

## Calculation of One-dimensional Forward Modelling of Helicopter-borne Electromagnetic Data and a Sensitivity Matrix Using Fast Hankel Transforms

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Received 19 February 2014; Received in revised form 22 May 2014; Accepted 30 May 2014

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### Abstract

The helicopter-borne electromagnetic (HEM) frequency-domain exploration method is an airborne electromagnetic (AEM) technique that is widely used for vast and rough areas for resistivity imaging. The vast amount of digitized data flowing from the HEM method requires an efficient and accurate inversion algorithm. Generally, the inverse modelling of HEM data in the first step requires a precise and efficient technique provided by a forward modelling algorithm. The exact calculation of the sensitivity matrix or Jacobian is also of the utmost importance. As such, the main objective of this study is to design an efficient algorithm for the forward modelling of HEM frequency-domain data for the configuration of horizontal coplanar (HCP) coils using fast Hankel transforms (FHTs). An attempt is also made to use an analytical approach to derive the required equations for the Jacobian matrix. To achieve these goals, an elaborated algorithm for the simultaneous calculation of the forward computation and sensitivity matrix is provided. Finally, using two synthetic models, the accuracy of the calculations of the proposed algorithm is verified. A comparison indicates that the obtained results of forward modelling are highly consistent with those reported in Simon et al. (2009) for a four-layer model. Furthermore, the comparison of the results for the sensitivity matrix for a two-layer model with those obtained from software is being used by the BGR Centre in Germany, showing that the proposed algorithm enjoys a high degree of accuracy in calculating this matrix.

**Keywords:** *fast Hankel transforms, forward modelling, frequency domain data, HCP coils system, HEM method, sensitivity matrix.*

### 1. Introduction

Helicopter-borne electromagnetic (HEM) methods are used in the exploration of subsurface resistivity distributions across wide areas. Under such methods, an alternating current within the transmitter coil generates an

intense primary magnetic field ( $H_p$ ) directed towards the ground. This alternating magnetic field in turn generates eddy currents in the conductive earth structures which themselves generate a secondary magnetic field ( $H_s$ )

which can be detected by a receiver coil. Finally, the real and imaginary components of these normalized secondary (with respect to the primary magnetic field, i.e.,  $Z=H_s/H_p$ ) magnetic fields are measured in parts per million (ppm) at several frequencies by receiver coils. By the processing and inversion of these data, some information regarding the resistivity distribution of the area can be gathered [1]. Many people have studied the inversion of multi-frequency HEM data, including the research work of Sengpiel and Siemon (2000), Zhang et al. (2000), Farquharson et al. (2003), Huang and Fraser (2003), Siemon et al. (2009a), Arab-Amiri et al. (2010), Shirzaditabar et al. (2010 and 2011) and Abedi et al. (2013) [2,3,4,5,6,7,8,9,10]. During the process of inverse modelling, the data will be converted to a model described by a set of unknown parameters. The main idea behind this process is to find a model whereby its responses (forward calculated data) fit the observed data in a least squares sense. Thus, the first step in this process involves finding an accurate and reliable tool for generating data from the proposed models. This condition is satisfied through forward modelling. If the forward calculation is not sufficiently accurate, then approaching the final model such that its response will fit adequately with the real field data will not be possible. Since forward modelling is needed in each iteration of inversion, so the high speed of its calculation is required to perform the inversion of data rapidly. Thus, the algorithm that is used for forward modelling must simultaneously exhibit an adequate speed and accurate computation.

The EM response of a layered half-space earth model for dipole source excitation is provided by Frischknecht (1967), Ward (1967), Wait (1982) and Ward and Hohmann (1997). The integral within the response of this model is solved numerically by the Laplace transform [11], an extension of the Bessel function [1] and the method of fast Hankel transforms (FHTs) [12]. The solution of this complex integral has been investigated by a few researchers using the above methods, although no comprehensive explanation of the methodology has been addressed in the published literature. In this study, the usage

and formulation of the FHTs method to solve this integral in order to acquire the secondary field components of the measurement is explicitly explained. As the relation of the observed data with the model parameters is nonlinear, the derivatives of the forward data with respect to the model parameters are required in an iterative solution. These derivatives form elements of a matrix (which is known as the ‘sensitivity matrix’) which again has not been explained comprehensively in the relevant published literature as regards how these elements are calculated. In this study, an attempt is made to comprehensively explain all the steps and details of the calculation of these derivatives. In attaining higher precision forward calculations, analytic differentiation is used to calculate the sensitivity matrix.

In addition to the greater speed of forward modelling, the computation of the integral and the sensitivity matrix are being done simultaneously. Ultimately, the precision of the forward calculation is evaluated with the synthetic data provided by a published paper [6] for a four-layered model. The sensitivity matrix for a two-layered model is also derived and is then compared with that obtained by the software used at the BGR<sup>1</sup> Centre.

## 2. Forward modelling of HEM data

As was stated, the first step in converting the data into a model closely resembling the conditions underground is forward modelling. Under this process, a hypothetical model with known parameters will be used to find the synthetic data at any specific location of the receiver. The relation between the model’s parameters and the measured data can generally be obtained through the following relation.

$$d = f(m) \tag{1}$$

in which, assuming  $M$  data and  $P$  model parameters,  $d = (d_1, d_2, d_3, \dots, d_M)$  is the data vector,  $m = (m_1, m_2, m_3, \dots, m_P)$  is the model parameters vector, and  $f$  is the forward function under investigation. In one-dimensional modelling for a  $N$ -layered earth model, as can be seen in Figure 1,  $\rho_i$  and  $t_i$ ,

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respectively, denote the resistivity (ohm-meter) and thickness of the earth layers (metres), comprising a model parameter vector. Thus, the number of the model's parameters is  $P=2N-1$ . The data vector is also composed of the real part ( $R$ ) and the imaginary part ( $Q$ ) of the ratio of the secondary magnetic field to the primary magnetic field in ppm<sup>1</sup>. Hence,

$$m = [\rho_1, \rho_2, \rho_3, \dots, \rho_N, t_1, t_2, t_3, \dots, t_{N-1}]^T \quad (2)$$

also

$$d = [Z_1, Z_2, Z_3, \dots, Z_M]^T \quad (3)$$

in which

$$Z = R+iQ \quad (4)$$

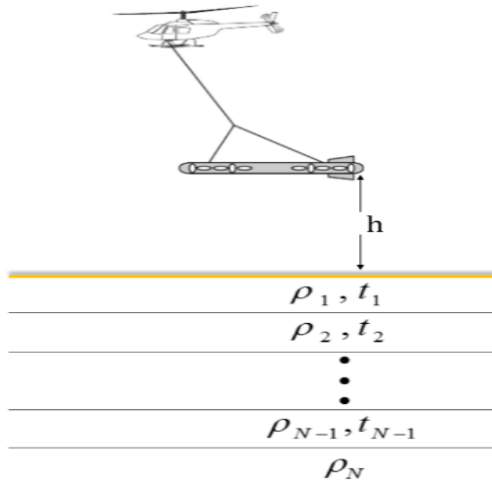


Fig. 1. A basic diagram of a one-dimensional N-layer half-space earth model.

Among the various configurations of transmitter and receiver coils, three of them - namely horizontal coplanar (HCP), vertical coplanar (VCP) and vertical coaxial (VCA) systems - are frequently used in practical HEM surveys [13]. In this paper, however, the HCP coil system is considered as providing the most reliable modelling work on a multi-layered earth model (e.g., Huang and Fraser (2003) and Siemon et al. (2009a)) and as such this configuration is used. The EM response of a layered half-space earth model for dipole source excitation in the form of a HCP system at a height  $h$  above the layered half-space is given by the forward operator in the following relation [14].

$$Z(r)[ppm] = \frac{Z_s}{Z_p} = R + iQ = r^3 \int_0^\infty R_0(\omega, \lambda, \rho, t) \frac{\lambda^s}{A_0} e^{-2A_0 h} J_0(\lambda r) d\lambda \quad (5)$$

in which  $Z_s$  and  $Z_p$  are secondary and primary magnetic fields, respectively, at the receiver coil location,  $h$  is the bird's height from the ground,  $R$  and  $Q$  are in-phase (real) and out-of-phase (imaginary) components of data,  $i = \sqrt{-1}$  is the imaginary number,  $r$  is the distance between the transmitter and the receiver coils,  $\lambda$  is the integral's variable,  $J_0$  is a zero-order Bessel function of the first kind, and  $\omega$  is the angular frequency.  $A_0$  can be calculating by the following formula.

$$A_0 = \left( \frac{i\omega\mu_0}{\rho_0} + \lambda^2 - \omega^2 \varepsilon_0 \mu_0 \mu_{r1} \right)^{1/2} \quad (6)$$

in which  $\rho_0$ ,  $\mu_0$ , and  $\varepsilon_0$ , are, respectively, the resistivity, magnetic permeability and electric permittivity of the free space, and  $\mu_{r1} = \mu_1/\mu_0$  is the relative magnetic permeability of the first layer that has its own magnetic permeability ( $\mu_1$ ). If the area under investigation has no noticeable magnetic anomaly, it can be assumed that  $\mu_0 \approx \mu_n$  or, in other words, that the relative magnetic permeability of all the layers of the model is equal to one [5]. To find the parameters of the layered earth, we need the reflection coefficient  $R_0$ , which can be obtained through the following relation [14].

$$R_0 = \frac{\mu_0 B_1 - \mu_1 A_0}{\mu_0 B_1 + \mu_1 A_0} = \frac{B_1 - \mu_{r1} A_0}{B_1 + \mu_{r1} A_0} \quad (7)$$

The parameter  $B_1$  and - following that - the reflection coefficient  $R_0$  are then given by the following recursive relation.

$$B_1 = A_1 \frac{B_2 + A_1 \tanh(A_1 t_1)}{A_1 + B_2 \tanh(A_1 t_1)} \quad (8)$$

In addition, for the  $n$ th layer

$$B_n = A_n \frac{B_{n+1} + A_n \tanh(A_n t_n)}{A_n + B_{n+1} \tanh(A_n t_n)} \quad (9)$$

$$A_n = \left( \frac{i\omega\mu_0}{\rho_n} + \lambda^2 - \omega^2 \varepsilon_0 \mu_0 \right)^{1/2} \quad (10)$$

While for the last layer

1. Parts per million

$$A_N = B_N = \left( \frac{i\omega\mu_0}{\rho_N} + \lambda^2 - \omega^2 \varepsilon_0 \mu_0 \right)^{1/2} \quad (11)$$

Using Relations 8-11, one can accurately determine the reflection coefficient  $R_0$  of Relation 7. However, the forward equation in Relation 5 is a form of Hankel integral, which follows from the general form below [15].

$$f(r) = \int_0^\infty k(\lambda) J_n(\lambda r) d\lambda \quad (12)$$

In this integral,  $k(\lambda)$  is the transformed kernel function and  $J_n(\lambda r)$  is the  $n^{\text{th}}$ -order Bessel function of the first kind. Various numerical methods have been proposed for solving this integral form. In this study, the FHTs method with 120-fold weighted coefficients introduced by Guptasarma and Singh (1997) is used to solve Equation 5. Using these coefficients, the above-mentioned integral can be written as a sum of multiple products.

$$f(r) = \frac{1}{r} \sum_{i=1}^n k(\lambda_i) W_i \quad (13)$$

Using this method, the forward equation in Relation 5 would become,

$$Z = R + iQ = r^2 \sum_{i=1}^{120} k(\lambda) W_i \quad (14)$$

By comparing Relation 13 with Relation 5, the kernel function  $k(\lambda)$  is given as follows.

$$k(\lambda) = R_0 \frac{\lambda^3}{A_0} e^{-2A_0 h} \quad (15)$$

Next, using Relation 14, we will be able to find the forward modelling response of the model.

### 3. Sensitivity matrix calculations

In most geophysical problems, the relation between the data and the model's parameters is a nonlinear one. In a problem with  $M$  data and  $P$  parameters, the relation between each datum  $d_i$  with the model's parameters  $m_i$  is described using a nonlinear function  $f_i$ .

$$d_i = f_i(m_1, m_2, m_3, \dots, m_p) = f_i(m) \quad (16)$$

Relation 5 shows that the forward equation under investigation is fully nonlinear. The

main idea in solving this kind of problem would be to use a Taylor series expansion of the forward function  $f_i(m)$  around an initial guess to linear the equation in the neighbourhood of this initial model. In the next step, by improving on this model and repeating a similar operation, we will obtain a model whose response fits with the observed data in a least squares sense. Thus, by neglecting derivatives of the second-order and higher, we will have the Taylor expansion of the following equation [16].

$$\Delta d = J \cdot \Delta m \quad (17)$$

where  $\Delta d$  is a vector of difference between the measured (real) data and the response of the initial model,  $\Delta m$  is the model's improvement in each iteration (i.e., the difference between the updated and initial model parameters) and  $J$  is the Jacobian partial derivative or sensitivity matrix, which can be obtained through the following relation.

$$J_{ij} = \frac{\partial f_i(m)}{\partial m_j}, \quad i = 1, 2, 3, \dots, M, \quad j = 1, 2, 3, \dots, P \quad (18)$$

Or, in other words

$$J_{M \times P} \begin{bmatrix} \frac{\partial f_1}{\partial m_1} & \frac{\partial f_1}{\partial m_2} & \dots & \frac{\partial f_1}{\partial m_p} \\ \frac{\partial f_2}{\partial m_1} & \frac{\partial f_2}{\partial m_2} & \dots & \frac{\partial f_2}{\partial m_p} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial m_1} & \frac{\partial f_M}{\partial m_2} & \dots & \frac{\partial f_M}{\partial m_p} \end{bmatrix} \quad (19)$$

As can be observed in Relation 18, the elements of the sensitivity matrix comprise the partial derivatives of the forward equation with respect to the model's parameters. The inversion methods based on the linearization of nonlinear functions critically depend on the ability to estimate the Jacobian matrix. In most inverse modelling problems, an approximation technique, such as the finite-difference method, is usually used to calculate sensitivity matrix elements. It has been shown that these techniques not only produce errors but also that they are computationally intensive [5]. As such, in order to avoid these problems, in this study an analytical differentiation method is used to compute the partial derivatives and to attain

higher accuracy. As an  $N$ -layer earth model (Fig. 1) is considered, all of  $f_i$  s are the same and are derived from Relation 5. By looking at this relation, it should be clear that the only term dependent on the model's parameters is the reflection coefficient  $R_0$ . Hence,

$$\frac{\partial f_i}{\partial m_i} = r^3 \int_0^\infty \left( \frac{\partial R_0}{\partial m_i} \right)_0 \frac{\lambda^3}{A_0} e^{-2A_0 h} J_0(\lambda r) d\lambda \quad (20)$$

Furthermore, with consideration to Relation 4, we have

$$\frac{\partial R_0}{\partial m_j} = \frac{\partial R_0}{\partial B_1} \cdot \frac{\partial B_1}{\partial m_j} \quad (21)$$

$$\frac{\partial R_0}{\partial B_1} = \frac{2A_0}{(B_1 + A_0)^2} \quad (22)$$

in which  $m_j$  could be either  $\rho_j$  or  $t_j$ . Using Relations 8-11 and the chain differentiation rule, we have

$$\frac{\partial B_1}{\partial \rho_j} = \frac{\partial B_1}{\partial B_2} \cdot \frac{\partial B_2}{\partial B_3} \cdot \frac{\partial B_3}{\partial B_4} \cdots \frac{\partial B_{j-1}}{\partial B_j} \cdot \frac{\partial B_j}{\partial A_j} \cdot \frac{\partial A_j}{\partial \rho_j} \quad (23)$$

$$\frac{\partial B_1}{\partial t_j} = \frac{\partial B_1}{\partial B_2} \cdot \frac{\partial B_2}{\partial B_3} \cdot \frac{\partial B_3}{\partial B_4} \cdots \frac{\partial B_{j-1}}{\partial B_j} \cdot \frac{\partial B_j}{\partial t_j} \quad (24)$$

For the calculation of the derivatives of type  $\frac{\delta B_n}{\delta B_{n+1}}$  that appear in the above relations, and with respect to Relations 9 and 10, one can show that

$$\frac{\partial B_n}{\partial B_{n+1}} = \frac{A_n^2}{\cosh^2(A_n t_n) [A_n + \tanh^2(A_n t_n)]^2} \quad (25)$$

and also

$$\frac{\partial B_j}{\partial A_j} = \frac{[B_{j+1} + 2A_j \tanh(A_j t_j) + A_j^2 t_j \sec^2 h^2(A_j t_j)]}{V} \quad (26)$$

$$\frac{[1 + t_j B_{j+1} \sec^2 h^2(A_j t_j)]U}{V^2}$$

where

$$V = A_j + B_{j+1} A_j \tanh(A_j t_j) \quad (27)$$

$$U = A_j B_{j+1} + A_j^2 t_j \tan(A_j t_j) \quad (28)$$

It must be noted that in the last layer  $B_N = A_N$ , and hence when  $j=N$ , we have

$$\frac{\partial B_j}{\partial A_j} = 1 \quad (29)$$

For calculating the last terms of Relations 23 and 24, using Relations 9 and 10, we have

$$\frac{\partial A_j}{\partial \rho_j} = \frac{-i \omega \mu_0}{2 \rho_j A_j} \quad (30)$$

$$\frac{\partial B_1}{\partial t_j} = \frac{[A_j^3 \sec^2 h^2(A_j t_j)]V - [A_j B_{j+1} \sec^2 h^2(A_j t_j)]U}{V^2} \quad (31)$$

Finally, using the recursive Relations 21-31,  $\frac{\partial R_0}{\partial m_j}$  could be obtained and substituted in

Eq. 20. The integral in Relation 20 is also in the form of Hankel integrals of Relation 12. Hence, in calculating it, the FHTs with 120-fold weighted coefficients introduced by Guptasarma and Singh (1997) are used. With respect to Relation 13, we will have the following:

$$\frac{\partial f_i}{\partial m_j} = r^2 \sum_{i=1}^{120} k(\lambda) W_i \quad (32)$$

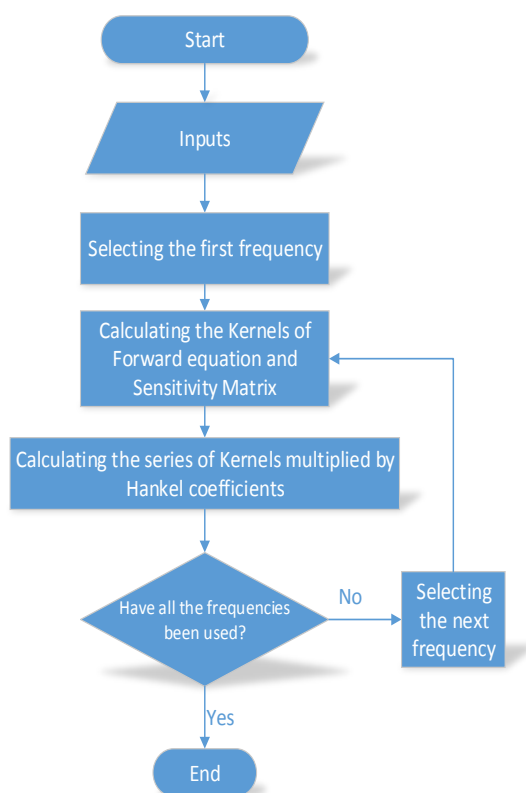
where its Kernel function is

$$k(\lambda) = \left( \frac{\partial R_0}{\partial m_j} \right)_0 \frac{\lambda^3}{A_0} e^{-2A_0 h} \quad (33)$$

#### 4. An algorithm for the simultaneous calculation of the forward modelling and sensitivity matrix

Considering the huge amount of data involved in the HEM method, time conservation is an important factor in computations. Since, in practice, many parameters in the calculation of the sensitivity matrix and forward modelling are the same as in inverse modelling and the approach for solving both of them employs FHTs, an efficient algorithm for the simultaneous calculation of both of them (Fig. 2) is proposed and its related computer codes are written in the MATLAB environment.

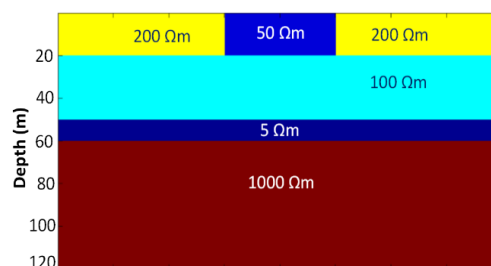
In this algorithm, as Figure 2 illustrates, first all of the required input parameters, including the number of layers, the resistivity and thickness of each layer, the frequencies being used, the flight altitude, the coils distance and the Hankel coefficients are recalled. Next, using Relations 15 and 33, the kernel functions must be calculated for the selected frequency. Finally, using the multiplication sigma of Relations 14 and 32, the values of the forward (synthetic) data and the elements of the sensitivity matrix will be computed and saved. The process will be applied to all the functional frequencies.



**Fig. 2. A simple flowchart of the proposed algorithm for the simultaneous calculation of the forward modelling and sensitivity matrix.**

### 5. Results and discussion

In this section, the results of the proposed algorithm are compared with those obtained by the BGR Centre’s software using two synthetic models. First, the results of the forward modelling for the model in Figure 3 will be investigated.



**Fig. 3. A sketch of the layering of a four-layered synthetic model.**

The synthetic data of this model has been considered by Siemon and his colleagues [6]. The aforementioned model is a four-layered model with a respective specific resistivity of 200, 100, 5 and 1,000 ohm-meter. The first, second and third layers have each a thickness of 20, 30 and 10 metres, respectively. The functional frequencies are 387, 1,820, 8,225, 41,550 and 133,200 Hz. The distance between the coils and the helicopter’s flight altitude, respectively, are 8 and 30 metres. In Table 1, we have outlined the results of 1D forward modelling for the proposed algorithm and those provided by the BGR Centre software [6] at a site located on the left part of the model.

**Table 1. The results of 1D forward modelling for the proposed algorithm and those produced by the BGR software.**

Forward modeling	Data of the proposed algorithm		Data of the BGR Software	
Frequency(Hz)	R(ppm)	Q(ppm)	R(ppm)	Q(ppm)
387	21.8	68.36	21.8	68.36
1820	129.1	164.35	129.1	164.4
8225	280.42	291.45	280.4	291.5
41550	734.95	747.46	734.7	747.4
133200	1506.18	1046.81	1506	1047

Figure 4 another way of comparing the real and imaginary components of the synthetic data which have been computed by the 1D forward modelling of the proposed algorithm and from the BGR Centre software.

With respect to Table 1 and Figure 4, one can conclude that the results of the proposed algorithm are in close agreement with those of the BGR Centre software.

To study the accuracy of the results sensitivity matrix, we performed calculations for a two-layered model as in Figure 5. In this model, the resistivity of the first and second

layers, respectively, is 100 and 20 ohm-meter. The thickness of the first layer is 40 metres. The flight altitude and the distance between the coils have been set to 30 and 8 metres,

respectively. It has also been supposed that the data have been surveyed in five frequencies: 900, 1,800, 7,200, 57,600, 133,200 Hz.

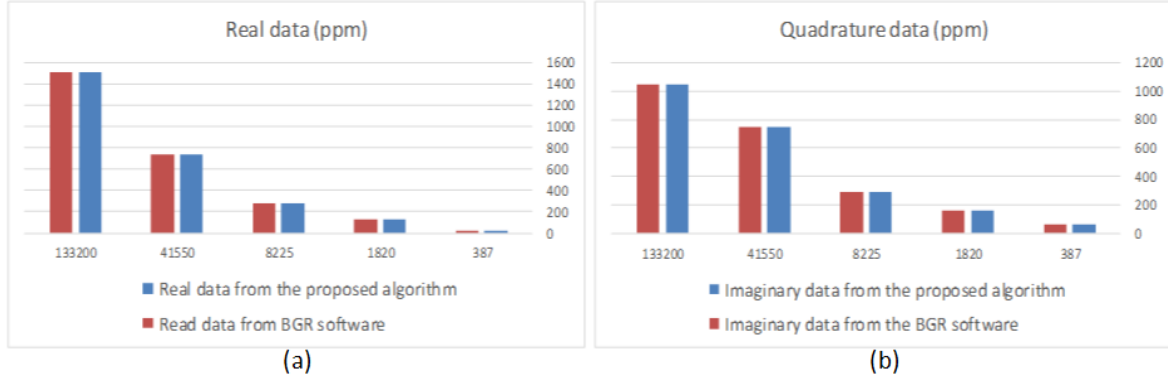


Fig. 4. Comparison of the 1D forward data from the proposed algorithm and the BGR software: a) real data, and b) imaginary data.

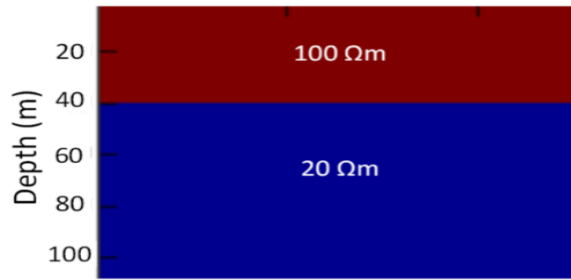


Fig. 5. A sketch of the layering of the two-layered model.

The proposed algorithm produced the sensitivity matrix as Relation 34. In addition, the BGR Centre software gives the sensitivity matrix as Relation 35.

$$J = \begin{bmatrix} -0.168630052 & -2.307556506 & -1.233126203 \\ -0.382572998 & -2.817244513 & -2.310850482 \\ -1.774434761 & -1.695683674 & -4.461238981 \\ -8.093572747 & 0.322332069 & 1.667932774 \\ -8.364728 & -0.045660611 & -0.093729348 \\ -0.622167035 & -0.83915142 & -1.505919931 \\ -1.076951435 & -0.126277744 & -1.711578762 \\ -2.75713792 & 1.796732719 & 0.562416015 \\ -1.942099699 & -0.155828341 & 0.53428438 \\ 0.24964686 & -0.028788255 & -0.387795832 \end{bmatrix} \quad (34)$$

$$J = \begin{bmatrix} -0.168630043 & -2.307556368 & -1.233126056 \\ -0.38257298 & -2.817244316 & -2.310850236 \\ -1.774434695 & -1.695683537 & -4.461238644 \\ -8.09357294 & 0.322332169 & 1.667933246 \\ -8.364727538 & -0.04566057 & -0.093729256 \\ -0.622167018 & -0.839151475 & -1.505919817 \\ -1.076951407 & -0.126277799 & -1.711578662 \\ -2.757137865 & 1.796732616 & 0.56241588 \\ -1.942099815 & -0.155828423 & 0.534284438 \\ 0.249646814 & -0.02878823 & -0.387795493 \end{bmatrix} \quad (35)$$

Since there are 10 data in five frequencies (comprising five real and five imaginary components) and the number of parameters is three, the resulting matrix is a  $10 \times 3$  matrix whose first and second columns are, respectively, the derivatives of each resistivity of 100 and 20 ohm-meter, and its third column is the derivative of the 40 metre thickness of the first layer. Furthermore, the first five rows of the matrix are the derivatives of the real parts, and the second five rows those of the imaginary parts. A comparison of the two resulting matrices shows that the results from the proposed algorithm are in high conformity with those of the BGR Centre software.

## 6. Conclusion

In this study, first, the relations of the 1D forward modelling of helicopter-borne frequency domain electromagnetic data were investigated. As there is no analytic solution for the integrals in these relations, the fast transforms using Guptasarma and Singh's (1997) digital linear filter coefficients have been used to solve them numerically. Next, by the analytic calculation of the partial derivatives of the forward equation with respect to the model's parameters and the FHTs method, the required relations for constructing the sensitivity matrix were obtained. Following this, an efficient algorithm for the simultaneous calculation of the two parts was elaborately designed to speed up the required computation. Finally, the obtained results of this study were compared with those of the BGR Centre software for two synthetic models. This comparison has shown that the results of the proposed algorithm for the 1D forward modelling and sensitivity matrix calculations are in strong agreement with those of the BGR software.

## Acknowledgements

The authors are grateful to Dr Bernhard Siemon for providing the results of the BGR Centre software computations for the required models.

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