Probabilistic Design - The Future of Rock Engineering

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Abstract

A brief background to the development of the rock engineering design process is given, showing that since the development of the science of mathematics, deterministic methods have been used to perform various calculations. The variability of rock properties and support characteristics have always been known. However, they were not explicitly used in design but compensated for by the use of a safety factor, i.e. making a design more stringent than required by the calculations. The problem with this procedure is that the effect of increasing a safety factor on the overall stability of the design cannot be known because the range of variability is not incorporated in the design. The only way to overcome this problem is to make use of the science of probability. In doing that, the ranges of rock qualities are explicitly included in the design and the probability of failure is exposed. Examples of common rock engineering calculations in mining are provided, showing that the probabilistic designs are not difficult or time consuming to perform and yield much more useful outcomes than merely using a safety factor.

Keywords: Rock Engineering, Deterministic Design, Reliability, Point Estimate method (PEM)

Introduction

Rock control has been practised ever since the first hole in the ground was made by man. Even in the very ancient times, it is for instance conceivable that loose fragments of rock would have been removed before they could cause injury.

The subterranean city of Derinkuyu in Turkey, excavated more than 3000 years ago, had arched roofs and shaped pillars. In the 2000 year old Roman limestone mines underneath Paris, roof control pillars built of stone still offer support today.

There is no evidence today that those designs were based on mathematics. That only came much later.

With the development of the science of mathematics and the wider teaching of mathematics, it became possible to develop quantified laws of nature and based on that, the extensions to predict the behaviour of rock.

It became possible to calculate the weight of blocks of rock and then to calculate the strength required for supports to withstand that weight.

Once computers became available and accessible, it became possible to develop and use highly sophisticated mathematical routines and complex mining or other excavation lay-outs. Yet, all the mathematical routines that are used are based on explicitly defined singular input conditions.

It is universally appreciated that rock is variable in nature, given to imperfections. The most common way to accommodate that, is to incorporate a safety factor in designs, in other words to multiply the outcome of a resistance calculation by some factor to compensate for unknown deviations from the average input numbers that were used.

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This is a reasonable and under most conditions a safe approach, but not an optimum one. In this way, we do somewhat account for variability, but in a manner that we cannot quantify: a safety factor of 2, for instance, means that the average support is twice as strong as the average load, but it certainly does not follow that the overall design is twice as likely to be stable. In those areas where the load is greater than the average, and the support resistance less than the average, we simply don't know how stable the design is.

With the deterministic approach, we can only say for certain what the situation is at the point where the average load coincides with the average of the support resistance, but without even knowing where that point is!

For too long, as geotechnical engineers, we have been reluctant to incorporate the science of probability in our designs. This may in part be due to a suspicious view of statistics caused by the irresponsible use thereof in advertising and other areas of society. It may also simply be due to the fact that it is not included in all rock engineering training programs and consequently, rock engineers are not always equipped with the necessary skills. Yet, if properly used, it is a vital tool in our arsenal. It is the only science which we can use to quantify uncertainty and to shed light on the probability of failure of our designs.

This paper will show examples of how the science of probability can be used in common designs on a more rational basis than with the deterministic design procedures based on average values with a compensating safety factor.

Background of Probabilistic Design

The notion of using probabilistic design procedures in rock engineering is not new. There are several examples of scientific procedures to use probabilistic concepts in rock engineering design in literature.

In South Africa, Stacey [1] presented a simple probabilistic approach to handle rockfalls in deep level gold mine workings while Esterhuizen and Streuders [2] presented a probabilistic procedure to evaluate the probability of instability arising due to keyblock failure, based on the distribution of discontinuities in the roof. Hoek [3] gives an example of using the probability of failure concept to evaluate the stability of a rock slope. Hantz et al [4] advocate the combined use of fundamental rock engineering concepts, history and probabilistic procedures to assess the rockfall hazard, using a potential rock slope failure in the Grenoble area as an illustrative example. Tono et al [5] discuss the application of statistics to handle imprecise data to evaluate the reliability of tunnel linings. Esterhuizen (2003)'s JBlock program [6] has been available for some time and is included in rock engineering curricula at certain universities. Harr [7] presents several statistical procedures used for civil engineering design – this work is unique in the sense that it explains statistics from a practical engineering perspective as opposed to a classical statistical perspective.

There are several more examples; the above are included to indicate that examples of several applications of probabilistic procedures in several branches of the broader rock engineering discipline already exist. Yet, after several decades, even though the application has been demonstrated to be superior to simple deterministic design in several areas, reliability based design is not in general use in the field of rock engineering. There could be several reasons for this.

The most important obstacle may well be that the application thereof will force the general public as well as engineers to come to terms with the fact that explicit probabilities of failure will be stated in designs. The probability of failure *per se* is not new. It has always been there, it has always been an integral part of any design, but numbers have not been calculated and brought into the open.

Stacey [8] points out that the wider adoption of the concept of the probability of failure will require the participation of management at executive level to state design objectives in those terms, but that this has not happened. Hoek [9] offers some hope by stating "…..there does appear to be a slow but steady trend in society to accept the concepts of risk analysis more readily than has been the case in the past".

The definition of acceptable risk levels will require serious debate. In the mining industry, the trend internationally is to aim for *zero harm*. It is doubtful whether "zero" in the public perception coincides with the true meaning of the word in scientific terms. It is perhaps more appropriate to equate *zero harm* to acceptable harm levels in the home environment – which is not true zero and in itself difficult to determine as those risks are not the same the world over.

Basis of Reliability based Design

Reliability based design in rock engineering is based on the fundamental acceptance that rock qualities are variable and so are the qualities of artificial support and even adherence to design dimensions. The design is then based not on average values as in deterministic design processes, but on the distributions of the input variables. The output is then also in the form of a distribution instead of a single number. The output distribution is then very simply used to determine the probability that failure will occur.

In order to perform a reliability based design, the variabilities of the various input parameters also have to be known. This does not necessarily require more work to be done for data collection. In most cases, the data have already been gathered in order to calculate average numbers for the deterministic input. It merely requires one or two additional calculations to be performed, which is no trouble at all with commonly used software like Microsoft Excel.

At the core of the process, the fundamental deterministic algorithms are still there. Essentially, the input ranges are merely used with the same algorithm as would be used for the deterministic calculation, to calculate a number of outputs and then to base decisions on the range of outputs. The single number resulting from a deterministic calculation is merely extended into a range, based on the range of input variables.

The nature of the output distribution is a function of the nature of the input variables. For instance, the wider the spread of the input variables, the wider the spread of the output. The wider the range of the outputs, the less certain the design and the higher the probability of failure. This is the fundamental improvement brought about by using a probabilistic approach: with the deterministic approach, the range of possible outcomes cannot be quantified and hence the uncertainty is not

known while with the probabilistic approach, it is highlighted.
These concepts will become clearer with the examples which follow.

Handy Techniques

There are a number of handy, relatively simple techniques that can be used to handle the probabilistic routines. Most of these can be performed at least at entry level with commonly available software like Microsoft Excel. There are also a number of tailors made statistical packages available, one of the more popular ones being @Risk, an add-on module for Excel.
There are several good handbooks that explain the procedures in detail, Harr [7] being particularly useful for engineering applications. In this paper, only some examples are presented as illustration of the ease with which the procedures can be performed and the usefulness of the approach.

The Point Estimate Method (PEM)

The PEM is useful in situations where the deterministic algorithms to be used are relatively simple and where the distributions of the input parameters are known. It can then be used to determine the distribution of the safety factor and the probability of failure.
In essence, a small number of safety factor calculations are performed by varying the input according to the distributions. The number of calculations to be performed are 2^n, where n is the number of variable inputs. The calculations are performed using the mean plus or minus one standard deviation for each variable. The following example will illustrate the process.

PEM Example

The probability of planar failure for the following situation is to be investigated, see Figure 1.
The area of the plane per running metre is 6.9 m^2. The plane has an inclination of 60^0 and the weight of the material that may fail is 275.26 kN per running metre.
The equation for the safety factor, assuming that the slope is dry and unsupported, is:

$$SF = \frac{Ac + W\cos(\alpha)\tan(\phi)}{W\sin(\alpha)} \tag{1}$$

where,
A = area of plane
W = weight of plane
c = cohesion
α = inclination of plane
ϕ = angle of friction of plane.

In this example, the cohesion and angle of friction are variables. The mean and standard deviation of the cohesion is 29 kPa and 5 kPa respectively and that of the angle of friction 32^0 and 4^0 respectively. As there are two independent variables, the number of calculations required is $2^2 = 4$.
For each calculation, the constants (in this case A, W and α) will not be changed, while the following will be used for c and ϕ:
c + standard deviation: 34 kPa
c – standard deviation: 24 kPa
ϕ + standard deviation: 36^0
ϕ – standard deviation: 32^0
The calculations are then tabulated as follows:

Table 1: Calculations for PEM example

SF(c,f)	$SF_{i,j}$	$SF_{i,j}$ /4	$(SF_{i,j})^2$	$(SF_{i,j})^2/4$
FS++	1.404	0.351	1.970	0.493
FS+-	1.291	0.323	1.667	0.417
FS-+	1.114	0.279	1.242	0.310
FS--	1.002	0.250	1.003	0.251
Σ		1.203		1.471

The mean safety factor is the mean of the 4 calculated safety factors, or 1.2. The standard deviation of the safety factors is found by:

$$\sigma_{SF} = \sqrt{\overline{SF^2} - \left(\overline{SF}\right)^2} = \sqrt{1.471^2 - 1.203^2 = 0.024} = 0.155 \quad (2)$$

With the deterministic calculation, the safety factor can be known but the distribution of safety factors brought about by the variability of the input, is not known.

The probability of failure is the probability that the safety factor is less than 1. This is easily found as a standard Excel function,

NORMDIST (x, mean, standard deviation)

= *NORMDIST* (1.0,1.203,0.155) = 0.095

With the deterministic procedure, the failure probability of 9.5% is not known, although it is real. It is merely hidden.

Overlapping Distributions of Capacity and Demand

Once it is recognised that both the load on any system (the demand) and the strength (capacity) are variable, and the means and standard deviations are known, it is possible to construct frequency distributions for them, such as in Figure 3 in the example to follow. In the area where the distributions overlap, the demand exceeds the capacity and failure will occur. The area of overlap relative to the total area, is then the probability that failure will occur. This can be quantified.

The area of overlap, or the probability of failure, is

$$A = p_f = .5 - \psi(\beta) \quad (3)$$

$$\beta = \frac{\overline{C} - \overline{D}}{\sqrt{S_c^2 + S_d^2}} \quad (4)$$

For $\beta > 2.2$,

$$\psi(\beta) = .5 - \frac{1}{\beta}(2\pi)^{-0,5} \exp\left(-\frac{\beta^2}{2}\right) \quad (5)$$

and for $\beta \leq 2.2$ statistical tables are used. The mathematical description of the tables, albeit less academically pure, can also be used instead of the tables with better accuracy:

$$\psi(\beta) = .0006\beta^6 - .0096\beta^5 + .0563\beta^4 - .1379\beta^3 + .0423\beta^2 + .3893\beta + .0004 \quad (6)$$

Overlapping Distributions Example

Consider the case where mine pillars are used as the primary support. If the mean strength of the pillars is 12 MPa and the standard deviation, caused by variability of pillar height and width, is 3 MPa while the mean pillar load is 8 MPa and the standard deviation 2 MPa, then the frequency distributions will be as shown in Figure 2.

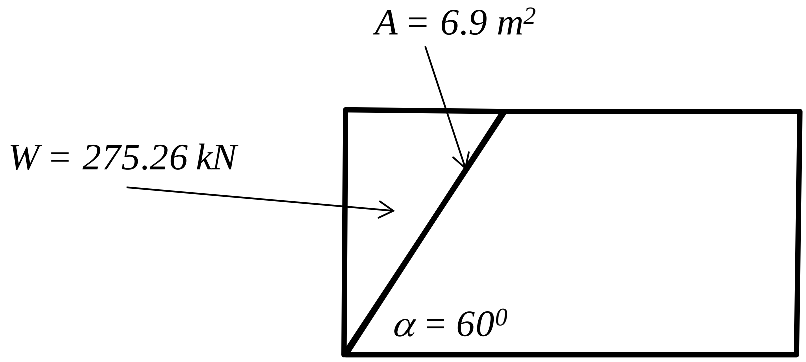

Figure 1: Cross section through a plane that has the potential to fail.

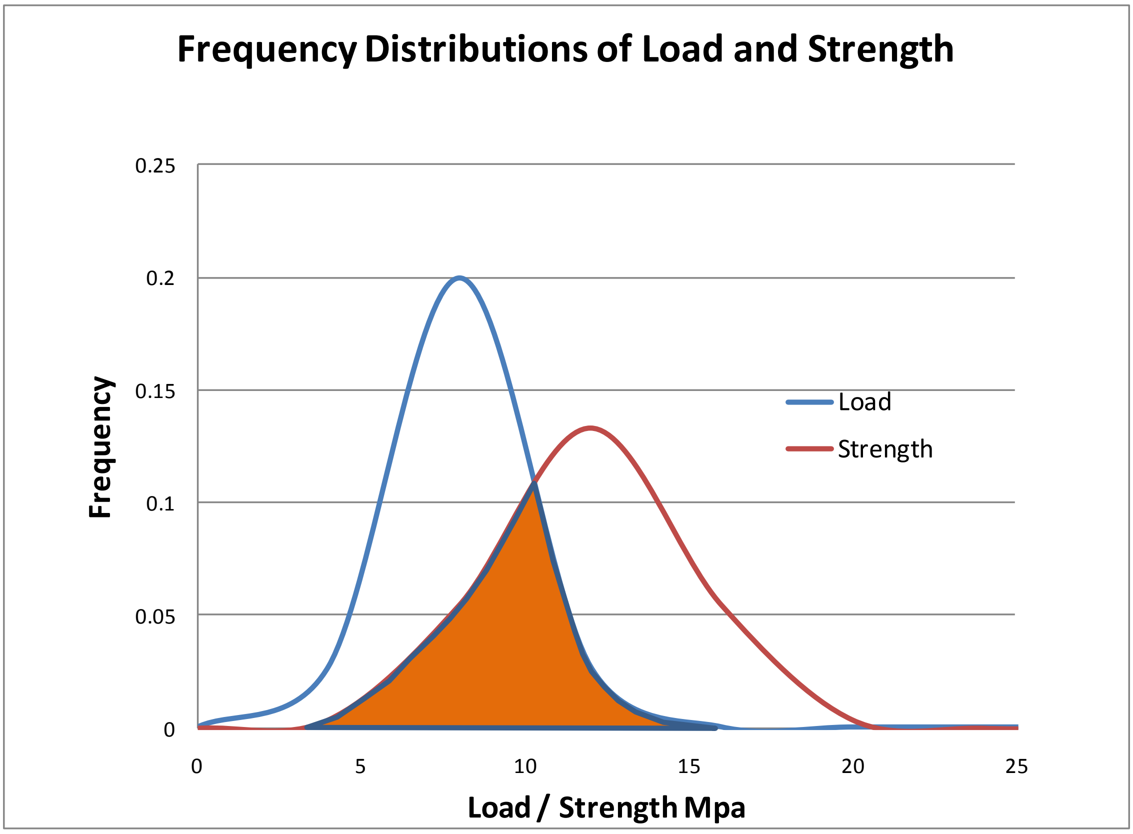

Figure 2: Frequency distributions for pillar load and strength.

According to Equation 4, β = 1.11 and then Equation 7 yields a value for $\psi(\beta)$ of 0.37. The area of overlap is 0.13 according to Equation 3 and the probability of failure is then 13%. Note that the deterministic safety factor, the ratio of strength to load, using just the average values, is 1.5.

Monte Carlo Technique

The Monte Carlo technique had its origins with the development of the nuclear bomb in the United States during the second world war, where scientists had to consider the effect of a large number of variables on the probability of a chain reaction occurring.

It is fundamentally simple. Assuming that the distribution characteristics of each of the input parameters is known, random input numbers are chosen from each distribution and for each set of input numbers, an output using a known algorithm is calculated, see Figure 3. This process is repeated several hundreds of thousand times. The outputs are then collated in a distribution and the final outcome is the distribution of outputs with a known mean and standard deviation. For the purpose of determining a failure probability, this simply means that the cumulative frequency of safety factors less than 1.0 need to be found.

This technique would be close to impossible to perform without the use of modern computers, but with computers it is relatively simple. Custom made software, like the popular Excel add-on @ Risk, are available. Even without those, it is possible to perform the procedure in Excel with some limitations.

The most important limitations when using Excel are that the algorithm has to be written such that each variable only occurs once in the equation and that the standard deviation of the output cannot be calculated due to the limitation of that procedure in Excel. However, the mean, most frequent value and importantly, the probability of failure, can be calculated.

In Excel, the process entails defining the characteristics of the input distributions (these need not be normal distributions) and then the basic algorithm to calculate for instance the safety factor has to be provided. Then, the safety factor is calculated any number of times using the random function in conjunction with the input distribution characteristics and stored in a matrix. The frequency distribution of the output can then be plotted, basic parameters like the mean and most frequent value determined and the probability of a safety factor being less than 1.0 (in other words the probability of failure) can be determined.

The author uses this process for a number of applications and even with more than 2 million output calculations, the execution time is minutes and certainly not hours on a commonly used computer without special additions.

Monte Carlo Example

Where a mine roof is supported using the suspension principle, i.e. weak material in the immediate roof is simply suspended from a stronger layer higher up by means of roof bolts, see Figure 4, the safety factor of the support system can be expressed by:

$$SF = \frac{n\pi d_h \left(\frac{l_b}{t_w} - 1 \right) \tau}{\rho g B s} \qquad (7)$$

where,

n = number of bolts per row

d_h = diameter of hole

l_b = length of hole

t_w = thickness of weak layer

τ = shear strength of resin/rock contact plane

B = road width

s = spacing between rows of roofbolts

Note that Equation 7 is based on the pessimistic assumption that the sidewalls of the roadways offer no support to the roof, to cater for the situation where joints occur at the edges of the roadway.

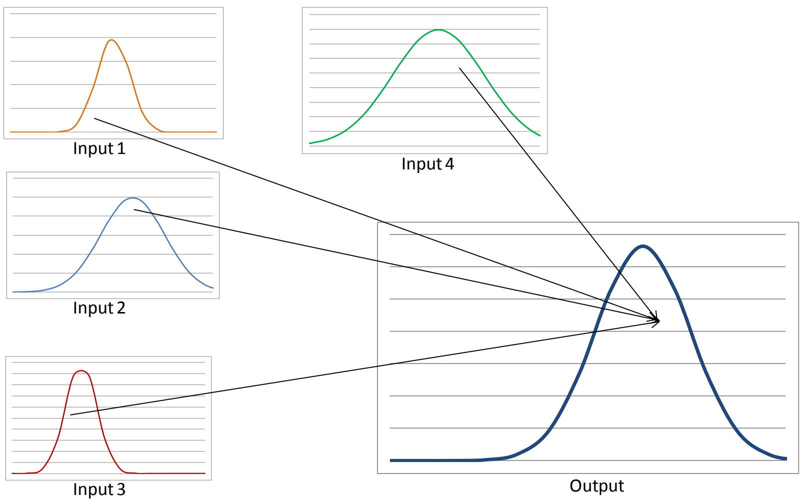

Figure 3: Schematic explanation of the Monte Carlo technique

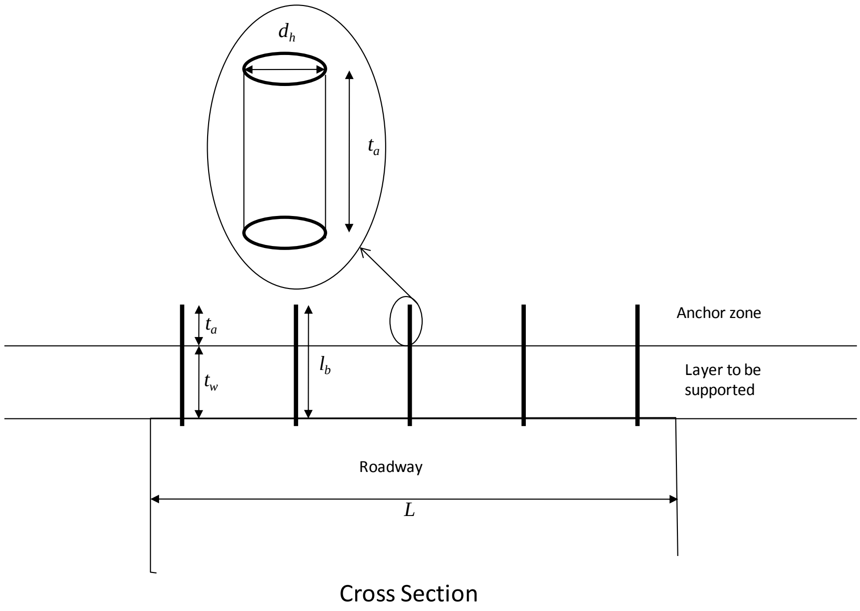

Figure 4: Simplified cross section through a roadway showing the installed support and explaining the symbols used in Equation 8.

In Equation 7, all the parameters except the number of bolts per row, *n*, can be regarded as variable. Table 2 contains reasonable numbers for the distributions of the input.

Table 2: Distribution characteristics of input parameters for roof support design

Parameter	Mean value	Standard Deviation
Hole length	1.55 m	0.1 m
Thickness of weak layer	1.0 m	0.2 m
Hole diameter	0.028 m	0.001 m
Resin/rock shear strength	2 100 kPa	450 kPa
Row spacing	1.7 m	0.3 m
Road width	6.6 m	1.1 m

The easiest way to handle a number of variables in a single equation is to perform a Monte Carlo simulation. This was done for the distributions in Table 2 as input into Equation 7 with 4 bolts per row. The distribution of safety factors resulting from this is shown in Figure 5. The probability of failure was found to be 29%, notwithstanding the fact that the deterministic safety factor was 1.5, which is regarded as acceptable for roof support design.

The results are only valid for the situation where joints occur at the edges of the roadways, which is not the general case, rather a worst case scenario. If the sidewalls are indeed regarded as support, in other words in the situation where joints do not occur at the edges, a simple way of performing the calculation is to artificially increase the number of bolts per row by 2, in other words add one roofbolt in the place of each sidewall.

This more realistic general view results in an increased safety factor for the situation, to a value of 2.2. The distribution of safety factors is also more favourable, see Figure 6. However, the probability of failure is still significant at 14%.

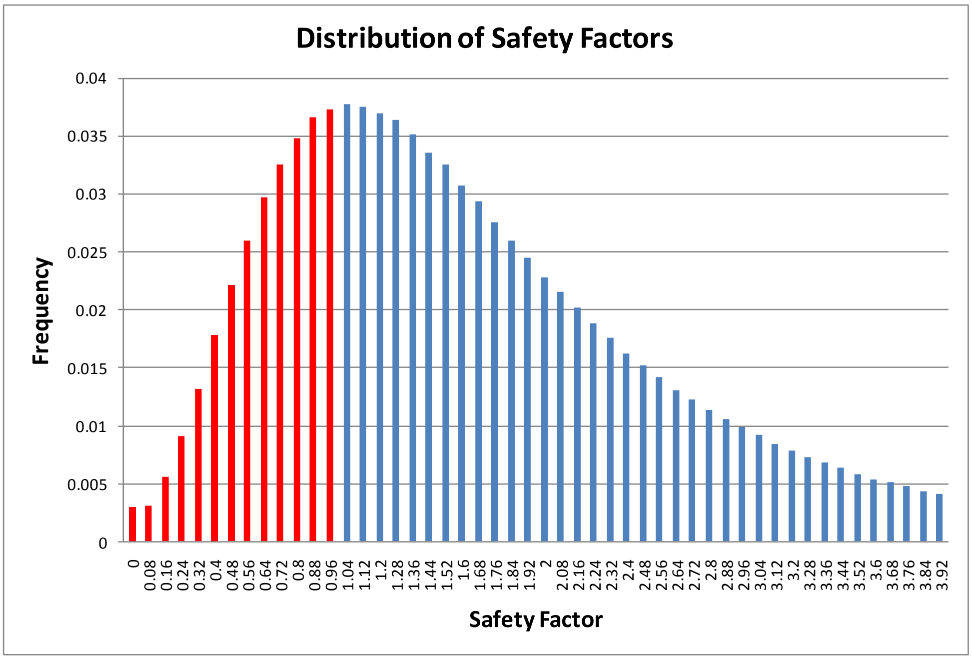

Figure 5: Distribution of roof support safety factors. The safety factors less than 1.0, where failre can be expected, are shown in red.

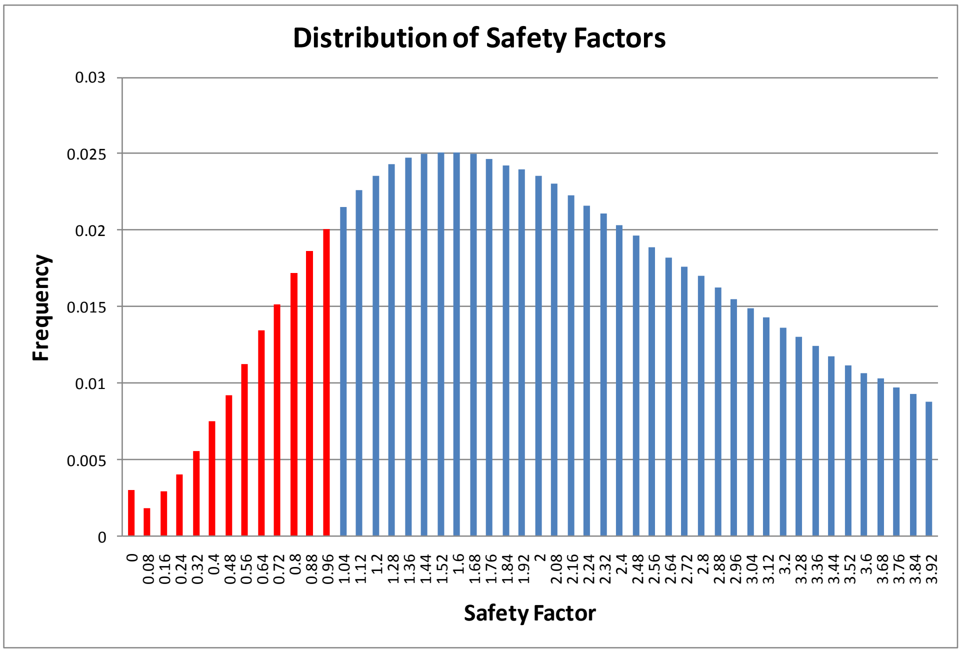

Figure 6: Distribution of roof support safety factors for the situation where the sidewalls are also regarded as supports. The safety factors less than 1.0, where failure can be expected, are shown in red.

These examples indicate that even with an acceptable deterministic safety factor of 1.5, significant failure can still be expected using reasonable values for the variability of the input parameters. This will not be known if the probabilistic procedure is not used, although the failure probability will exist in the background. It will be there, but it will not be known. The only way to expose it, is to perform the probabilistic procedure described here.

Conclusions

The most popular method to cater for the variability in rock engineering design has been to include a factor of safety. With the factor of safety, a design is simply made a number of times as strong as a calculation based on average values would require.

However, this does not really compensate for variability, mainly because the range of input is not used to determine a suitable factor of safety. Therefore the effect of variability is not quantified, and importantly, it is not known whether or not the output is increased sufficiently to include most of the impact of the input variability.

The methods to incorporate variability as an integral part of calculations are not overly complex or time consuming. The statistical methods are proven, well documented and easy to use.

In the examples provided in this paper, it was shown that day-to-day calculations can easily be performed using the probabilistic methods. The output is substantially improved and much more real information becomes available to make rational decision making much more defensible.

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