Flexural Toppling Failure in Rock Slopes: From Theory to Applications

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Abstract

Toppling failure is one of the most common modes of failure of rock slopes in layered rock strata. Flexural toppling is one of the well-known modes of the failure. This type of failure occurs due to bending stress. In this article, a brief yet comprehensive review of toppling failure is presented. Firstly, the conditions and general mechanism of the failure are described. Then, experimental, theoretical and numerical modeling of the failure is summarized. Next, several case histories are analyzed and the results with the existing theoretical models were compared and presented. Finally, some practical recommendations as how to use these models are made.

Keywords: Rock Slope, Toppling Failure, Monograms

Introduction

Toppling failure is one of the most common hazardous instability of rock slopes that engineers and researchers are facing with. Flexural toppling failure is one of the specific modes of toppling failure which occurs due to bending stresses. In order to properly describe the mechanism of such a failure, it is presumed that rock mass contains a set of parallel discontinuities dipping steeply against the excavated face. As such, the rock mass behaves like a series of superimposed inclined cantilever rock columns with a potential of flexural topping failure. The body force of each rock column can be analyzed into two components; normal to and parallel with the rock column longitudinal axis. The normal component causes the rock column to bend and to transfer the load to the underlying stratum. The bending moment, due to the self weight, produces tensile and compressive stresses in every cross-sectional area of each rock column. Once the tensile stress exceeds the rock column tensile strength, failure of rock mass will be initiated. Hence, to compute the factor of safety of the rock slope against flexural toppling failure, the magnitude of maximum tensile stress in rock columns must be determined. Theoretically, the rock columns are “statically indeterminate problems” and the maximum tensile stress cannot be determined only by equations of equilibrium. Therefore, in order to properly compute the factor of safety of rock slope against flexural toppling failure, boundary conditions and/or principle of compatibility relationship must be satisfied. In case of dynamic loading, some temporary loads will be added to the rock columns. These loads usually are produced by earthquake; however, blasting, huge construction equipments, wind etc. also may produce dynamic loads. In flexural toppling failure, the dynamic loads must be considered as additional to the existing tensile stress in rock columns which may accelerate the failure process of the rock mass.

In this paper, some case histories of the failure are presented and analyzed by existing theoretical methods and the results are compared.

A Review of Flexural Toppling Failure

In the field of rock mechanics and rock engineering, Müller in 1968, was first who mentioned overturning of rock columns and/or rock blocks [1]. He

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suggested that block rotation or toppling may have been a contributory factor in the failure of the north face of the Vaiont slide. Müller further encouraged Hofmann to carry out a number of model studies to investigate block rotation [2]. In 1971, Ashby analyzed overturning of rock blocks systematically and suggested some criteria for its instabilities. He was the first to suggest the name of “toppling” failure for rock slopes [3]. However, Ashby’s studies only related to blocky and columnar toppling. The first classification of toppling was offered by Goodman and Bray in 1976 [4]. But, before that, some case studies and physical modeling about the failure can be found in literature review. In actual case studies of toppling failures of rock mass, De Freitas and Watters in 1973, Bukovansky et al. in 1974 and Heslop in 1974 presented some new findings [5-7]. In laboratory research works, in 1970 Erguvanli and Goodman constructed a physical model to perform toppling failure tests by means of base friction apparatus [8]. The results of this study were published later by Goodman in 1972 and Goodman and Erguvanli in 1989 [9]. Also, in 1974, Hafman presented some data about experimental modeling of this type of failure [2]. However, in literature review before 1976, no method was available to analyze the flexural toppling failure. Goodman and Bray in 1976 divided toppling failure into main (flexural, block, block-flexure) and secondary types [4]. The classification has become widely accepted by other researchers. In their paper, they suggested the following relation as a necessary criterion to assess potential of main toppling failures and a step by step approach to analyze blocky or columnar toppling instability.

$$(90-\delta)+\phi \prec \alpha \qquad (1)$$

where,
α : Inclination of face slope
ϕ : Friction angle between the layers
δ : Inclination of the layers

The method of Goodman and Bray was modified by Cruden [10]. From 1976 till now, on the basis of this classification, many physical modeling, numerical approaches, computers programming, design charts and case histories of toppling failure have been published. In physical modeling, toppling failure has been modeled by Whyte in 1973, Soto in 1974, Kawamoto et al. in 1983, Aydan and Kawamoto in 1987 and 1992, Stewart in 2005, Adhikary et al. in 1997 and 2007, Majdi and Amini in 2008, Amini et al. in 2008 [11-19]. In numerical approaches, toppling failure has been modeled by Cundall in 1971, Byrne in 1974, Hammett in 1974, Ishida et al. in 1987, Nichol in 1988, Pritchard and Savigny in 1990, Adhikary et al. in 1996, 1997, 1998, 2002, Benko in 1997, Kimber et al. in 1998, Mitani et al. in 2004, Riahi in 2008, Brideau and Stead in 2009 [20-34]. For analysis of toppling failure with analytic solution "step by step" method of Goodman and Bray is famous and have been programmed by several authors [35-36]. Step by step method of Goodman and Bray for block toppling has been converted to design charts several times [36-38]. Also, Aydan in 1989 and Adhikary et al. in 1997 presented some nomograms for analysis of flexural toppling failure on the basis of Aydan and Kawamoto step by step method [14, 16]. These nomograms are so simple and applicable for engineers. In the field of case histories of toppling failure, the researches of Wyllie in 1980, Shimizu et al. in 1993, Tu et al. in 2007 are well-known [39-41]. In the field of flexural toppling failure, Aydan and Kawamoto was first to present a theoretical method to analyze slopes and underground openings prone to flexural toppling failures, on the basis of limiting equilibrium method utilizing the bending theory of cantilever beam in 1992 with the consideration of gravity, earthquake and water pressure [14]. On the basis of the method, the intercolumn forces between rock columns with potential of flexural toppling failure can be computed as follows:

$$P_{i-1} = \frac{P_{i+1}\left(\eta h_i - \mu \cdot \frac{t_i}{2}\right) + W_i \cdot \cos\delta \cdot \frac{(h_i + h_{i-1})}{4} - \frac{2I_i}{t_i} \cdot \left(\frac{\sigma_t}{F_S} + \frac{W_i \cdot \sin\delta}{t_i}\right)}{\eta h_{i-1} + \mu \cdot \frac{t_i}{2}} \tag{2}$$

where,

W_i : Weight of rock column i

t_i Thickness of rock column i

h_i, h_{i-1} :Lengths of rock column i

P_i, P_{i-1} :Inter-column normal forces acting on the column i

x_i, x_{i-1} : Application points of P_i, P_{i-1}

T_{i+1}, T_{i-1} : Inter-column shear forces acting on the rock bolt

η : Dimensionless ratio x_i / h_i

E_i :Dynamic load acting on rock column i

μ : Frictional coefficient between the layers

β :Inclination of dynamic load

F_s : Factor of safety

I_i : Moment of inertia of rock column i

So, the factor of safety of a slope can be computed by equation 1 with step by step and trial and errors method. This method were verified by experimental modeling (base friction and centrifuge) outcomes several times [14, 16]. In 2009, Amini on the basis of equation of equilibrium and compatibility law presented a new method to analyze and compute the safety factor of flexural toppling failure [19]. They proved that rock mass with potential of flexural toppling failure can be modeled with a single beam-column with length Ψ. The parameter Ψ can be computed as follows.

$$\Psi = \left[\frac{b - \left(b^2 - 4ac\right)^{0.5}}{2a}\right] H = CH$$

$$a = \frac{\tan(\delta - \varphi) \cdot \cos^2\varphi}{\tan(\delta - \varphi) + \tan(\theta - \delta + \varphi)}$$

$$b = \frac{2\cos(\theta - \delta + \varphi) \cdot \cos\varphi}{\sin\theta}, \quad c = \left[\frac{\cos(\theta - \delta + \varphi)}{\sin\theta}\right]^2 \tag{3}$$

Where,

H : Slope height

Ψ : Calculated Lengths of rock columns used for computation of inter-column's resultant force

φ : Angle between total failure plane and the line of normal to discontinuities

θ : Angle between face slope with respect to the horizontal.

So, factor of safety of rock slopes against flexural toppling failure can be determined as follows:

$$F_s = \frac{t \cdot \sigma_t}{3C^2 \cdot H^2 \cdot \gamma \cdot Cos\delta} \tag{4}$$

where;

γ : Unit weight of intact rock samples

t : Average thickness of rock columns

It is incapable of taking into account the interlayer slip between layers. So it may present an underestimated factor of safety for toe of slope. The results of this method are compared with physical modeling (base friction and centrifuge modeling). The comparisons showed that the experimental and theoretical outcomes are close.

In the field of dynamic toppling Amini et al. in 2009 carried out some dynamic physical modeling of active and passive toppling failure [42]. They used the pseudo-static analytical methods (Aydan and Kawamoto and Amini et al. methods) for assessment of toppling failure in dynamic condition.

Case Histories

As it was mentioned, earlier in this paper, some physical, numerical and theoretical modeling's along with some case studies have been presented to analyze flexural toppling failure. But, there is not any unique scheme for analysis of the failure. Some researchers suggest that the kinematic method should be used to assess the failure [4, 39]. Some of them prefer limiting equilibrium methods [14,16,19]. Most of these methods were verified by physical modeling's results but their validities for analysis of real flexural toppling failure are still questionable for most engineers. Since pure flexural toppling failure is a rarity, no comparative studies can be found to confirm the results of these methods in comparison with the real ones. In this section some typical and well-documented flexural toppling failures gathered from various sites and from literature review as well are analyzed by the existing methods. Then the results are compared with the real ones.

First Series of Case Studies
(Gathered from Field Observations)

There are two well known mountain ranges, namely; Zagros in south west and Alborz in north of Iran. Several main vehicle and railway routes and tunnels pass through layered strata of both mountains. Hence, both flanks of each mountain range are prone to flexural and blocky-flexural toppling failure. After a vast site visiting, 5 locations with typical flexural toppling failure potential were selected for this research purposes. The information related to rock slope instability of these sites were gathered from large accessible documents through ministry of road and transportation. The documents well illustrated that there were several slope failures in these sites where proper assessing and stabilizing of them were always questionable. Also, the route accident documents clearly reflect several type of rock fall hazards among which flexural and blocky-flexural toppling failure of the rocks are very common. The geometry of these slopes and discontinuities of the rock masses were gathered during some site investigations and were analyzed as presented in figs. 1 and 2. The geomechanical properties of the intact rocks and joints were determined in Chase-Mandro and Tehran University rock mechanics laboratories. The results are presented in table 1. As it can be seen from figs. 1 and 2, all the prospected cases are in limiting equilibrium conditions; so that the in-situ factor of safety in all cases are equal to 1. Therefore, the validity of limiting equilibrium methods outcomes can be checked by these case studies.

Table 1: Geomechanical and geometrical characteristics of case studies (series 1)

Parameters / Case Studies		Location (country) -----	Slope angle $(^0)$	Dip of layers $(^0)$	Height of slope (m)	Friction angle of layers $(^0)$	Layer thickness (m)	Angel between failure plane and normal to layers $(^0)$	Density of rock sample (KN/m^3)	Tensile strength of rock columns (MPa)
1	Galandrood	Iran	80	40	16.5	26	0.3	0	27	5-10
2	Chalous	Iran	70	44	25.5	30	0.46	5	26.2	2-4
3	Angooran	Iran	80	75	30.5	30	1.75	0	26	4-7
4	Shahrekord	Iran	90	80	5.5	27	0.15	0	26	2.6-6
5	Firuzkuh	Iran	70	55	8.5	27	0.35	0	26	2-5
6	Kandovan	Iran	85	50	7.7	27	0.3	0	26	2-5

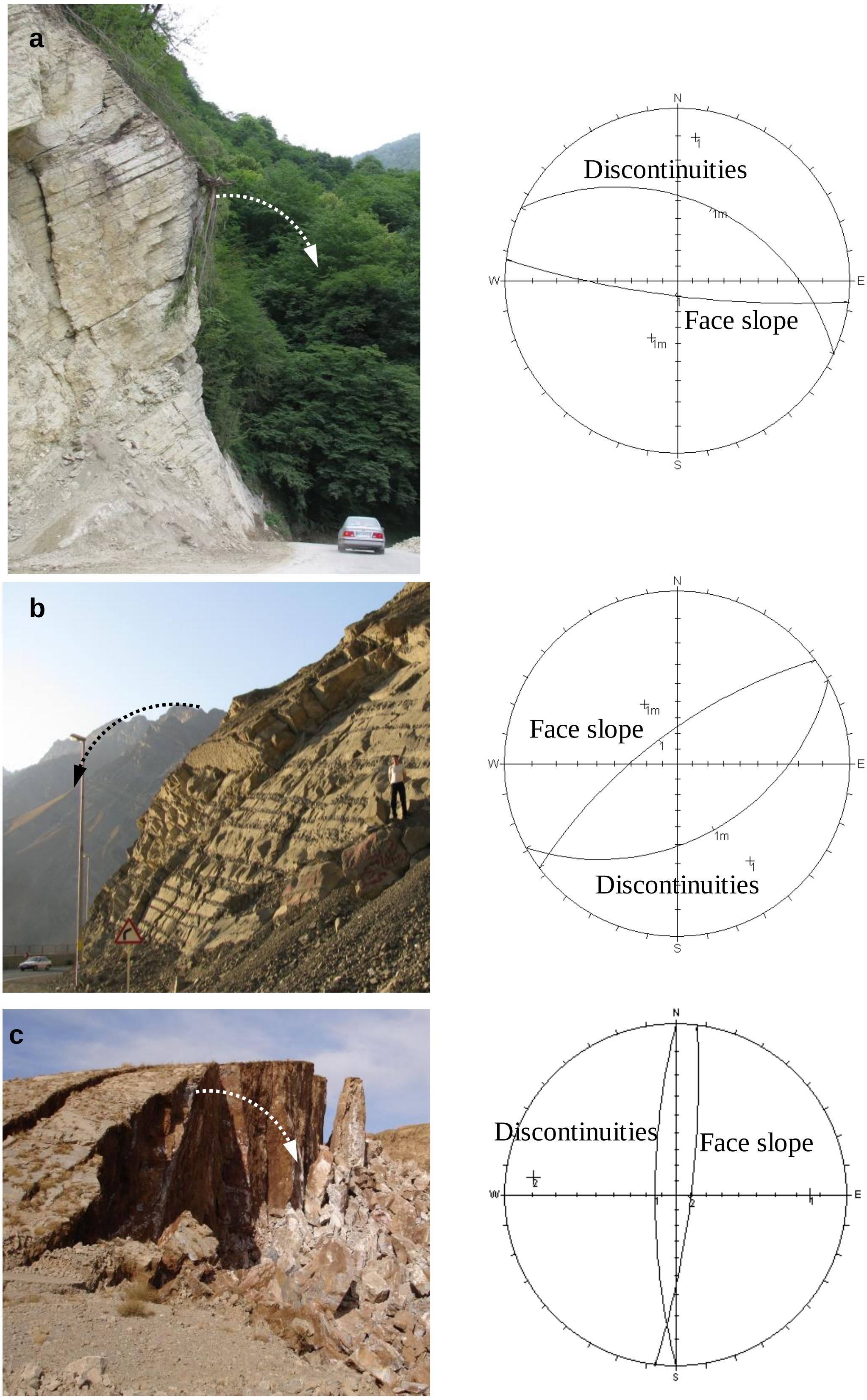

Figure 1: Photos and streonet diagrams of the case studies a)Galandrood b)Chalous C)Angooran

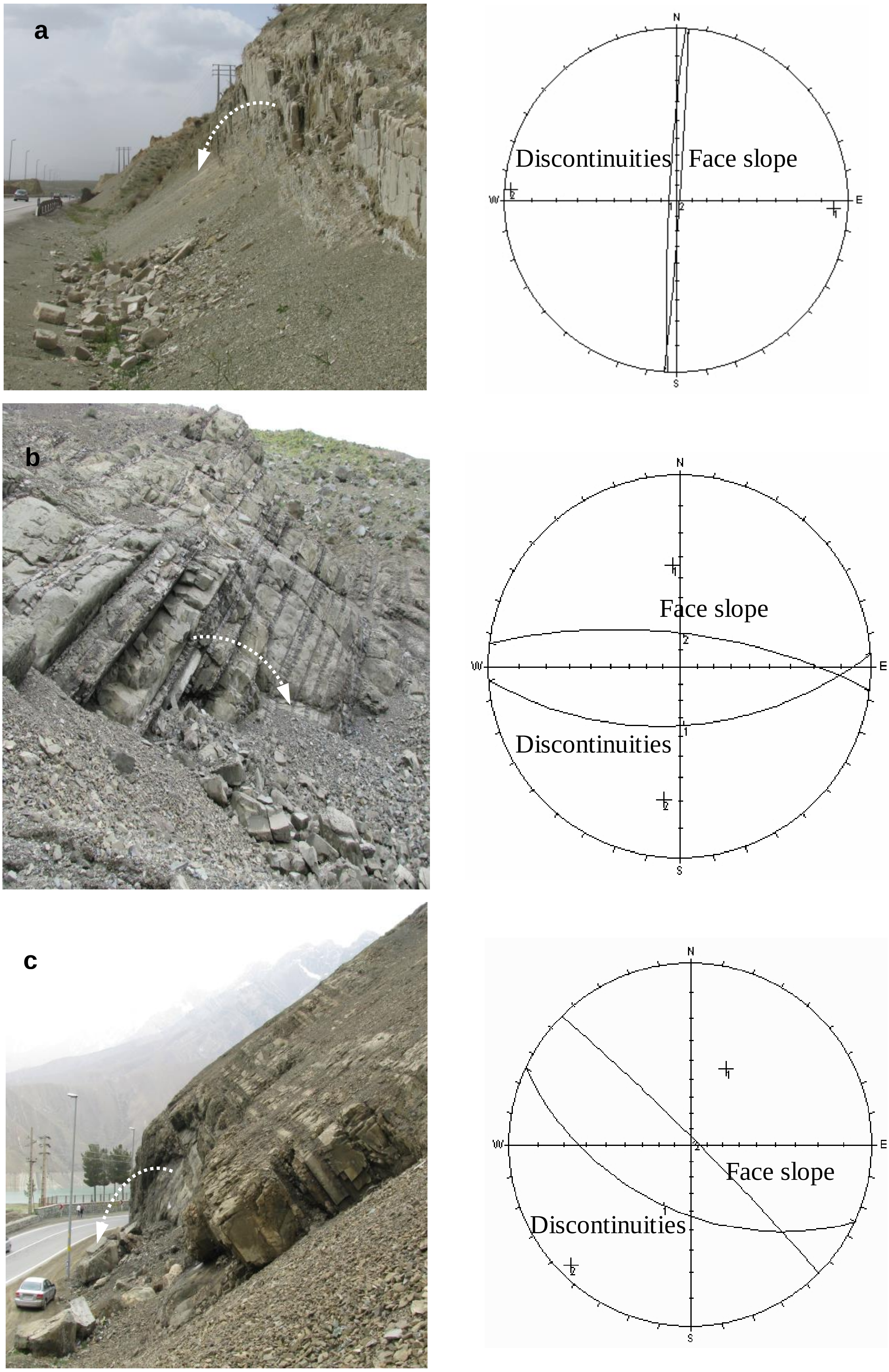

Figure 2: Photos and streonet diagrams of the case studies a)Shahrekord b)Firuzkuh c)Kandovan

Table 2: Geomechanical and geometrical characteristics of case studies (series 2)

Case studies \ Parameters		Location (country)	Face slope angle	Dip of layers	Height of slope	Friction angle of layers	Layer thickness	Density of rock sample	Tensile strength of rock columns
		-----	$(^0)$	$(^0)$	(m)	$(^0)$	(m)	(KN/m^3)	(MPa)
1	Eibar	Spain	135	56	35	30	0.1	25	5-10
2	Azumi	Japan	105	40	40	30	1	25	2-5
3	Kitamatado	Japan	115	45	25	30	1	25	2-5
4	Siwalik	Nepal	135	60	95	30	1.5	20	5-10
5	Fengtan	China	117	37	135	30	1	27	3-5
6*	Savage 1	Australian	135	70	107	30	1	25	5
7*	Savage 2	Australian	143	70	107	30	1	25	5

*case studies 6 and 7 are before and after stabilization, respectively.

Second Series of Case Studies (Gathered from Literature Review)

The first series of the case studies, gathered by the authors, who obtained several rock samples and tested in 2 different laboratories to decrease the laboratorial personnel's errors. However, all the aforementioned case studies were obtained from one country (Iran). Hence, the method and the corresponding results may not be universally applicable. Therefore, in order to properly evaluate the outcomes of the existing methods, the data of most accessible well documented case histories of flexural toppling failure were gathered from literature review as well [6,40,41,43,44]. The specifications of these examples are presented in table 2. Since the locations of these case studies are from different geological formation, hence, the assessment results are more reliable.

Analysis of the Case Histories by the Existing Methods

To perform a comprehensive comparative analysis of flexural toppling failure the following analytical approaches have been examined:

Goodman-Bray Method

As it was mentioned earlier in this paper, Goodman and Bray in 1976 suggested a criterion as necessary condition to analyze active flexural toppling failure on the basis of kinematic rules. To assess the results of this approach, the active case histories were analyzed by this method. Since all of these case histories failed due to pure flexural toppling, hence the criterion suggested by Goodman and Bray should be satisfied. The results of this evaluation are presented in fig. 3. As it can be seen from this figure, there are not good correlations between theoretical prediction and the results of the given case studies. Therefore, most failures are located in theoretical stable area. Thus, this criterion can only be used as a primary tool to assess the potential of flexural toppling failure. It is important to mention that this approach, that is, Eq. (1) can only be used as the necessary criterion but not sufficient one to evaluate the flexural toppling failure. In other words, if the Eq. (1) is not satisfied; the slope is stable against static flexural toppling failure. Because the shear resistance between the two adjacent layers is more than the shear flow force due to bending, so the layers cannot slip on the common boundary of each other. On the contrary, if the relation is satisfied, the slope has the potential of flexural toppling failure. As a result, one cannot easily appreciate the governing condition to see whether the slope is stable or it fails. Hence, the method does not provide a decisive factor of safety to be representative for the rock mass condition.

Aydan-Kawamoto Method

As was mentioned, safety factor of a rock slope against flexural toppling failure can be determined by Aydan and Kawamoto method with step-by-step and trial and error methods. However, using this method needs lengthy process which is time consuming manually and it requires a special computer programming tool. Hence, the aforementioned researchers provided a FORTRAN code to facilitate the process. The results of this method for the case histories presented in this research are illustrated in tables 3,4. As it can be seen from this table, though the method slightly overestimates as compared with the real situation, however, it represents the condition satisfactorily.

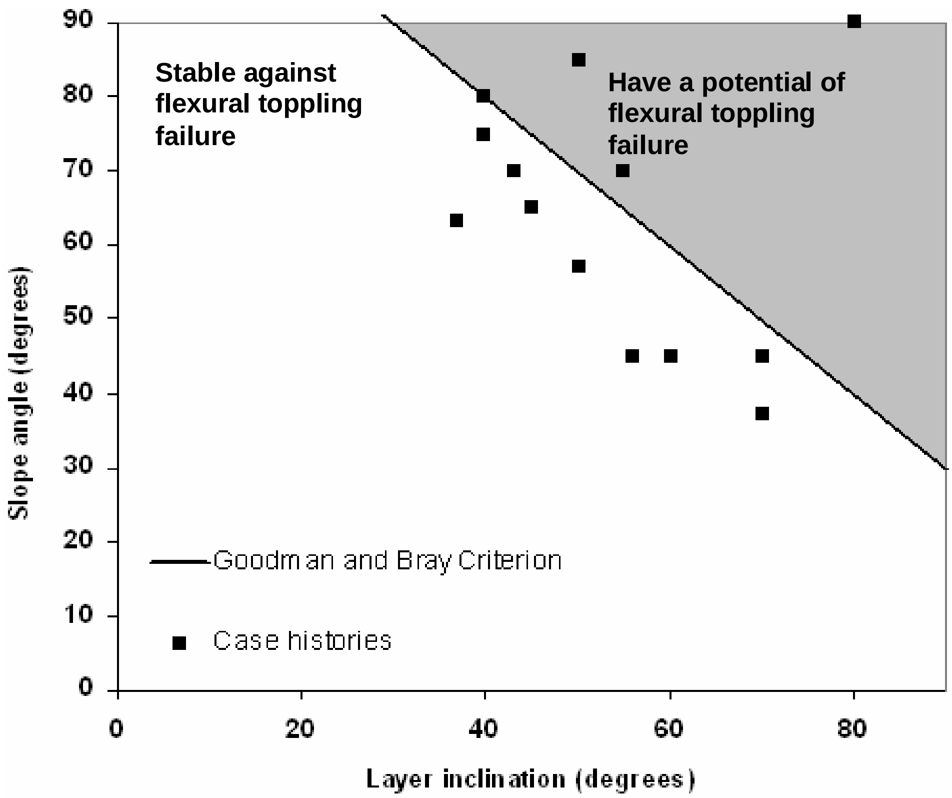

Figure 3: Assessment of the case histories of flexural toppling failure on the basis of Goodman and Bray method

Number	1	2	3	4	5	6
Case studies	Galandrood	Chalous	Angooran	Shahrekord	Firuzkuh	Kandovan
Actual height of slope(m)	16.5	25.5	30.5	5.5	8.5	7.7
Prediction height of slope(m)	24-29	29-35	32-37	7-12	10-15	9.5-14

Table 4: Safety factors of case histories on the basis of Aydan and Kawamoto method (series 2)

Number	1	2	3	4	5	6
Case studies	Eibar	Azumi	Kitamatado	Siwalik	Fetangan	Savage 1
Actual height of slope(m)	25	35	40	135	95	107
Prediction height of slope(m)	26-32	37-43	49-58	137-210	97-117	115-128

Table 5: Safety factors of case histories on the basis of Amini et al. method (series 1)

Number	1	2	3	4	5	6
Case studies	Galandrood	Chalous	Angoran-zanjan	Shahrekord	Firuzkuh	Kandovan
Safety factors	1.2-2.4	0.82-1.65	1.1-1.92	1.6-3.7	2-5	1.26-3.16

Table 6: Safety factors of case histories on the basis of Amini et al. method (series 2)

Number	1	2	3	4	5	6	7
Case studies	Eibar	Azumi	Kitamatado	Fetangan	Savage1	Savag2	Siwalik
Safety factors	0.49-0.98	0.44-1.09	1.89-4.72	1.2-2	0.47	1.89	0.7-1.6

Majdi-Amini; Amini et al. Methods

On the basis of this method, the safety factor of rock mass against flexural toppling failure can be computed directly. The results of this method were verified by many physical models such as base friction and centrifuge methods and tilting tests [19]. By this method several case histories have been analyzed and the results were compared with real conditions and presented in tables 5,6. As it can be seen from the tables, good correlations exist between the suggested theoretical method and the experimental results.

Concluding Remarks

A brief yet comprehensive overview of the flexural toppling failure from theory to applications has been made. The results of comparative studies on the basis of a vast range of case histories obtained from different geographical part of Iran have been presented. From this appraisal, the following concluding remarks are drawn:

1. Goodman and Bray‘s method, Eq. (1), is an essential criterion and may fulfill the necessary condition for flexural toppling failure yet not sufficient one. However, this criterion could be used for primary assessment of flexural toppling failure.
2. Theoretically, there is good agreement between Aydan-Kawamoto’s method and actual mechanism of flexural toppling failure. In this method, the slips between the layers are taken into account. However, based on this method, the safety factor of rock mass against flexural

toppling failure can only be computed on the basis of step by step in trial and error scheme thus it poses quiet lengthy process which is time consuming approach. Also, the exact value of parameter η is not clear.

3. The method proposed by Majdi and Amini and extended further by Amini and Majdi and Aydan are easy to use in practice and the exact safety factor of rock mass against flexural toppling can also directly be computed. However, the results may lead to a conservative design.

4. It is suggested that the active and passive flexural toppling failure to be analyzed so that the total failure plane be assumed 10 and 30 degrees above normal to the discontinuities, respectively.

5. Numerical methods such as; DEM, DFEM, FEM etc. are so powerful tools to analyze the deformation of layers with potential of flexural toppling failure; however, it is too difficult to model the ruptures of layers under tensile strength by these methods.

6. Decreasing of the face slope inclination is one way to increase the safety factor of rock slopes against flexural toppling failures. If the face slope is 10-15 degrees above normal to the discontinuities, the slope will be stable against flexural toppling failure.

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