

Pillar Design in the Hard Rock Mines of South Africa

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Abstract

This paper gives an overview of the difficulties associated with the design of hard rock pillars in South African mines. Recent examples of large scale pillar collapses in South Africa suggest that these were caused by weak partings which traversed the pillars. Currently two different methods are used to determine the strength of pillars, namely, empirical equations derived from back analyses of failed and stable cases and numerical modeling tools using appropriate failure criteria. It is illustrated in the paper that both techniques have their limitations and additional work is required to obtain a better understanding of pillar strength.

Empirical methods based on observations of pillar behaviour in a given geotechnical setting are popular and easy to use, but care should be exercised that the results are not inappropriately extrapolated beyond the environment in which they are established. An example is the Hedley and Grant formula (derived for the Canadian uranium mines) that has been used for many years in the South African platinum and chrome mines (albeit with some adaptation of the K-value). Very few collapses have been reported in South Africa for layouts designed using this formula, suggesting that in some cases it might yield estimates of pillar strength that are too conservative.

As an alternative, some engineers strongly advocate the use of numerical techniques to determine pillar strength. A close examination unfortunately reveals that these techniques also rely on many assumptions. An area where numerical modeling is invaluable, however, is to determine pillar stresses accurately and to study specific pillar failure mechanisms, such as the influence of weak partings on pillar strength.

Keywords: Weak Partings, Hard Rock Pillar Design, Empirical Technique, Numerical Method, Pillar Strength

1-Introduction

Appropriate pillar design is a fundamental building block of mine design to ensure safe and economic extraction of valuable national resources. It is therefore worthwhile to take a critical look at the tools currently available to rock engineers to conduct these designs. This paper focuses only on stable pillars and does not address the issues associated with crush/yield pillars. The design of stable pillars is currently very topical in the shallow hard rock mining sector in South Africa and this paper will highlight a number of examples from this country. Hard rock pillar design nevertheless appears to be of universal interest as shown by the examples described below.

Zipf (2000) describes collapses of room and pillar mines in the United States. The term, *catastrophic pillar failure* or CPF, is used to describe the mechanism whereby a few pillars fail initially, their load is then transferred to adjacent pillars, which also

fail. This may result in a “pillar run” and hundreds of pillars may fail in the process. A number of examples of CPF collapses in “metal” mines are given in Zipf’s paper and apparently at least four such examples have occurred in the United States since 1972. One of the more recent examples is a large pillar collapse in a room-and-pillar base metal mine, described in Dismuke et al. (1994). Figure 1 illustrates the area of the collapse. The failure began in four centrally located pillars and spread rapidly to include almost 100 pillars. The pillar width was 8.5 m and the room width was 9.7 m. The pillar height was about 12 m, resulting in a width-height ratio of 0.70. The extraction ratio was approximately 78%. Based on the examples available to him, Zipf made the comment that mines experiencing CPF generally exhibit the following characteristics (quoted directly from his paper):

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- “(1) Extraction ratios are usually more than 60%. A high extraction ratio will put pillar stress close to peak strength and provide ample expansion room for the failed pillar material.
- (2) Width-height ratios of pillars are always less than 3 for coal mine failures, usually much less than 1 in metal-mine failures, and less than about 2 for nonmetal mine failures. A low width-height ratio ensures that the failed pillar material can easily expand into the surrounding openings and that the failed pillar will have little residual load-bearing capacity.
- (3) The number of pillars across the panel width is always at least five and usually more than 10, which typically ensures that pillars have reached their full tributary area load. Minimum panel widths for CPF are at least 80 m.
- (4) Substantial barrier pillars with width-height ratios more than 10 are absent from the mine layout.
- (5) Although CPF seems more prevalent in shallow mines less than 100 m deep, this may be only a reflection of the prevalence of shallow room-and-pillar coal mines.”

An extensive database of hard-rock pillar failures was compiled by Lunder and Pakalnis (1997) who analyzed 178 case histories from hard-rock mines of which 98 were located in the Canadian Shield. Of the pillars investigated, 76 were classed as stable; 62 were classed as failed; and 40 were classed as unstable. Many of these pillars were rib or sill pillars from steeply dipping ore bodies. They proposed that the pillar strength can be adequately expressed by two factor of safety (FOS) lines. Pillars with a $FOS < 1$ fail while those with a $FOS > 1.4$ are stable. The region $1 < FOS < 1.4$ is referred to as unstable and pillars in this region are prone to spalling and slabbing, but have not completely failed. The data collected by these authors are given below in Figures 19 and 20.

Esterhuizen (2006) conducted an evaluation of the strength of slender pillars.

Figure 2 illustrates published case histories of failed pillars from hard rock metal mines. This figure illustrates that the pillar strength becomes highly variable as the width to height ratio decreases. For the w/h ratio of 1, the pillar strength is not expected to exceed 65% of the laboratory strength (but it can also be significantly lower than this value). Esterhuizen suggests that a larger factor of safety is required for these pillars to account for the increased variability in strength. Numerical modelling indicated that for slender pillars, the difference between the pillar load at the onset of spalling and the ultimate pillar strength can be small. This implies that slender pillars are at or near the point of failure at the onset of brittle spalling.

Although a large number of additional papers on pillar design can be found in literature, the objective of this paper is not to give an overview of all these papers, but rather to highlight the difficulties associated with pillar design in South Africa and to emphasize the need for additional research. Some examples of pillar failures in South Africa are therefore described below and the commonly used pillar design methodologies are critically examined.

2-The Coalbrook disaster and the evolution of pillar design in South Africa

Although this paper focuses on pillar design in hard rock mines, it is worthwhile to investigate the evolution of pillar design in South Africa in general. This will shed some light on the apparent lack of appropriate pillar design tools for shallow hard rock mines.

Initially, pillar dimensions and mining spans in South African coal mines were based on experience obtained through trial and error. This resulted in a number of collapses. The first report of a coal pillar collapse in South Africa was in 1904 at the Witbank colliery (Madden, 1991). Since that date, 81 pillar collapses have been recorded in 31 collieries in the Transvaal and Orange Free State. Between 1904 and

1965, there were 50 pillar collapses (Salamon and Wilson, 1965). The research into coal pillar strength in South Africa gained momentum after the Coalbrook disaster. As described by Van der Merwe (2006), the major need for coal mine research in South Africa was identified when the Coalbrook disaster occurred on 21 January 1960. In total, 437 workers lost their lives when pillars over an area of 324 hectares collapsed (Figure 3). Following the disaster, the South African government sponsored research into coal mine safety

by forming the Coal Mines Research Controlling Council (CMRCC). Research into coal pillar strength received top priority from the Council for Scientific and Industrial Research (CSIR) and the Chamber of Mines Research Organization (COMRO). A large *in situ* testing program to determine pillar strength was conducted by the CSIR (Bieniawski and Van Heerden, 1975). Tests were also conducted by the Chamber of Mines as described by Wagner (1974).

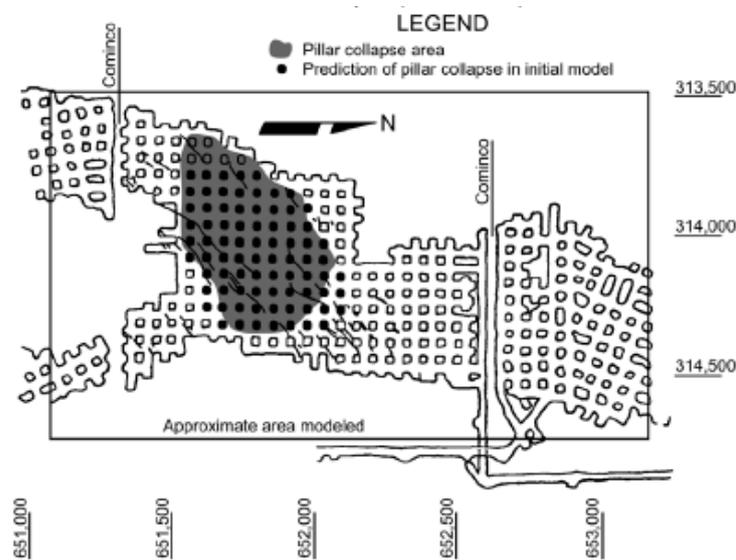


Fig 1. Collapse of a number of pillars in a base metal mine in Missouri (after Dismuke et al., 1994 and also published in Zipf, 2001).

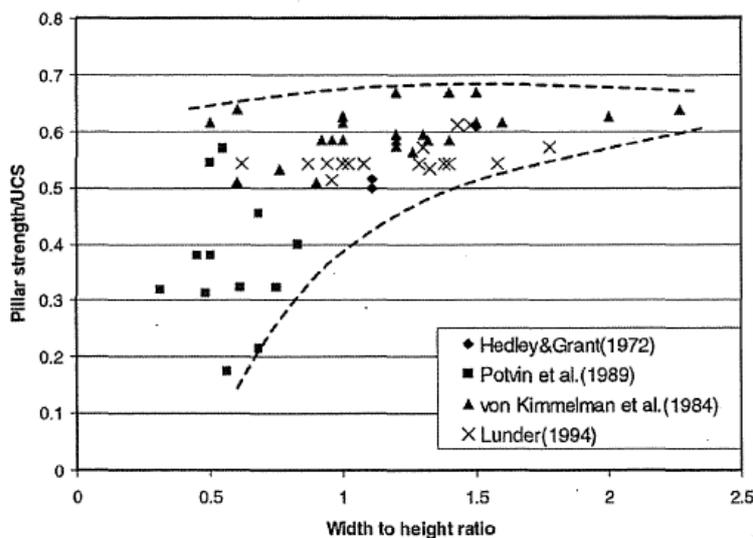


Fig 2. Pillar stability graph showing examples of failed pillars from hard rock mines (after Esterhuizen, 2006).

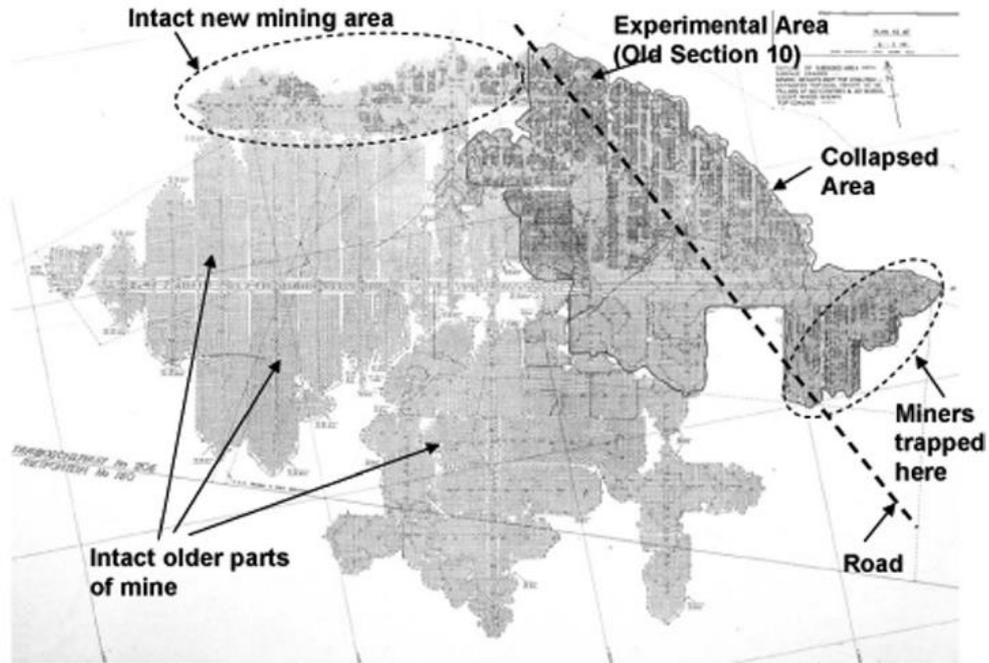


Fig 3. Coalbrook at the time of collapse. The collapsed area is the outlined area in the eastern part of the mine (after Van der Merwe, 2006).



Fig 4. An example of poor pillar cutting in a platinum mine in the Bushveld Complex.

A key study on coal pillar strength was conducted by Salamon and Munro (1967) and the findings as described in their paper are still being used today. It was postulated that the strength of the pillars can be expressed, in the given range of dimensions, as a power function of the height and width. Of interest is that this “power law” was originally proposed by Greenwald, Howard and Hartmann (1939). Salamon and Munro derived their equation by estimating the value of the constant K and the powers of width and height by the method of maximum likelihood. The derivation was empirical and was based on data from stable and collapsed cases. The analyzed data included 125 cases of which 27 were collapsed pillars. It is noteworthy that they warned in the paper against the following mistake commonly made in rock engineering:

“The work described in this paper is essentially empirical, and the results, therefore, should not be extrapolated beyond the range of the data which were used to derive them.”

Although the formula is well known, it is repeated below for completeness:

$$\text{Strength}_{(coal)} = K \frac{w^a}{h^b}, \quad (1)$$

where K reflects the fitted ‘strength’ of a metre cube of coal (7.2 MPa), w is the width of the (square) pillar and h is the height in metres. The parameters a and b are equal to 0.46 and 0.66 respectively. The pillar volume is given by $V = w^2 h \text{ m}^3$. Defining the width: height ratio, $R = w/h$, equation (1) can be expressed in the alternative form (Madden, 1988)

$$\text{Strength}_{(coal)} = KV^{(a-b)/3} R^{(a+2b)/3} = KV^{-0.0667} R^{0.5933}. \quad (2)$$

It can be seen from equation (2) that if $a = b$, the pillar strength is independent of the pillar volume whereas if $b > a$, as in equation (1), the pillar strength is predicted to decrease as the pillar size is increased even if the pillar shape is unchanged.

Did the use of equation(1) improve the pillar designs? Wagner and Madden (1984)

reported that since 1967, approximately 1100 million tons of coal had been mined in South Africa and they estimated that 1.2 million pillars were left underground during that time. During the same period, 13 cases of pillar collapses were reported involving a total of about 4000 pillars. This corresponded to a probability of failure of only 0.003. In spite of the satisfactory performance of the pillar design procedure, three areas requiring further research were identified. These were:

- Regional differences in coal seam strength
- Effect of mining method on pillar strength
- Strength of squat coal pillars

Regarding the strength of the squat pillars, the original database used by Salamon contained no pillar with a width to height ratio greater than 3.8. Evidence collected in the field suggested that beyond a critical width to height ratio, the pillar strength exceeds that suggested by equation (1). Salamon (1982) proposed therefore that when the width to height ratio exceeds a critical ratio, the pillar strength formula should be replaced by the following:

$$\text{Strength}_{(coal)} = KV^{-0.0667} R_0^{0.5933} \left\{ \frac{0.5933}{\varepsilon} \left[\left(\frac{R}{R_0} \right)^\varepsilon - 1 \right] + 1 \right\} \quad (3)$$

where

K = the strength of a unit cube of coal

V = the pillar volume (m^3)

R = the pillar width to height ratio

R_0 = the critical width to height ratio

ε = rate of pillar strength increase

From field data, no evidence was available of a collapse of a pillar with a width to height ratio greater than 4. Therefore the critical width to height ratio was selected as 5. A value of 2.5 was chosen for ε as it was considerably lower than that obtained from laboratory tests on sandstone.

Regardless of the apparent success of the coal pillar power strength formula, some criticism can be raised regarding its

applicability. Virtually all laboratory and field evidence indicates that the w:h strengthening curve has a zero or positively upwards curvature (Ryder and Jager, 2002). The power formula forces downward curvature and this leads to the inelegant form of the squat pillar formula. An objection raised by Bieniawski is that according to the power law formulation, the cube strength ($w = h$) would continue to decrease indefinitely with side length. This is considered unreasonable (Hustrulid, 1976).

Based on these criticisms, an alternative 'linear' equation, with no volumetric size effect, was proposed which directly expresses the strengthening effect of the width:height (w/h) ratio (Bieniawski, 1992).

$$Strength_{(coal)} = K \left(0.64 + 0.36 \frac{w}{h} \right) \quad (4)$$

As noted above, equation (1) implies that pillars with self-similar dimensions (the same ratio of w to h) will have different strengths and that the predicted strength decreases as the pillar volume is increased. By contrast, equation (4) has no size effect in that the strength depends only on the ratio of w to h . York and Canbulat (1998) compared the relative goodness of fit power formula versus linear forms for both coal and hard rock materials and concluded that the latter behaved at least as well.

Further work regarding coal pillar design was conducted by other workers (e.g. Van der Merwe, 2006), but it is beyond the scope of this paper and the reader is referred to this reference for additional information. In conclusion, Ryder and Jager (2002) state:

"The power law and its derivatives are perhaps too entrenched in coal engineering to warrant withdrawing from them at this time, but in hard rock engineering, the simpler and probably more realistic linear forms are advocated for general use."

Regarding the gold mining industry in South Africa, the pillar problem was a completely different issue owing to the great depth of these mines and the

associated seismicity. Leaving small pillars or remnants in stopes is in fact detrimental to stability as these may become a source of seismicity. This was a key motivation for the early adoption of the longwall mining method as it minimized the formation of remnants (Hill, 1948). These longwalls remained vulnerable to seismicity and one method to reduce this risk was to leave strips of ground behind as strike stabilizing pillars. A key function of these pillars is to reduce the levels of energy release rate (Ryder and Jager, 2002). Owing to the size of the gold mining industry and the role it played in the South African economy, significant research into the behaviour of these pillars were conducted over the years. Their widths typically vary from 30 m to 40 m and at a typical stoping width of 1.5 m, this gives a width to height ratio of at least 20. These pillars are therefore considered "indestructible". Seismic problems are nevertheless experienced when these pillars "punch" into the footwall and a simple empirical formula is used during design to ensure that the average stress on these pillars does not exceed a specified multiple of the strength of the rock in the footwall or hanging wall. So-called bracket pillars are also used to clamp seismically active geological structures in the gold mines. As the nature and function of the bracket and stabilizing pillars are different to those found in shallow hard rock mines, these topics are not discussed in this paper and the reader is referred to Ryder and Jager (2002) for additional information.

In comparison to the gold and coal mining industries in South Africa, the platinum and chrome mines were the proverbial "step children" and have never received the same attention in terms of research efforts and funding. Fortunately, the mines in the Bushveld Complex have never had catastrophic pillar failures analogous to the Coalbrook disaster or rockbursts to kick start research and it was only with the rapidly increasing price of platinum from 1999 onwards that research activity started

to receive more attention. In terms of rock engineering knowledge, much was therefore “borrowed” from the other mining sectors and overseas research findings. A contributing factor may also have been that pillar cutting in the hard rock industry is not as precisely controlled as in the coal industry and it is therefore rather difficult to repeat the statistical approach followed by Salamon and Munro (1967). An example is shown in Figure 4 which illustrates a mapped survey of typical pillar cutting in one of the Bushveld mines where the pillars can be seen to have a wide variety of shapes. Gay et al (1982) stated that at that time:

“The design of pillar layouts in shallow to medium depth chromium and platinum mines has not reached the same advanced stage as has the design of pillars in coal mines”

Three reasons for this were given in the paper namely:

1. Very little is known about the strength of small pillars composed of chromitite or pyroxenitic platinum ore.
2. Because of the very competent and stiff strata in the hanging wall, it is difficult to determine the pillar loads accurately.
3. The presence and shear along near vertical faults can change the loading condition from a stiff displacement controlled system to a soft load controlled system.

A large collapse involving pillars at the No 6 Shaft, Bafokeng North mine is described by Kotze (1995). The collapse occurred in February 1975. Luckily nobody was injured during this collapse as it occurred overnight. This collapse is referred to as the “Hospital collapse” by mine personnel as the mine’s hospital was located on surface above the collapsed area and some damage was sustained by the hospital building owing to the resulting subsidence. The building was repaired and it is still being used today. Extraction of the reef in this area was by room and pillar methods, with rooms approximately 25 m wide and pillars nominally 5 m square, providing an

extraction of 96%. It appears as if the pillars were cut smaller than this as Kotze estimated from the mine plans that the equivalent “average” pillar width in this area was 3.6 m with an associated extraction of about 98 percent. Unfortunately no mention is made in the paper of the pillar height. Local hanging wall support was provided by timber sticks and mat packs. The strike span of the stope at the time of the collapse was approximately 400 m with the average depth of the overburden was 58 m. The extent of the collapse was of the order of 350 m on strike and 350 m on dip and the outline of the collapsed region is shown in Figure 5. The upper and lower limits of the fall were bounded by two faults. Up to the time of the collapse there were no signs of pillars scaling and no timber poles were broken, suggesting that conditions were indeed stable. This condition, however, changed overnight with the entire rock mass to surface moving down between the two faults shown in Figure 5. No doubt this observation resulted to the hypothesis given in Gay et al’s paper that the presence and shear along near vertical faults can change the loading condition from a stiff displacement controlled system to a soft load controlled system. Kotze used numerical modeling and stress measurements in pillars adjacent to the collapse area conducted by the CSIR in 1975 to estimate a Merensky pillar strength of 75 MPa (for a pillar with an effective width of 4.3 m). This value is of course only applicable to the particular width to height ratio of the pillars (which is unfortunately not discussed in the paper).

3-Empirical Methods to Estimate HARD ROCK pillar Strength

Martin and Maybee (2000) give a very good overview of the different empirical strength formulas that were developed to predict pillar strength. A comparison of these formulas is given in Figure 6. These curves were calculated for a pillar height of 5 m.

Regarding South Africa, based on the success of the Salamon and Munro strength formula in the coal mines, it was natural for the shallow hard rock mines to adopt a similar power-law strength formula. The necessary research was, unfortunately, never conducted to develop and calibrate a formula for local conditions. Instead, the Hedley and Grant (1972) formula developed for the Canadian uranium mines was adopted. Only the K-value was modified to reflect local rock strengths. This approach seemed to work well and over the years it has become firmly entrenched. It will not be incorrect to state that it is currently the "industry accepted" method for designing pillars in shallow hard rock mines in South Africa. Ryder and Jager (2002) comment that this original formulation: "for want of local data, have subsequently been applied in South African hard-rock mines". This is a good example of what frequently happens in rock engineering. Over time, initial assumptions and interim solutions get entrenched as common practice and the original assumptions are rarely revisited or questioned. Based on this wide acceptance of the Hedley and Grant formulation and lack of local research into more appropriate formulations, it will be worthwhile to examine the assumptions made in the original paper.

The uranium mines in the Elliot Lake area used a stope-and-pillar layout to mine the orebody. Narrow pillars about 250 ft (76 m) were left along dip (see Figure 7). The pillars on which the analyses was conducted was typically 10 - 20 ft wide ($\approx 3 - 6$ m) and 8 - 20 ft high ($\approx 2.5 - 6$ m). The width to height ratio of most of the pillars was close to 1 and only a very few (3 in the database) had a width to height ratio of 2.5. It is clear that this original formulation was derived for slender rib pillars and it can be questioned whether it is applicable to square pillars with a width to height ratio greater than 2.5. The original data used by Hedley and Grant is reproduced in Figure 8. Although of poor

visual quality, this table is reproduced in its original form owing to its historical significance. It is immediately obvious that the dataset used was very small (28 pillars). This should be compared to the coal database of Salamon and Munro (1967) which included 125 pillars of which 27 were collapsed pillars. The width to height ratio of the failed pillars varied from 1.1 to 1.5. Only 3 of these pillars were "crushed" and 2 were "partially crushed". Of further concern is that it is stated in the paper that: "The information on complete pillar crushing was obtained second-hand because it happened in mines which are closed." This work was conducted in the days before computer-based numerical modelling could be used to determine pillar stress. The approach followed was therefore to use tributary area theory which relates the pillar stress to the pre-mining stress and the extraction ratio by:

$$\sigma_P = \frac{S_0}{1 - e} \quad (5)$$

where

σ_P = pillar stress

S_0 = pre-mining stress normal to the orebody

e = extraction ratio

For workings inclined at an angle α to the horizontal, the normal stress S_0 is a combination of the vertical stress component S_v and the horizontal stress component S_h :

$$S_0 = S_v \cos^2 \alpha + S_h \sin^2 \alpha \quad (6)$$

The vertical stress was simply assumed to be a function of the weight of the overlying strata and the horizontal stress perpendicular to strike was assumed to be 3000 psi based on measurements in two mines. The stress given in the table in Figure 8 for each pillar is therefore only a rough estimate. The methodology followed in the paper to derive the pillar strength formula can be summarized as follows:

The first step was to adopt the power law strength formulation used by Salamon and Munro (1972). In the notation of Hedley and Grant it is given as:

$$Q_u = K \frac{w^a}{h^b} \quad (7)$$

where

Q_u = pillar strength, psi

w = pillar width, ft

h = pillar height, ft

K = strength of 1 ft cube, psi

a and b are constants

Hedley and Grant acknowledge that this equation refers to square pillars, whereas those in the uranium mines are long and narrow. Their assumption was therefore that the strength of the slender pillars will not be much greater than a square pillar of width equaling the minimum width of the long pillar. Secondly, from extrapolation of laboratory tests, it was estimated that the value of K is 26 000 psi for a 1-ft cube.

Thirdly, appropriate values for parameters “ a ” and “ b ” had to be derived. Three different sets of values were available to them at that stage in the literature. The value for “ a ” was relatively constant at 0.5 and Hedley and Grant therefore decided to also adopt this value. As “ b ” varied more, a new value was computed and their approach was to focus on the three failed pillars in the database. For each of these pillars, the tributary area stress in the table (Figure 8) was assumed to be the pillar strength. This value, as well as the K -value and $a = 0.5$ were inserted into equation (7). For each pillar, the value of “ b ” was solved. The three values ranged from 0.736 to 0.768 with a mean of 0.75. This value was adopted and it resulted in the now familiar Hedley and Grant formulation.

Clearly the formulation above is based on a large number of assumptions and the applicability of this formulation to the design of hard rock pillars in the Bushveld Complex in South Africa becomes highly questionable. The first use of this formula in a South African mine is not clear, but Ozbay et al (1995) stated that it was “popularized by Wagner and Salamon (1979) as quoted by Kersten (1984)”. Kersten used it to design pillars for Agnes Gold Mine. Subsequently, it has been used to design a large number of bord and pillar

layouts in the country with an appropriate modification of the value of K . The rule frequently used in South Africa is to estimate K at between one-third and two-thirds of the UCS of the pillar material (e.g. see Ozbay et al. 1995). Almost no collapses in the Bushveld have been reported to date using this formulation (except where weak clay layers are present in the pillar, see sections below), so the uncomfortable question therefore remains: Why does it work and are the current designs perhaps too conservative? One hypothesis is that the “squat” behaviour of hard rock pillars may occur significant earlier than the $w:h$ ratio of 5 assumed for coal pillars (Ryder and Jager, 2002).

As a first attempt to develop a new formula for the South African mines, Watson (Watson et al, 2008) derived new values for the power law formulation given in equation (7). He used a maximum likelihood evaluation similar to that used by Salamon and Munro (1967). His database consisted of 179 Merensky Reef pillars of which 109 were stable. The width to height ratio of the pillars in the database ranged from 1 to 8, with the majority between 3 and 6. Only one pillar had a width to height ratio of less than 1. The values derived are $K = 86$ MPa, $a = 0.76$ and $b = 0.36$. It is interesting to note that the “ b ” parameter is much lower than for the Hedley and Grant formula. The formula predicts pillar strengths that are much greater than the traditional Hedley and Grant formulation with K values assumed a third of the UCS (see Figure 9). Unfortunately, it is not known if this formula has been tested in any trial mining sections.

4-Corrections for rectangular pillars

Equation (7) is applicable to square pillars. Another awkward assumption commonly made when designing pillars is to calculate the “effective width” for rectangular shaped pillars. Holland and Gaddy (1957) states that only the minimum lateral dimension, w , affects the strength of a

pillar, while the other dimension, L , has no effect. Wagner (1974) suggested that the effective width of a pillar should take the form:

$$w_{eff} = \frac{4A}{C} = \frac{2wL}{w+L}, \tag{8}$$



Fig 5. Extent of the collapse (dotted line) at No 6 Shaft, Bafokeng North mine.

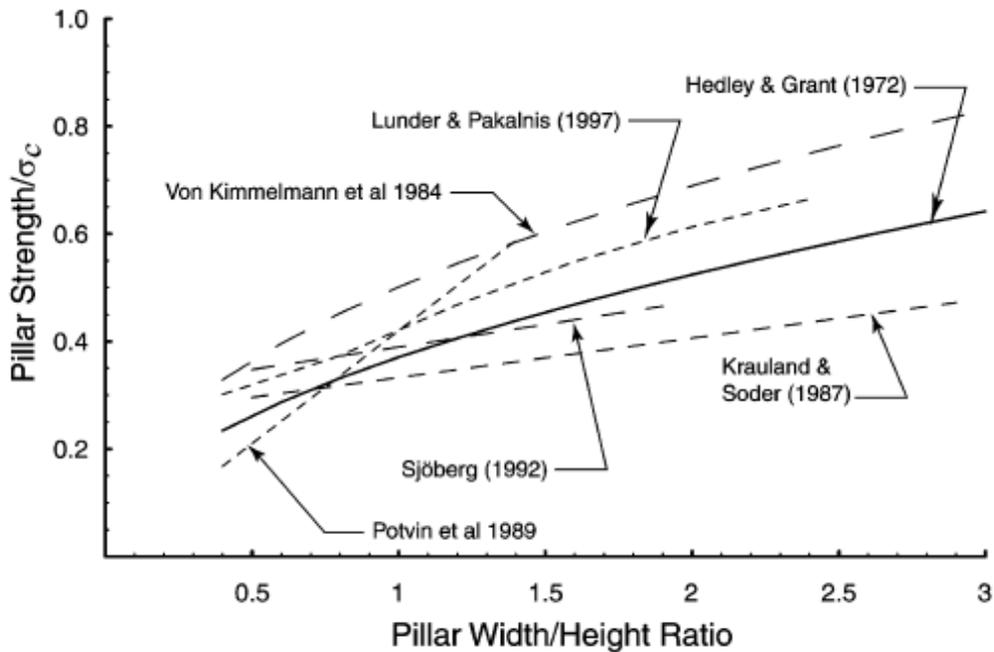


Fig 6. Comparison of empirical pillar strength formulae (after Martin and Maybee, 2000).

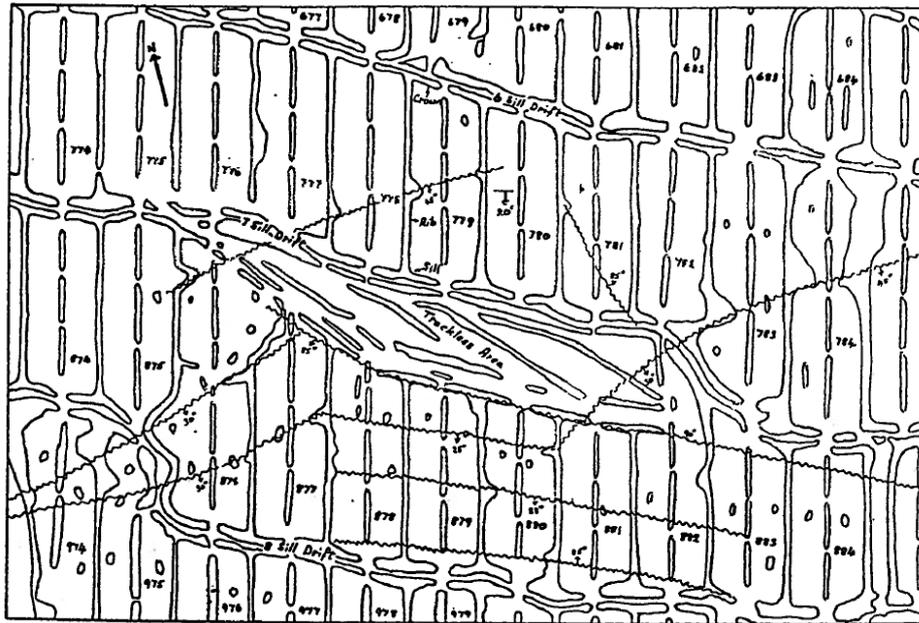


Fig 7. Typical layout of a mine in the Elliot Lake district (after Hedley, Roxburgh and Muppalaneni, 1983).

TABLE 3 — Dimensions and Estimated Stress and Strength of Pillars

Depth ft	Dip deg.	Extraction %	Pillar		Estimated Pillar	
			Width ft	Height ft	Stress psi	Strength psi
Stable Pillars						
500	17	85	10	10	5000	14600
700	17	85	10	10	6400	14600
800	26	65	20	18	3800	13300
850	20	85	10	10	7600	14600
950	11	85	10	10	7500	14600
1000	22	65	20	18	4000	13300
1050	15	85	10	10	8500	14600
1200	18	85	10	10	9400	14600
1300	20	65	20	20	4600	12300
1600	20	60	20	18	4800	13300
1600	20	65	18	18	5400	12600
1600	22	75	20	14	7600	16000
1700	22	65	40	20	5800	17400
1700	22	60	22	20	5000	12900
1700	12	75	20	14	7600	16000
1800	5	75	20	14	8000	16000
1900	23	65	19	18	6400	13000
2200	25	65	20	20	7200	12300
2400	11	65	20	8	7600	24400
2500	9	65	20	8	7900	24400
2700	13	65	20	8	8600	24400
2900	12	70	15	9	10500	19400
2900	12	75	20	9	12600	22400
Partially Failed Pillars						
1400	20	85	10	10	11400	14600
2400	18	80	10	9	13400	15800
Crushed Pillars						
2800	12	80	10	9	15200	15800
2900	12	80	10	9	15700	15800
3400	5	80	15	10	18500	17900

Fig 8. A reproduction of the original dataset used by Hedley and Grant (1972) to calibrate their power law formulation for pillar strength. Although of poor visual quality, this table is reproduced here in its original form owing to its historical significance.

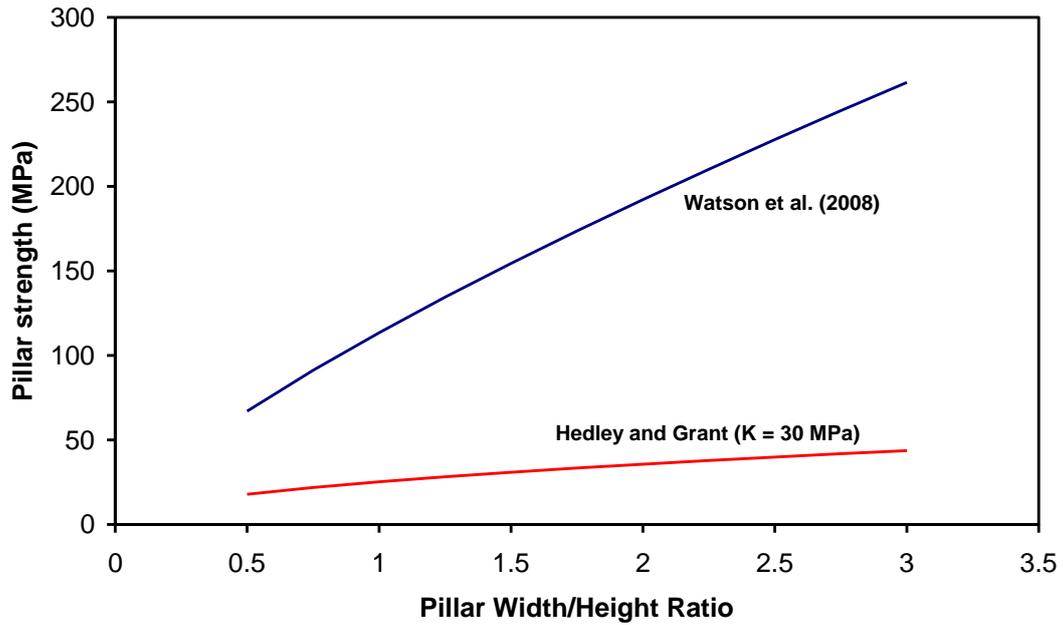


Fig 9. A comparison of the Merensky pillar strength predicted by the new Watson formula and the traditional Hedley and Grant using a K-value of 30 MPa (a third of the average UCS obtained from laboratory tests of Merensky Reef samples from Impala 2A Shaft).



Fig 10. Typical multi-layer composition of pillars in the Bushveld Complex exploiting the LG6 and LG6A chromitite layers.



Fig 11. Presence of weak clay layers in the pillars at Wonderkop mine. This photograph illustrates the presence of a clay layer between the LG6a chromitite and the pyroxenite below it.

Where A is the cross-sectional area of the pillar and C is the perimeter. This formula is based on observations made by Wagner during large scale underground tests namely that: “the strength of the circumferential portions of a pillar is virtually independent of the width-to-height ratio, whereas the strength of the centre increases with increasing ratio.” The effective width as defined by (8) approaches a finite value of twice the minimum pillar width for very long and narrow pillars. It is not clear if this assumption is correct and Ryder and Ozbay (1990) suggested an alternative shape strengthening factor of the form $f = 1.0/1.1/1.2/1.3$ for pillars having w_1/w_2 ratios of $1/2/4/\infty$. This implies that Wagner’s perimeter rule may be overestimating the strengthening effect of very long pillars.

Unfortunately no good experimental evidence is available regarding the effect of pillar shape on strength for hard rock pillars in South Africa and this area requires further research. Somewhat concerning is that the whole empirical design philosophy rests on a huge number of unproven assumptions and pillar strength is clearly an area that requires systematic research in future.

4-Recent Examples of Pillar Failures

To illustrate the inherent dangers of using empirical design formulas for rock masses in environments for which they were not originally designed, consider three case studies of recent mine collapses in the South African Bushveld Complex. Spencer (1999) reported on the failure of the pillar system and the subsequent closure of the Wonderkop Chrome Mine in May 1998 (stopping only commenced in September 1996). The mine is located close to Rustenburg. The mine exploited the lower group chromitite seams, namely the LG6 and LG6a. An internal pyroxenite waste band is found between these two reefs and this results in the pillars having a multi-layered appearance (Figure 10) wherever these two reefs are mined. The Wonderkop Mine was the most easterly situated LG6 mining operation in the Rustenburg area and situated adjacent to the Spruitfontein dome (an upfold structure which separates the Rustenburg section from the Marikana section). Its close proximity to the Spruitfontein dome has influenced the structure of the LG6 and LG6a seams and this has resulted in thick clay layers (up to 300 mm in some places) traversing the pillars in some areas (Figure 11). The position and thickness of this weak layer is highly variable (e.g. see Figure 12).

The original pillar design at the mine was conducted using the Hedley and Grant pillar strength formula. The pillar sizes were 12 m x 6 m giving an “effective width” of 8 m according to Wagner’s perimeter rule. K was assumed to be 27.3 MPa, which was a third of the laboratory strength of the rock (using samples obtained from another mining section). Owing to the complex multilayer structure of the pillars, it is not clear which of these layers were tested. The stoping width was 2 m, so the width to height ratio was at least 3 if the smallest dimension of the pillars is considered. When using the “effective width” of the pillars as 8 m, the strength of the pillars was estimated to be 45.9 MPa.

The first underground inspection of the pillars by the consulting rock engineer was conducted in July 1997. During this visit it was noted that some joints were beginning to open at the corners of some pillars. Some joints were also opening up at the sides of the pillars and in a few cases sliding along the clay layer was noted. Figure 13 illustrates the layout and the positions where pillar failure was observed. Following these observations, steps were taken to introduce a system of barrier pillars (Figure 14) with a width to height ratio of at least 10 (it is generally believed that at this ratio the pillar becomes indestructible). During the following nine months, the condition of the pillars continued to deteriorate. To strengthen the pillars along the main dip belt and road declines, two strategies were adopted namely waste stowing between the pillars and mesh and lacing of the pillars. The success of the mesh and lacing of the pillars appeared to be doubtful as the drilling process introduced additional water into the pillar which probably weakened it further. During April 1998, the failure process accelerated and the rate of closure in some areas increased to 1.8 mm/day. Numerous falls of ground occurred and management decided to cease operations at this stage. Recently, a back

analysis of this pillar failure was conducted by the authors using the TEXAN boundary element program (Napier and Malan, 2007) which can explicitly simulate the individual pillars and calculate the stresses on these pillars relatively accurately. The results are shown in Figure 15. Two face positions were simulated namely July 1997 (Figure 13) and July 1998 (Figure 14). From this study, the Hedley and Grant formulation was used to back-calculate the K -value for the pillars. A value as low as 6 MPa was obtained, which is in agreement with earlier back analysis studies by Spencer which estimated a value as low as 4.6 MPa. It should nevertheless be noted that a power law strength formula might not be applicable to these pillars owing to the presence of the clay layer.

Two other large scale pillar collapses recently occurred in the Bushveld Complex. Detailed information regarding these collapses is not available in the public domain and therefore these mines will only be referred to as Mine A and Mine B.

Mine A is a platinum mine located in the eastern portion of the Bushveld Complex. At this mine, a clay layer is also present at the hangingwall/pillar contact (Figure 16). The reef exploited in this area is the UG2. The original mine design was conducted using the Hedley and Grant pillar strength formulation with a K -value of 35 MPa. The mining height was 2 m. In mid 2008, some concern was expressed regarding the stability of the pillars and a minor collapse occurred during this time. In an attempt to reinforce some of the pillars, many were supported using fibre reinforced shotcrete. This did not stop the deterioration, however, as shown in Figure 17 with the cracked shotcrete clearly visible. During December 2008, operations were temporarily suspended at the mine when the decline was affected by the instability. Similar to Wonderkop Mine, Mine B also exploits the LG6/LG6A chromitite reefs. The problem is essentially similar to that experienced at Wonderkop Mine with the

presence of a clay layer in some of the pillars. This resulted in collapses in parts of the mine. Typical pillar failure at the mine is shown in Figure 18. Experience has indicated that increasing the pillar sizes does not necessarily work in these cases. From these studies, the drawbacks of using empirical pillar strength formulas are obvious. The mechanism of failure in all

three cases is caused by the presence of clay layers which substantially weaken the pillars. The original empirical formulas were developed for different rock types and the application of these formulas outside the limits for which they were developed resulted in the large mine collapses described here.



Fig 12. Presence of weak clay layers in proximity to the LG6/LG6A chromitite reefs at Wonderkop Mine.

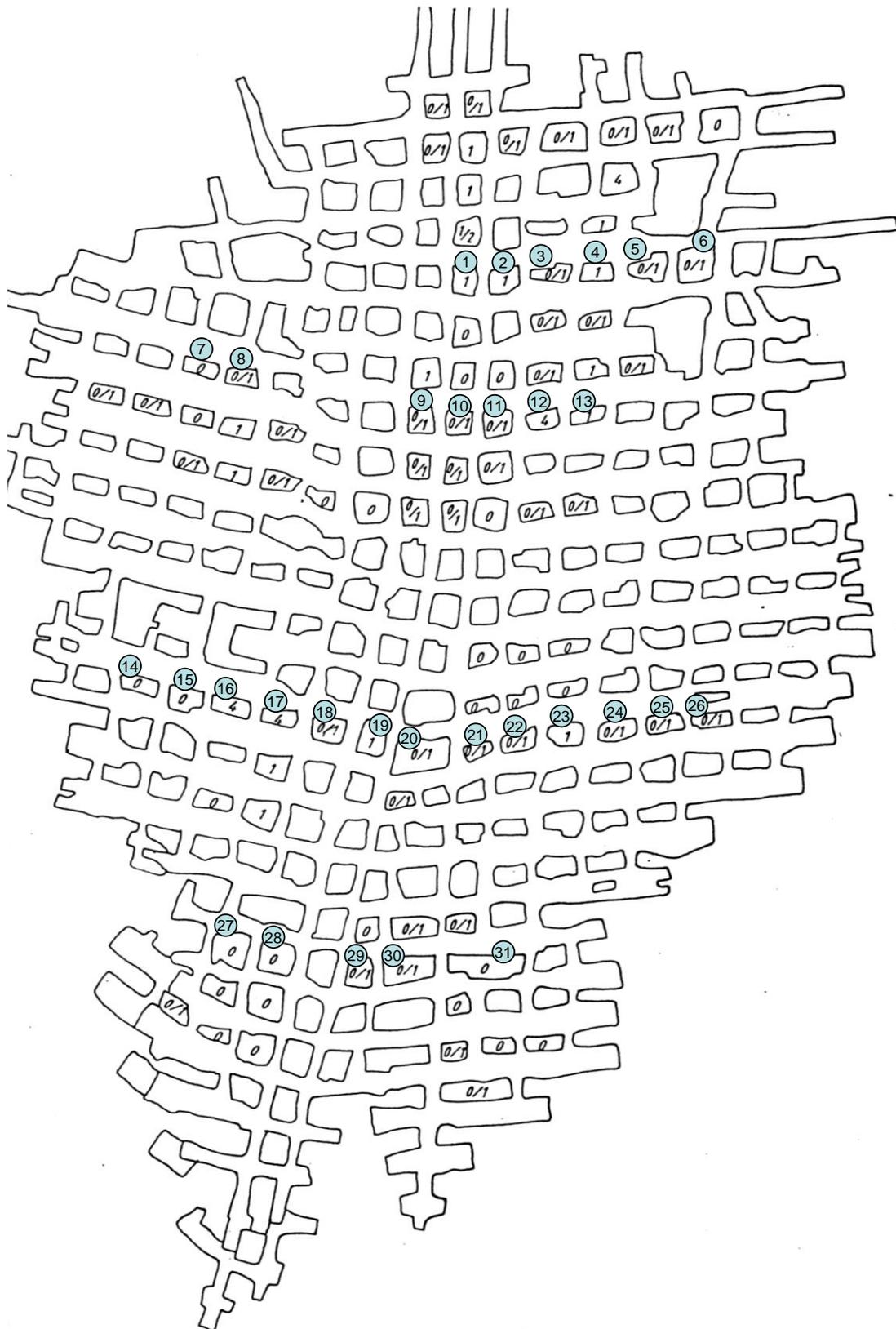


Fig 13. Failure condition of the pillars and the extent of mining during July 1997 (after Spencer, 1999). The failure codes used in this figure is as follows: 0 - No failure, 1 - Opening of joints at the corners, 2 - Opening of joints at the corners and along the sides, 3 - Material slabbing off the corners and sides, 4 - Horizontal movement occurring along the clay layer.

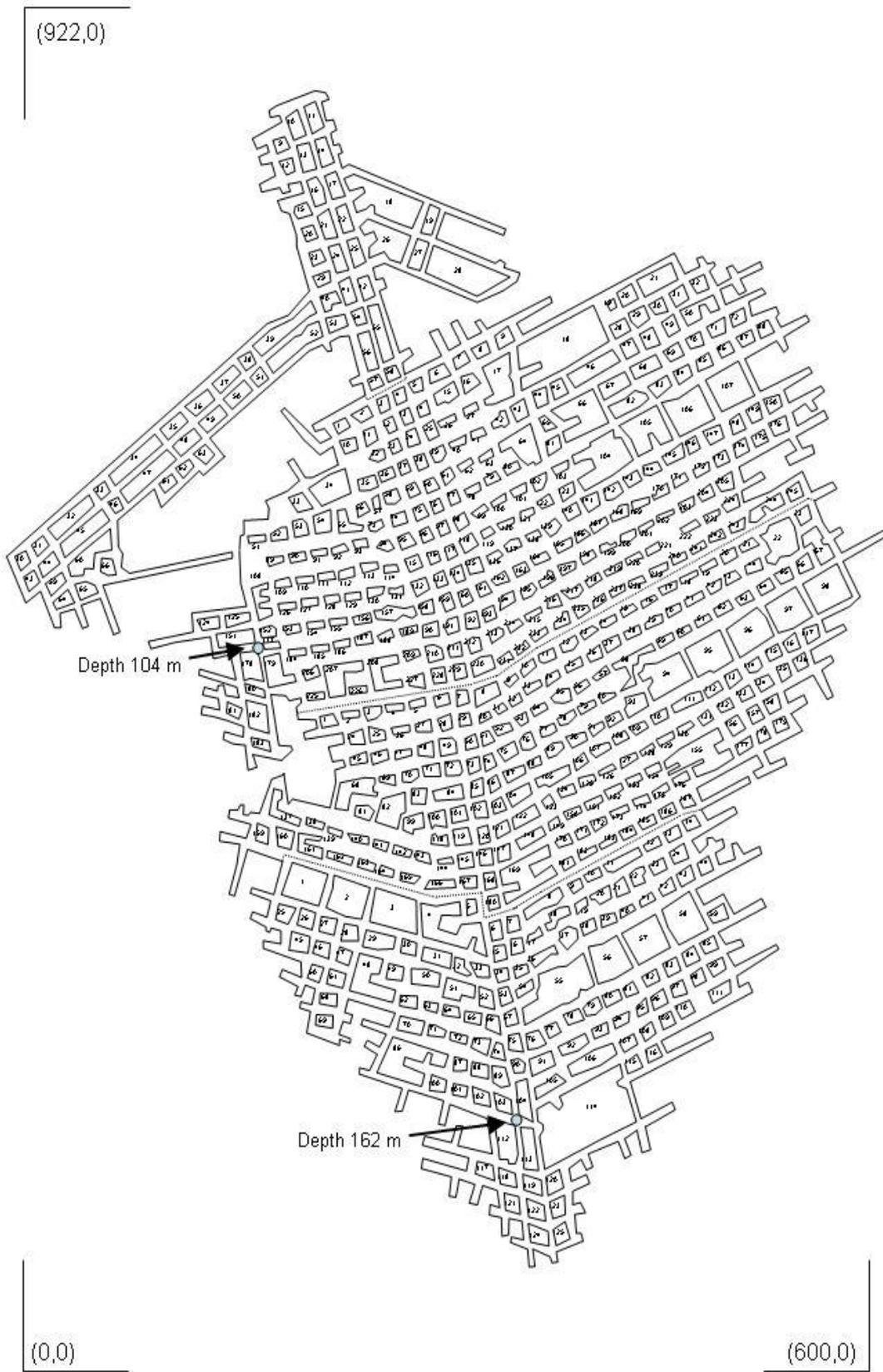


Fig 14. Extent of mining at Wonderkop Mine during July 1998.

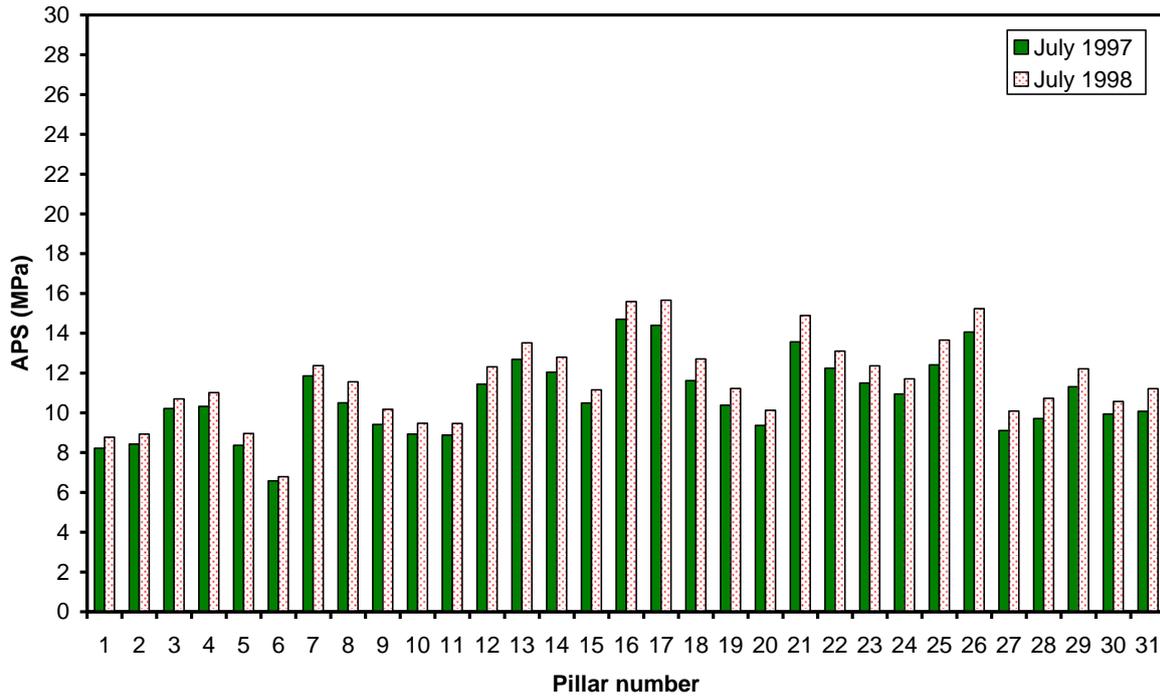


Fig 15. Simulated pillar stresses for selected pillars in the Wonderkop Mine (see Figure 11 for the positions of the pillars).



Fig 16. Typical pillar composition at Mine A (after Roberts and Clark-Mostert, 2010).



Fig 17. Pillar failure at Mine A after attempts to strengthen the pillars with shotcrete (photograph courtesy Dr Mike Roberts).



Fig 18. Interest mode of pillar failure at Mine B. For this pillar, a clay layer was found between the upper LG6A chrome and the pyroxenite below it. This slippery layer facilitates the fracturing of pyroxenite, causing it to scale out (left). The failures led to large amounts of convergence as can be seen in the photograph on the right.

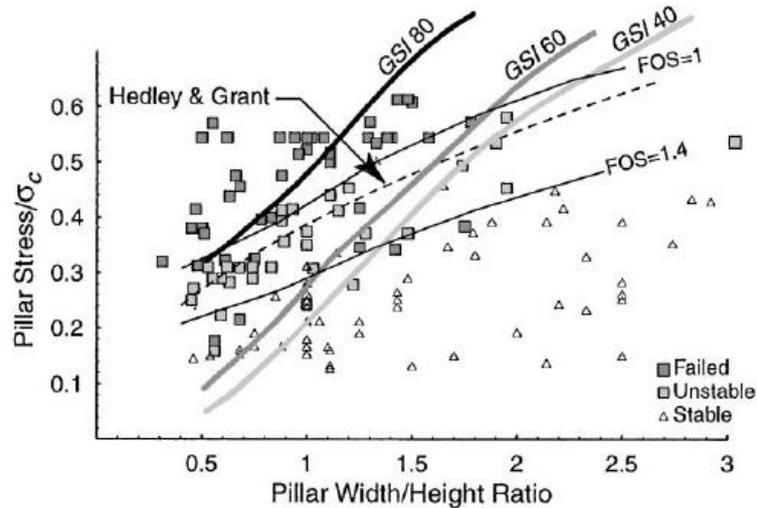


Fig 19: A comparison of the strength predicted by Hedley and Grant (1972), the data and FOS lines from Lunder and Pakalnis (1997) and the Phase 2 modelling for various GSI values (after Martin and Maybee, 2000).

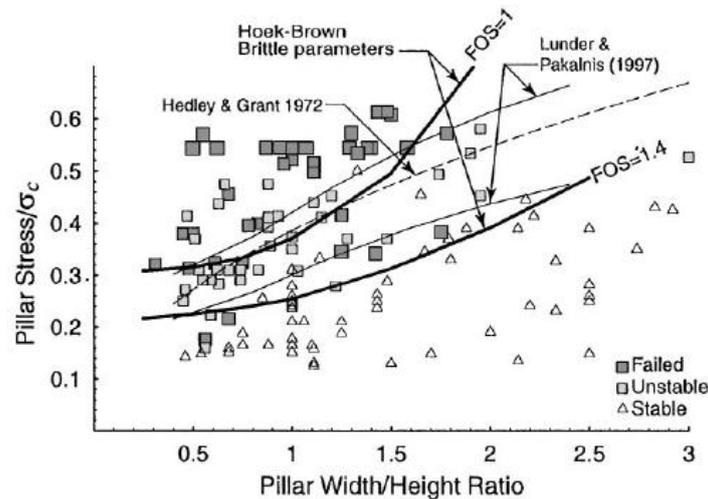


Fig 20: A comparison of the strength predicted by Hedley and Grant (1972), the data and FOS lines from Lunder and Pakalnis (1997) and modeling results using the Hoek and Brown brittle parameters.



Fig 21: An example of a pillar which contains a single joint dipping at almost 45° through the pillar. This joint will have to be modeled explicitly if the behaviour of this pillar is to be correctly simulated by a numerical modeling code.

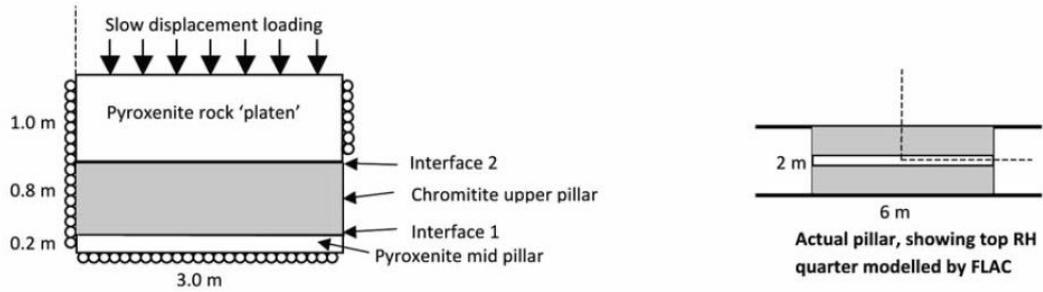


Fig 22. FLAC model to simulate the effect of weak interfaces in the pillar (after Potgieter and Malan, 2010).

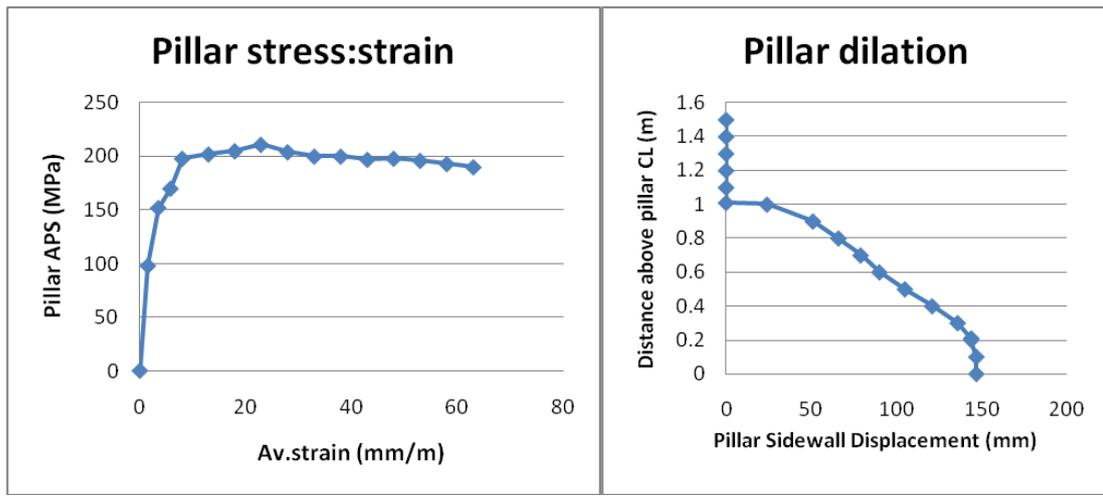


Fig 23. Pillar stress:strain and sidewall dilation (at peak strength point p), for interface friction angles of 30° (after Potgieter and Malan, 2010).

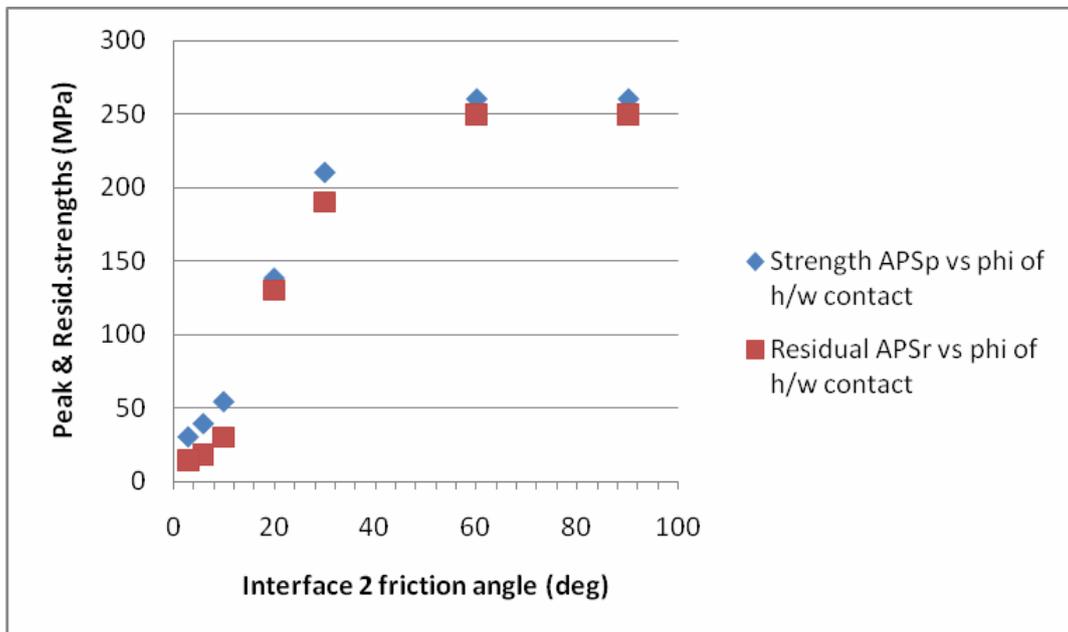


Fig 24. Peak and residual pillar strength versus hangingwall contact friction angle Φ_2 (after Potgieter and Malan, 2010).

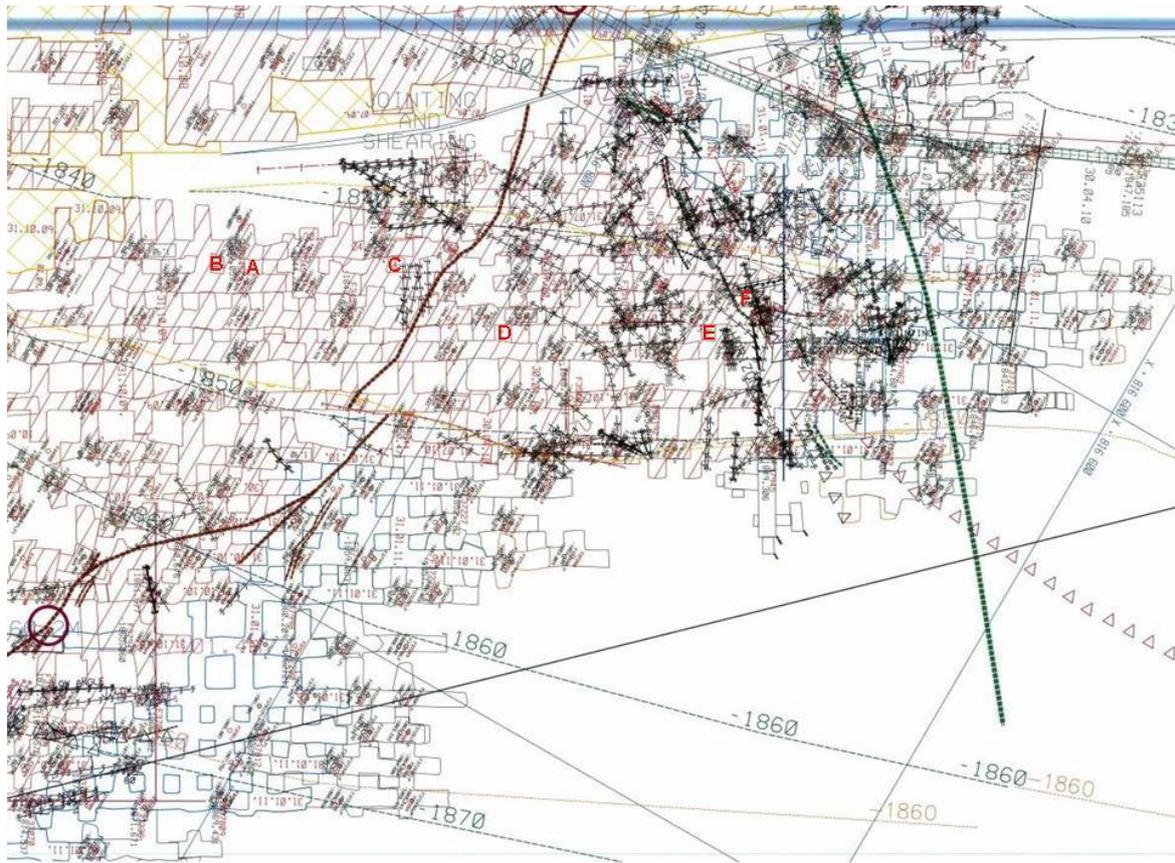


Fig 25. A bord and pillar layout in a platinum mine in South Africa.



Fig 26. Scaling on the edge of pillar A.

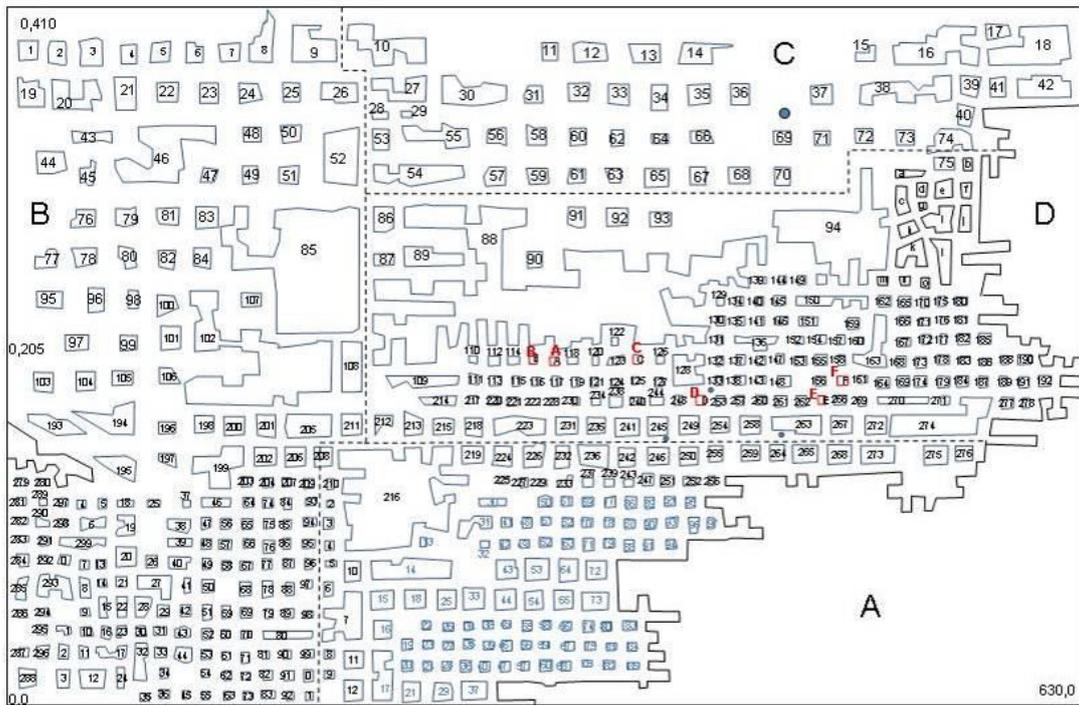


Fig 27. Representation of the pillar outlines with polygons to simplify the digitizing process.

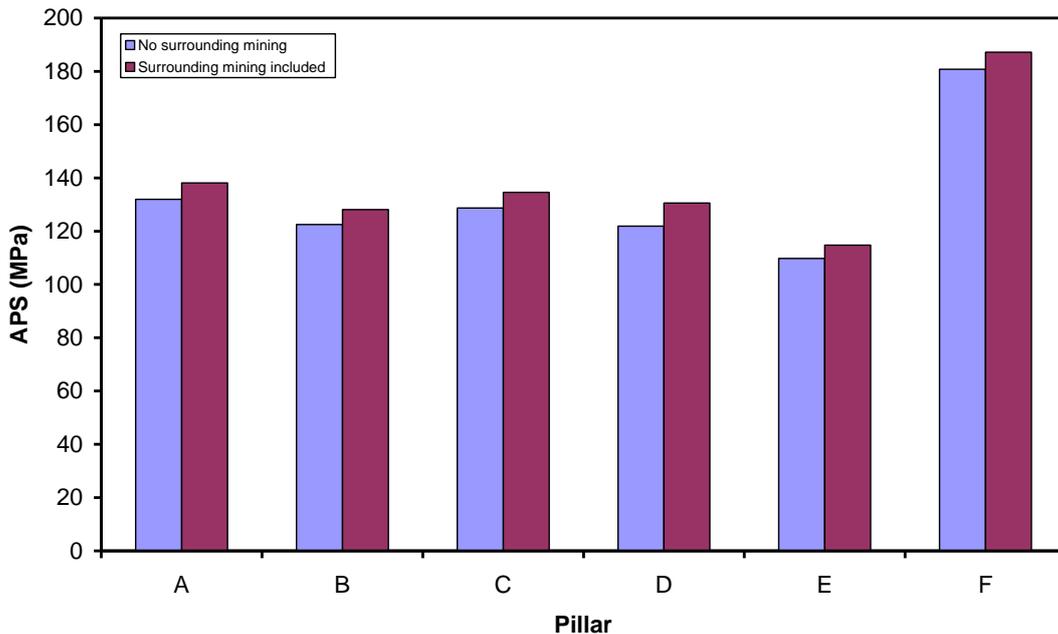


Fig 28. Simulated APS values of the pillars of interest.

5- Numerical Modelling to Estimate Pillar Strength

From the sections above, it is clear that the applicability of the current empirical formulas in hard rock mines is highly uncertain and additional verification and calibration work is required. The ore bodies in the Bushveld Complex are very

different to that for which the Hedley and Grant formulation was derived. Furthermore, the restricted range of slender w:h ratios used when deriving the equation is unfortunate and the choice of the appropriate K value is undefined and highly uncertain.

The alternative to the empirical approach is to use numerical modeling with appropriate failure criteria to determine pillar strength. A vast amount of literature is available on attempts to simulate pillar failure and it is not the objective of this paper to summarize all these findings. Focus will rather be placed on recent work that is applicable to the pillars in the Bushveld Complex.

Day and Godden (2000) presented a paper describing the design of panel pillars on Lonmin's platinum mines. They state that the validity of their approach is supported by extensive underground surveys and by computer back analysis studies. According to the authors, over 300 pillars per month were cut at Lonmin at that stage and these pillars behaved as expected. The method apparently works well up to width to height ratios of 5.5, but not at greater values owing to the onset of squat pillar effects. This seems rather disappointing as the expectation is that a numerical method with an appropriate constitutive model will "automatically" take care of the onset of squat pillar behaviour. Pillar strength was estimated by two-dimensional FLAC modeling using the following Hoek and Brown failure criterion:

$$\sigma_1 = \sigma_3 + \left(m\sigma_c\sigma_3 + s\sigma_c^2 \right)^{0.5} \quad (9)$$

where σ_c = uniaxial compressive strength (MPa) and m and s are constants that depends on the properties of the rock. The constant m was determined from laboratory testing and s equals to 1 for intact specimens. *In situ* values for the constants m and s were derived by application of rock mass quality ratings such as RMR using the equations of Priest and Brown (1983) for undisturbed rock masses:

$$m = m_i e^{\frac{RMR-100}{28}} \quad (10)$$

$$s = e^{\frac{RMR-100}{9}} \quad (11)$$

The authors derived *in situ* values for m and s for the UG2 and Merensky Reefs at Lonmin Mine. Typical values used in the modeling are as follows: UG2; $m = 25.83$,

$s = 0.51$ for a RMR of 94, Merensky (Type B); $m = 8.7$, $s = 0.57$. The resulting simulated pillar strengths seem plausible when the pillar width to height ratio is low. The authors nevertheless acknowledge that the squat pillar behaviour is not correctly simulated by this approach and that this will probably result in pillars being over-designed at depths exceeding 700 m.

A further concern regarding this approach is that it retains the flavour of a "semi-empirical" approach and the unmodified equations (10) and (11) may not be appropriate for the pillar material in the Bushveld Complex. Pells (2008) also expressed some concern about the Hoek and Brown failure criteria and quoted Mostyn and Douglas (2000) which provided a detailed critique of this failure criterion for intact rock.

"...there are inadequacies in the Hoek-Brown empirical failure criterion as currently proposed for intact rock and, by inference, as extended to rock mass strength. The parameter m_i can be misleading, as m_i does not appear to be related to rock type. The Hoek-Brown criterion can be generalized by allowing the exponent to vary. This change results in a better model of the experimental data." Martin and Maybee (2000) investigated the strength of hard rock pillars by using both empirical pillar strength formulae and numerical modeling using a Hoek-Brown failure envelope. Figure 19 illustrates a comparison between the pillar stability graph developed by Lunder and Pakalnis (1997), the Hedley and Grant equation and Phase 2 - Hoek and Brown modelling using different values of GSI. Figure 20 illustrates the same data using the Hoek and Brown brittle parameters. The conclusion reached by Martin and Maybee (2000) is that two-dimensional finite element analyses using conventional Hoek and Brown parameters for typical hard rock pillars predicted rib pillar failure envelopes that did not agree with empirical pillar failure envelopes. It appears that the conventional Hoek-Brown failure

envelopes over-predict the strength of hard rock pillars. This occurs because the failure process is fundamentally controlled by a cohesion loss process in which the frictional strength component is not mobilized. Their recommendation is that Hoek-Brown brittle parameters ($m_b = 0$ and $s = 0.11$) be used to simulate pillar strength.

Hoek, Kaiser and Bawden (1995) summarized the rock mass conditions for which the Hoek-Brown failure criterion can be applied. The criterion is only strictly applicable to intact rock or heavily jointed rock masses that can be considered to be homogenous and isotropic. For cases in which rock mass behaviour is controlled by a single discontinuity or joint set, a criterion that describes the shear strength of joints should rather be used (e.g. see the pillar shown in Figure 21). The implication of this is that for the three case studies of pillar failure in the Bushveld Complex discussed above, explicit simulation of the clay layer will be required. An example was conducted for the author by Dr John Ryder using the FLAC code and this is presented below to illustrate the value of modeling.

The pillar composition simulated was the LG6/LG6A "sandwich" shown in Figure 10. The qualitative effect of a strong pyroxenite layer within a chromitite pillar (with weak contacts, including weak hangingwall and footwall contacts) was therefore modelled in 2D. A generic model was built to investigate the effect of an inhomogeneous pillar with weak interfaces. Estimated in situ strain-softening parameters were drawn directly from studies carried out in the Bushveld Complex. The hanging wall and footwall were assigned the same properties as the pyroxenite layer, and symmetry was assumed for both the vertical and horizontal centerlines in the following layout in the FLAC finite difference code (Figure 22). The grid size was $0.1 \text{ m} \times 0.1 \text{ m}$. By applying slow displacement loading (velocity $5 \times 10^{-4} \text{ mm/step}$), complete

stress-strain and lateral deformation curves could be modelled (Figure 23). Lateral deformations showed no dramatic effects owing to the presence of the 'strong' layer of pyroxenite in the pillar, possibly because the modelled contrasts in strength, Poisson's ratio and dilatancy were not particularly marked. (Note that the horizontal scale in Figure 23b is in millimeters whereas the vertical scale is in metres). Likewise, the presence of a weak interface between the layer and the body of the pillar had virtually no effect, even if the friction angle of interface 1 was set as low as 6° . In contrast, low friction angles on the hanging wall contact (interface 2) had a powerful effect, reducing the peak strength p of the pillar by allowing lateral deformation and reduction of confinement, and reducing also the residual strength (Figure 24).

6-Case STUDY: Back analysis of Pillar Strength

To illustrate the value of a careful back analysis of pillar strength, an example of a bord and pillar section at a depth of approximately 1100 m is described in this section. Figure 25 illustrates the layout of the area. The small pillars are planned to be $6 \text{ m} \times 6 \text{ m}$ and the extraction ratio 75%. The pillars were carefully scrutinised during an underground visit to ensure that the plan agrees with the shape of the pillars observed underground. Pillars A, B, C, D, E and F were measured and photograph for use in the numerical modelling back analysis. As an example, pillar A is shown in Figure 26. In spite of the mining depth, the pillars appeared to be in a reasonably good condition. Prominent scaling of the sidewalls could nevertheless be observed on some of the pillars (but not all pillars) and this may be an indication that these pillars are loaded close to their peak strength. To back-calculate the pillar strength, numerical modelling of the pillar stress was required as there are large barrier pillars to protect the conveyer belts. Tributary area theory (TAT) will therefore

be a poor estimate of the stresses on the pillars. For the sake of interest, the TAT value on the 6 m x 6 m pillars for the 75% extraction ratio is given by

$$APS = \frac{\rho hg}{1-e} = \frac{2800 * 1100 * 10}{1-0.75} Pa = 123.2 MPa \quad (12)$$

where ρ is the density of the rock, h is the depth, g is gravitational acceleration and e is the extraction ratio.

The plan was carefully digitised and the layout simulated with the TEXAN code (Napier and Malan, 2007). The pillar outlines were represented by straight line polygons to simplify the digitising process. The parameters used are given in Table 1. The simulated APS values for the selected pillars are given in Figure 28. Two values are shown for each pillar, namely the simulated values if only the smaller region D is simulated and the simulated values if the surrounding mining areas A, B and C are included. Note that the APS values are larger with the surrounding areas included as the extraction ratio is high in these areas and it causes an additional transfer of load on the pillars of interest. The APS values of some of these pillars are also larger than the TAT value as some of these pillars are smaller than the specified 36 m². For these simulated APS values, minimum K-values for the Hedley and Grant formula could be calculated. These are shown in Table 2. It appears that a K-value of at least 90 MPa can be assumed for these pillars. This is significantly higher than the 45 MPa value that was traditionally assumed in these mines.

7-Conclusion

This paper gives an overview of the difficulties associated with determining the strength of hard rock pillars. Although a number of pillar design tools are available, pillar collapses still occur. Recent examples of large scale pillar collapses in South Africa were caused by weak partings which traversed the pillars. Currently, two

different methods to determine the strength of pillars are used namely, empirical equations derived from the back analyses of failed and stable cases and numerical modeling using appropriate failure criteria. Both techniques have their limitations and additional work is required to obtain a better understanding of pillar strength.

Empirical methods are popular and easy to use, but care should be exercised that the results are not extrapolated beyond the range of the data which were used to derive them. An example is the Hedley and Grant formula (derived for the Canadian uranium mines) that has been used for many years in the South African platinum and chrome mines (albeit with some adaptation of the K-value) to design pillar layouts in these mines. A careful study of the original publication by Hedley and Grant indicates that this formula is based on many assumptions that are not applicable to South African mines. Nevertheless, very few collapses have been reported and in some cases it appears that the Hedley and Grant formula might underestimate pillar strength significantly.

As an alternative, some engineers strongly advocate the use of numerical techniques to determine pillar strength. A close examination unfortunately reveals that these techniques also rely on many assumptions and extreme care needs to be exercised when using this approach. Pillar load is another unknown not discussed in this paper and care should also be exercised when this is estimated using numerical procedures (some difficulties will be outlined in Napier and Malan, 2011). An area where numerical modeling is invaluable, however, is to study specific pillar failure mechanisms, such as the influence of weak partings on pillar strength.

In conclusion, it appears that neither empirical techniques nor numerical modeling currently provide a solid basis to predict pillar strength. It is therefore recommended that both these techniques be utilized when addressing pillar design

problems in order to obtain the best possible insights. Owing to the uncertainties regarding pillar strength, pillar stress and loading stiffness, monitoring in trial mining sections (and even in established mining areas) is

considered to be an essential tool to assess the stability of pillar layouts in particular geotechnical areas. The need for additional research into pillar strength should also be emphasized strongly as this problem has clearly not yet been solved!

Table 1. Parameters used in the numerical model.

Parameter	Value
Young's Modulus	70 GPa
Poisson's ratio	0.28
k-ratio	1.5 (strike) 1 (dip)
Average overburden density	2800 kg/m ³

Table 2. Calculated minimum K-values.

Pillar	Area (m ²)	Perimeter (m)	Effective width (m)	Height (m)	APS (MPa) (surrounding mining included)	Minimum K-value (Hedley & Grant)
A	26.9	20.8	5.2	2.0	138.1	102.1
B	34.9	23.6	5.9	2.0	128.1	88.6
C	37.0	24.2	6.1	2.0	134.6	91.5
D	36.3	24.0	6.1	2.0	130.6	89.3
E	45.9	27.0	6.8	2.0	114.8	74.0
F	20.1	18.4	4.4	2.0	187.2	150.6

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