Cut-off Grade Optimization for Maximizing the Output Rate

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Abstract

In the open-pit mining, one of the first decisions that must be made in production planning stage, after completing the design of final pit limits, is determining of the processing plant cut-off grade. Since this grade has an essential effect on operations, choosing the optimum cut-off grade is of considerable importance. Different goals may be used for determining optimum cut-off grade. One of these goals may be maximizing the output rate (amount of product per year), which is very important, especially from marketing and market share points of view. Objective of this research is determining the optimum cut-off grade of processing plant in order to maximize output rate. For performing this optimization, an Operations Research (OR) model has been developed. The object function of this model is output rate that must be maximized. This model has two operational constraints namely mining and processing restrictions. For solving the model a heuristic method has been developed. Results of research show that the optimum cut-off grade for satisfying pre-stated goal is the balancing grade of mining and processing operations, and maximum production rate is a function of the maximum capacity of processing plant and average grade of ore that according to the above optimum cut-off grade must be sent to the plant.

Key words: Cut-off Grade, Open-Pit Mine, Production Planning, Output Rate, Balancing Grade, Maximum Output Cut- Off grade

1- Introduction

One of the most critical parameters in mining operation is the cut-off grade. Taylor defined cut-off grade as ‘‘any grade that, for any specific reason, is used to separate two courses of action, e.g. to mine or to leave, to mill or to dump . . .’’ [1], [2].

Somewhere else cut-off grade is defined as the grade that is used to discriminate between ore and waste within a given ore body. [3] The material in the deposit with the grade higher than cutoff grade is ore, which is sent to the processing plants; the material below the cutoff grade is sent to the waste dumps [3], [4].

Processing plant cut-off grade is an important parameter, which is determined after designing ultimate pit limits. Processing plant cut-off grade is defined as a grade that discriminates between ore and waste blocks within a given pit. If block grade in the pit is above cut-off grade it is classified as ore and if block grade is below cut-off grade it is classified as waste. Ore being the economical portion of the mineral deposit is sent to the mill or processing plant for crushing, grinding, and up-gradation of metal content [5].

As cut-off grade provides a basis for the determination of tonnes of ore and tonnes of waste, it directly affects the cash flows of a mining operation, based on the fact, that, higher cut-off grade leads to higher grades per ton of ore, hence, higher net present value (NPV) is realized depending upon the grade distribution of the mineral deposit [4].

Most researchers have used break-even cut-off grade criteria to define ore as a material that just will pay mining and processing costs. These methods are not optimum but the mine planner often seeks to optimize the cut-off grade of ore to maximize the net present value (NPV). [6]

The choice of the cutoff grade in mining influences the profitability and life of individual mines and thereby the quantity of a resource that is available to society. It is of such importance that it has been subject to regulation, most notably in South African gold mining. Though a major preoccupation of engineers, it is the subject of only a tiny literature in nonrenewable-resource economics. The

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optimal cut-off grade depends on all the salient technological features of mining, such as the capacity of extraction and of milling, the geometry and geology of the orebody, and the optimal grade of concentrate to send to the smelter [7].

Cut-off grade optimization can be performed considering different objectives. Maximizing net present value (NPV) is the most applicable objective. Work undertaken in the field of cut-off grade optimization has not advanced much beyond the work undertaken by Lane, which began in 1964 [8] and completed in 1988 [9]. His definitive work is based on the calculus of the Net Present Value criterion, which is the most widely understood, consistent, and appropriate method by which sequential cash flows arising from the extraction of mineral reserves from an exhaustible resource can be represented [10].

In addition to economical objectives, other goals may be defined for operations, as well. One of these goals may be maximizing output rate considering functional constraints. This objective is particularly important from marketing and market share points of view. A question, which may be arisen in marketing department of a mining firm, is that assuming constant capacities for the mining operation and processing plant, how much product can be produced in one year period? Or how much of market needs can be satisfied by a given mine and processing plant? The aim of this research is finding an answer for this question, and determining a cut-off grade for maximizing the amount of product per year or output rate.

2- Problem definition

As mentioned, problem under discussion in this paper is optimizing cut-off grade in order to maximize output rate. It is assumed that operation consists of two stages, mining and processing, and concentrate is the final product of operation. For solving this problem an Operations Research (OR) model with a maximization objective function and two functional constraints is formulated and then for solving the model a heuristic method has been developed.

2-1- Defining Parameters and Decision Variables of the Model

Parameters and decision variables that are used in the model are as follows:

Q_m:tonnage of total material in the pit, which is a constant.

g: Cut-off grade that is the main decision variable of model.

y: Recovery (yield) that is constant.

Q_h: tonnage of total ore in the pit that is increased as cut-off grade is decreased and vise versa. So Q_h is an absolutely descending function of g.

$\overline{g}$: Average grade of the ore that is increased as cut-off grade is increased and vice versa. So $\overline{g}$ is an absolutely ascending function of g.

Q_k: Tonnage of total product, which depends on amount of the ore and its average grade. Like Q_h, Q_k is an absolutely descending function of g.

g_{con}: Average grade of the final product that is a constant. Considering above mentioned definitions the following equation is satisfied:

$$Q_k = \frac{y\, Q_h\, \overline{g}}{g_{con}} \tag{1}$$

T: Production lifetime. This is one of the decision variables. Assuming constant capacities T is a descending function of g.

M: Ultimate capacity of the mining operation in terms of the tonnes of mined material (ore plus waste) per year, which is a constant. The amount of M depends on the capacity of drilling, blasting, loading, and hauling activities.

H: Ultimate capacity of the processing operation in terms of the tonnes of ore per year, which is a constant. The amount of H depends on the capacity of crushing, milling, concentrating, and other plant activities.

g_{mh}: The balancing cut-off grade of mining and processing. If processing cut-off grade would be equal to this amount, both mining and processing operations will operate at their maximum capacities. That is,

$$g = g_{mh} \Leftrightarrow \frac{Q_h}{Q_m} = \frac{H}{M} \tag{2}$$

Since H, M and Q_m are constants, but Q_h is a descending function of g, so in general,

$$\begin{cases} g \le g_{mh} \Leftrightarrow \dfrac{Q_h}{Q_m} \ge \dfrac{H}{M} \\ g \ge g_{mh} \Leftrightarrow \dfrac{Q_h}{Q_m} \le \dfrac{H}{M} \end{cases} \tag{3}$$

2-2- Formulating the model for problem

a. Objective function. As mentioned previously, in this problem we want to determine optimum cut-off grade which maximizes amount of product per year or output rate. Obviously, this is not an economical objective, but it is a functional goal that refers to the amount of the product that the mine is capable to supply per year. The objective function for problem can be written as:

$$\text{maximize} \quad Q_{k/y} = \frac{Q_k}{T} \tag{4}$$

where $Q_{k/y}$ is the amount of product per year, i.e. rate of output.

b. Constraints of model. Capacities of the mining and processing operations are the functional constraints of the model. These constraints can be formulated in terms of mathematical expressions as follows,

$$T \ge \frac{Q_m}{M} \quad , \quad T \ge \frac{Q_h}{H} \tag{5}$$

Therefore, the final model for this problem will be as figure 1.

$$\boxed{\begin{array}{ll} \text{maximize} & Q_{k/y} = \dfrac{Q_k}{T} \\ s.t. & T \ge \dfrac{Q_m}{M} \\ & T \ge \dfrac{Q_h}{H} \end{array}}$$

Figure1. Final model for problem.

3- Solving the Model

For solving the model two constraints can be rewritten as follows,

$$T \ge \max\left\{\frac{Q_m}{M}, \frac{Q_h}{H}\right\} \tag{6}$$

In boundary conditions this single constraint becomes,

$$T = \max\left\{\frac{Q_m}{M}, \frac{Q_h}{H}\right\} \tag{7}$$

Substituting this equation in objective function results,

$$Q_{k/y} = \frac{Q_k}{\max\{Q_m/M, Q_h/H\}} \tag{8}$$

Equation (8) can be rewritten as follows,

$$Q_{k/y} = \begin{cases} \dfrac{Q_k}{Q_m/M} & \text{if } Q_m/M \ge Q_h/H \\ \dfrac{Q_k}{Q_h/H} & \text{if } Q_m/M \le Q_h/H \end{cases} \tag{9}$$

or,

$$Q_{k/y} = \begin{cases} M\dfrac{Q_k}{Q_m} & \text{if } Q_h/Q_m \le H/M \\ H\dfrac{Q_k}{Q_h} & \text{if } Q_h/Q_m \ge H/M \end{cases} \tag{10}$$

refer to Equation (3) $Q_h/Q_m \le H/M$ means that $g \ge g_{mh}$ and $Q_h/Q_m \ge H/M$ means that $g \le g_{mh}$, so considering this subject and substituting Q_k from Equation (1), we have,

$$Q_{k/y} = \begin{cases} My\dfrac{\bar{g}}{g_{con}}\dfrac{Q_h}{Q_m} & \text{if } g \ge g_{mh} \\ Hy\dfrac{\bar{g}}{g_{con}} & \text{if } g \le g_{mh} \end{cases} \tag{11}$$

Therefore, maximum amount of $Q_{k/y}$ will be equal to maximum value of $My\left(\bar{g}/g_{con}\right)(Q_h/Q_m)$ or $Hy\left(\bar{g}/g_{con}\right)$, which one is higher. Then, for maximizing $Q_{k/y}$ we firstly, have to find the maximum value of $My\left(\bar{g}/g_{con}\right)(Q_h/Q_m)$ and $Hy\left(\bar{g}/g_{con}\right)$ in their allowed range of grade.

1- For maximizing $My\left(\bar{g}/g_{con}\right)(Q_h/Q_m)$, since Q_k is an absolutely descending function of g, so the highest value of $M\,Q_k/Q_m$ or $My\left(\bar{g}/g_{con}\right)(Q_h/Q_m)$ occurs at the lowest amount of its allowed range, i.e. $g = g_{mc}$. Then refer to Equation (2), we will have $Q_h/Q_m = H/M$. So,

$$\max\left\{My\,\frac{\bar{g}}{g_{con}}\frac{Q_h}{Q_m}\right\} = My\,\frac{\bar{g}_{mh}}{g_{con}}\frac{H}{M} = Hy\,\frac{\bar{g}_{mh}}{g_{con}} \tag{12}$$

where $\bar{g}_{mh}$ is average grade of ore when cut-off grade is set equal to g_{mh}.

2- On the other hand for maximizing $Hy\left(\bar{g}/g_{con}\right)$, since $\bar{g}$ is an absolutely ascending function of g, so the highest value of $Hy\left(\bar{g}/g_{con}\right)$ occurs at the highest amount of its allowed range, i.e. $g = g_{mc}$. Therefore,

$$\max\left\{Hy\left(\bar{g}/g_{con}\right)\right\} = Hy\left(\bar{g}_{mh}/g_{con}\right) \tag{13}$$

Comparing Equations (11), (12), and (13) results,

$$\max Q_{k/y} = Hy\,\frac{\bar{g}_{mh}}{g_{con}} \tag{14}$$

So, for maximizing output rate, $Q_{k/y}$, it is enough to set the processing cut-off grade equal to the mining and processing balancing cut-off grade, i.e. g_{mh}. Therefore, we can call g_{mh} as maximum output cut-off grade, and solution of the model will be,

$$\begin{cases} g_{opt} = g_{mh} \\ \text{maximum amount of } Q_{k/y} = Hy\left(\bar{g}_{mh}/g_{con}\right) \end{cases} \tag{15}$$

4- Illustrative example

Mineral inventory of a hypothetical mine is shown in Table 1. Assume that recovery (y) is 100 percent, product average grade (g_{con}) is 20%, and ultimate capacity of mining (M) and processing (H) are 100 and 50 tonnes per year, respectively. We want to determine the optimum cut-off grade of processing for maximizing output rate.

Cumulative tonnes and average grade of ore within mine as a function of cut-off grade is shown in Table 2.

Table 1. Mineral inventory of hypothetical mine.

Grade (%)	Quantity (tonnes)	Average Grade (%)
0.0-0.1	100	0.05
0.1-0.2	100	0.15
0.2-0.3	100	0.25
0.3-0.4	100	0.35
0.4-0.5	100	0.45
0.5-0.6	100	0.55
0.6-0.7	100	0.65
0.7-0.8	100	0.75
0.8-0.9	100	0.85
0.9-1.0	100	0.95

Table 2. Cumulative tonnes and average grade of ore within hypothetical mine as a function of cut-off grade.

g (%)	Q_h(tonnes)	$\bar{g}$ (%)	Q_k (tonnes)
0.0	1000	0.5	25
0.1	900	0.55	24.75
0.2	800	0.6	24
0.3	700	0.65	22.75
0.4	600	0.7	21
0.5	500	0.75	18.75
0.6	400	0.8	16
0.7	300	0.85	12.75
0.8	200	0.9	9
0.9	100	0.95	4.75

As can be seen total amount of materials within the mine, Q_m, is equal to 1000 tonnes. Model Fig. 1 for this problem will be as follows,

$$\boxed{\begin{aligned} &\text{maximize} \quad Q_{k/y} = \frac{Q_k}{T} \\ &s.t. \qquad T \geq \frac{1000}{100} = 10 \\ &\qquad\qquad T \geq \frac{Q_h}{50} \end{aligned}}$$

We use Equation (2) for determining g_{mh}. Since $H/M = 0.5$, then we have to find a cut-off grade for which $Q_h/Q_m = H/M = 0.5$. By searching Table 2 we will find that for g = 0.5% this equality is satisfied. Therefore, g_{mh} is equal to 0.5%, and for this value of g amount of $\bar{g}_{mh}$ and Q_h will be equal to 0.75% and 50 tonnes, respectively.

By substituting these quantities in Equation (15), we find,

$$\begin{cases} g_{opt} = 0.5\ \% \\ \text{maximum amount of } Q_{k/y} = 1.875 \text{ tonnes per year} \end{cases}$$

Amount of $Q_{k/y}$ for different values of g is shown in Table 3, and curve of $Q_{k/y}$ variation versus g is depicted in Figure 2.

Table 3. Values of $Q_{k/y}$ for different values of *g*.

g(%)	$\bar{g}$ (%)	$Q_{k/y}$ (tonnes/year)
0.0	0.5	1.25
0.1	0.55	1.375
0.2	0.60	1.5
0.3	0.65	1.625
0.4	0.70	1.75
0.5	0.75	1.875
0.6	0.80	1.6
0.7	0.85	1.275
0.8	0.90	0.9
0.9	0.95	0.475

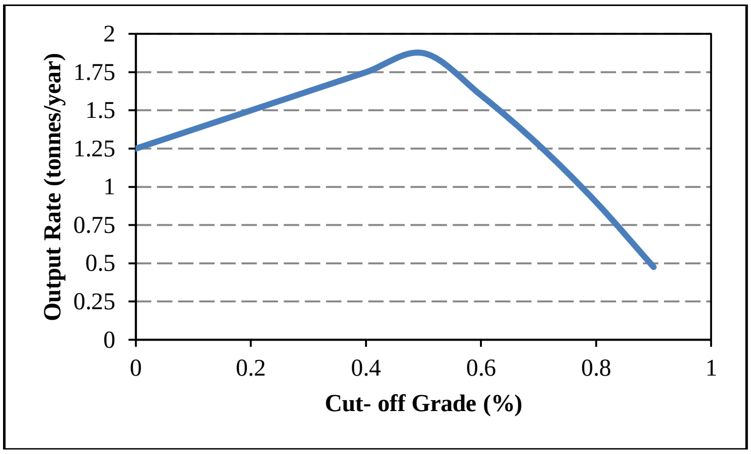

Figure 2. curve of $Q_{k/y}$ variation versus *g*.

5- Conclusions

Optimization of the processing plant cut-off grade is one of the most important items in production planning stage of an open pit mine. This optimization may be performed based on different objectives.

For example, in well-known Lane's algorithm for cut- off grade optimization, the objective function is maximizing net present value (NPV) that is an economical goal.

In this paper determination of the optimum cut-off grade in order to maximize output rate has been investigated. In the model that is developed here, the objective function is maximizing amount of product that can be produced per year, which in contrast with Lane's model, is not an economical objective, but is a functional goal.

The results of research revealed that in this case, optimum cut-off grade is equal to the balancing cut-off grade of mining and processing operations, which depends on mining and processing capacities, and tonnage- grade distribution of the ore in the pit.

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