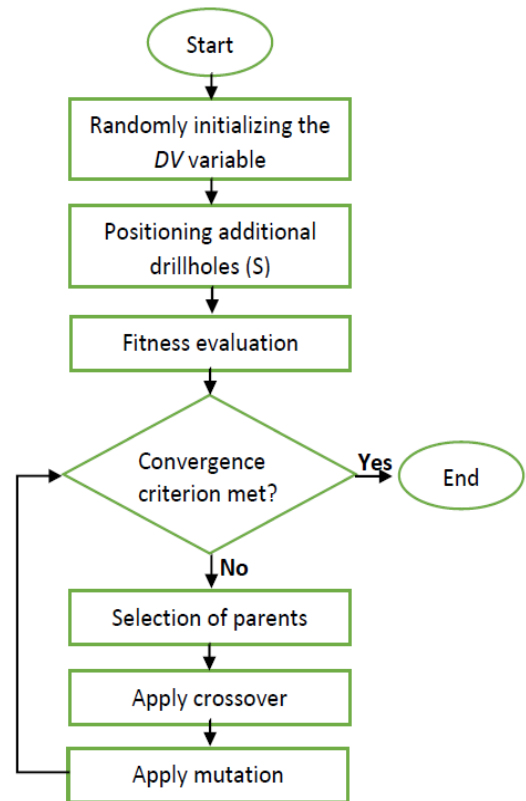


Fig. 1. Flowchart of a GA procedure for optimally locating additional drill holes

3. Case study

Jajarm Zu 2, an Iranian bauxite deposit, was selected for the purpose of validating the efficiency of the proposed algorithm. The deposit could be divided into 4 separate zones; lower Kaolin, shale bauxite, hard bauxite, and upper Kaolin; with hard bauxite being economically the most important zone. As a result of drilling 72 exploratory drillholes, a sum of 4439 m of core has been obtained but the geological and assaying data are available only for 574 m. Due to its economic importance, mineral resource evaluation is determined only based on the hard bauxite zone. For geostatistical analyses, samples should have equal size. Therefore, the assay

values have been composited into a constant length of 8 m. Figure 2 shows the frequency distribution of the regional variables of the grade of Al_2O_3 in hard bauxite zone of Zu 2. Deposit revealing that the distribution can be considered as Gaussian. For the structural analysis of Al_2O_3 , the directional and directionless experimental semivariograms were drawn, and since the data were not much, it was not possible to fit the model to directional ones and therefore, the deposit was assumed to be isotropic and the model fitted to only the directionless ones (Fig. 3). Figure 3 shows the spherical model fitted to Al_2O_3 directionless experimental semivariograms.

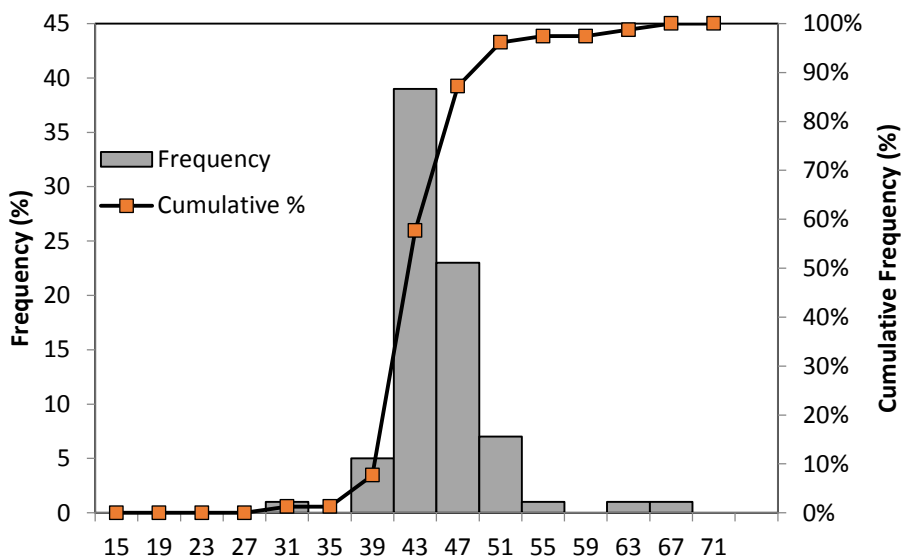


Fig. 2. Grade frequency distribution of composited cores in the HB zone

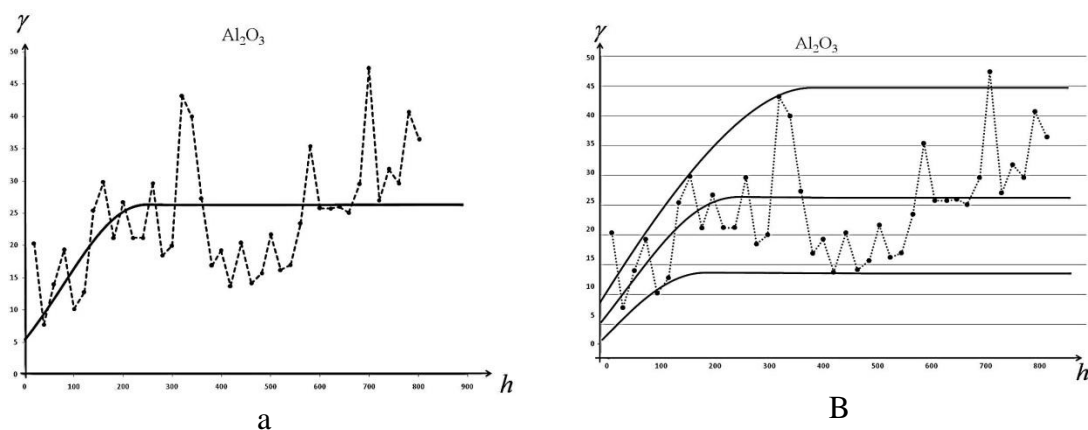


Fig. 3. Models fitted to experimental semivariograms a) crisp, b) fuzzy

As shown, conformity of a crisp variogram model has many ambiguities, specifically regarding its parameters; therefore, using a fuzzy variogram model can be beneficial in such a case. For this purpose, three lower, medium, and upper limit models were fitted to the experimental semivariogram instead of only one deterministic one (Fig. 3b) and the nugget effect, range and sill were defined for each model (Table 1). Triangular membership functions was used mainly because of the simplicity of the approach and the fact that it could easily be computationally implemented and also because it is sufficiently flexible to be able to adequately reflect the available information [28]. The parameters of the fitted fuzzy variogram model are defined by three triangular fuzzy numbers $\tilde{\theta}_1 = (2.3, 5.5, 8.5)$, related to the nugget effect, $\tilde{\theta}_2 = (11.3, 21, 36)$, sill $\tilde{\theta}_3 = (187, 245, 378)$, and range. To avoid more complexities, the upper and lower limit models were fitted spherically. A block model was constructed using block sizes of $5\text{m} \times 5\text{m} \times 5\text{m}$. Then the fuzzy grade of Al_2O_3 was calculated based on Bardossy's fuzzy kriging method (Eq. 5 and 6) using the Fuzzy Krig program. The "FuzzyKrig" program was prepared using the MATLAB (2013a) software at Kashan University, Kashan city, Iran (Fig. 4). The width of kriged values were

interpreted as an uncertainty measure originating from variogram uncertainty and depending on actual measurement values [16,17]. Figure 5 shows the width of the fuzzy estimated Al_2O_3 grade.

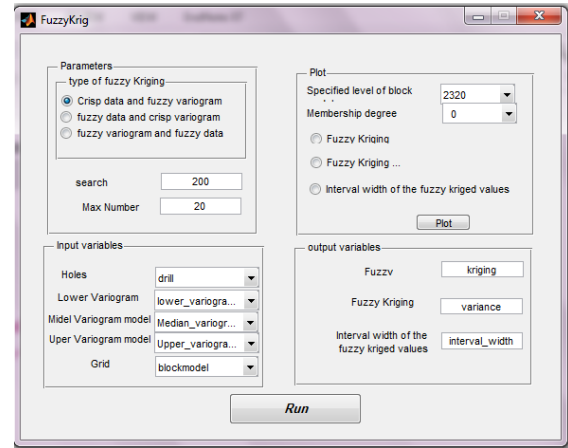


Fig. 4. Main GUI for the FuzzyKrig program

Table 1. Parameters of the fuzzy variogram model

	Sill	Range	Nugget effect
Lower limit	11.3	187	2.3
Crisp value	21	245	5.5
Upper limit	36	378	8.5

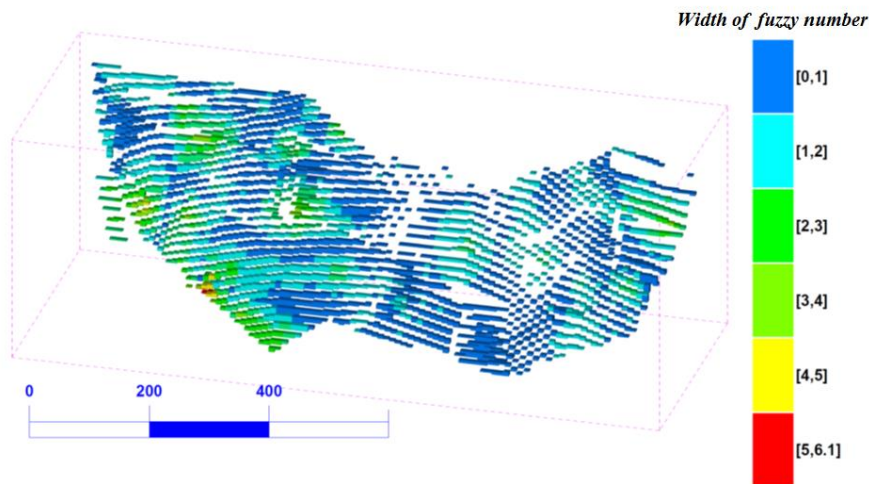


Fig. 5. Block model for the width of the fuzzy estimated Al_2O_3 grade

3.1. Points proposed for additional drillholes

The SamOptLoc program was developed using MATLAB (2013a) for optimally locating additional drillholes in Zu2 deposit. It

is capable of calculating additional drillholes' optimal locations (K) in two different cases: 1) the variogram model parameters are assumed to be crisp and the objective is to minimize the estimation variance, and 2) effects of the

epistemic uncertainties of the variogram model parameters and weighting with respect to grade are taken into account. Figures 5 and 6 show the results of the algorithm implementation for locating 20 vertical additional drillholes. The number of additional drillholes is determined based on the scatter plot of the number of additional drillholes against the best objective function value. As shown in Figure 5, the result of the optimally locating drillholes based on the objective function of the AKV minimization is the drillholes' distribution in places where its initial frequency is less. In the southern parts, where the initial drillholes' spacing from one another is low (15 m), no additional drillholes have been suggested. However, after adding the suggested ones in the northern parts, spacing has considerably decreased (from 50 to 25 m). Results of the implementation of the second algorithm (Fig. 7) are much farther

from those of the first algorithm. As shown in the second algorithm, the drillholes do not have a uniform distribution in the region and lie in places where, in addition to a reduced estimation variance, the ore's grade and the width of the fuzzy estimated grade are more compared to other places.

Therefore, an advantage of the proposed objective function (compared with the usual ones), is that in addition to having a direct relationship with the kriging variance, it depends highly on the width of the fuzzy estimated grade. This modification causes the additional drillholes' optimal pattern to have more concentration (instead of a uniform distribution) in places where ambiguities in the estimated grade are high. This therefore, causes the gathering of complementary information from these drillholes to have more influence on the mine's future decision making and a reduction in related risks.

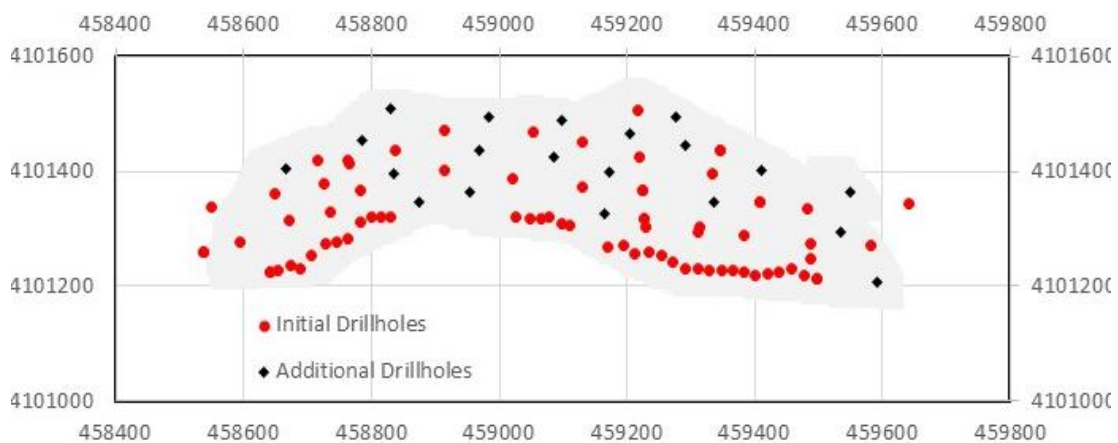


Fig. 6. Locations of the additional drillholes suggested based on the objective function of the estimation variance minimization

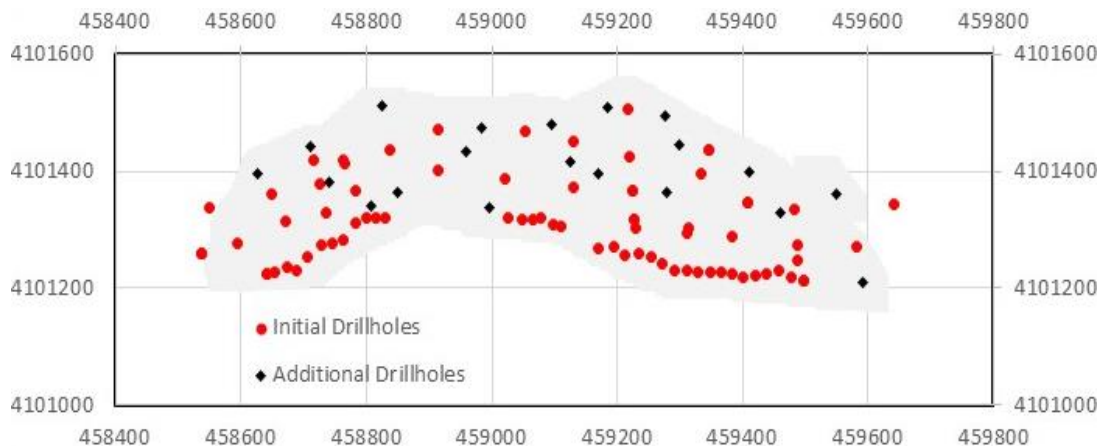


Fig. 7. Locations of the additional drillholes suggested based on the WKAV minimization

4. Conclusions

Different methods proposed for the optimal locating of additional samples and drillholes are appropriate tools, the inputs of which include the variogram model parameters, locations of the initial samples, and the geological block model. All these methods try to minimize the estimation variance (or the weighted estimation variance) as a criterion for uncertainties. Their capability is limited because their only criterion for uncertainty is the grade estimation variance, whereas the uncertainty caused by that of the model fitted to the experimental variogram is also considered. The theory of fuzzy sets can be used in kriging to show the uncertainties of the variogram model parameters which can be extracted (as the fuzzy subsets) from the experimental semivariogram. The uncertainty effects of the fitted model are modeled in the deposit using the fuzzy kriging method. One of the outputs of this method is a criterion called the width of the fuzzy number which can be used as a weighting criterion in calculating the deposit's weighted estimation variance as the objective function in the optimization studies of drilling points. This modification in the objective function will cause the additional drillholes' optimal pattern to have more concentration (instead of a uniform distribution) in places where ambiguities in the estimated grade are high and therefore, cause the gathering of complementary information from these drillholes to have more influence on the mine's future decision making and a reduction in related risks.

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