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International Journal of Mining and Geo-Engineering

IJMGE

IJMGE 59-4 (2025) 367-375

DOI: 10.22059/IJMGE.2025.393814.595242

Geomechanical characterization of an anisotropic fractured rock mass using fuzzy system and Monte Carlo simulation

Majid Noorian-Bidgoli [a, *]

[a] *Department of Mining Engineering, Faculty of Engineering, University of Kashan, Kashan, Iran.*

Article History:

Received: 20 April 2025.
Revised: 24 September 2025.
Accepted: 01 November 2025.

A B S T R A C T

Characterizing fractured rock masses is a major challenge in geotechnical and mining engineering because of uncertainties in both intact rock properties and fracture behavior. These uncertainties reduce confidence in the estimation of geomechanical parameters, which are critical for reliable design. Deterministic approaches often rely on mean values and therefore underestimate or overestimate variability, leading to unsafe or overly conservative outcomes. This study proposes an integrated framework that combines fuzzy logic with Monte Carlo simulation to quantify uncertainty in fractured rock mass properties. Seven key parameters—Poisson's ratio, Young's modulus, uniaxial compressive strength (UCS), cohesion, friction angle, and the Hoek–Brown constants m and s—were modeled as fuzzy variables using triangular membership functions. Degrees of membership (DoM) from 0.2 to 0.8 were applied to define parameter ranges, and Monte Carlo simulation with 25000 iterations was conducted for each DoM level to derive 95% confidence intervals. Results show that confidence intervals consistently narrow as DoM increases, indicating reduced uncertainty. Two quantitative indices—confidence interval width (CIW) and relative uncertainty (RU)—were introduced, confirming that higher DoM levels correspond to lower uncertainty. A sensitivity analysis further demonstrated that while normal distributions yield narrower intervals due to small standard deviations, log-normal and uniform distributions capture broader variability. Additional tests with intermediate DoM values confirmed the stability and robustness of the framework. Overall, the fuzzy–Monte Carlo approach offers a systematic and practical tool for uncertainty quantification in rock engineering. By incorporating variability into parameter estimation, the method enhances the reliability of numerical modeling and provides stronger support for reliability-based geotechnical design.

Keywords: *Uncertainty; Geomechanical characterization; Fractured rock mass; fuzzy; Monte Carlo simulation.*

1. Introduction

A fractured rock mass is a complex system composed of intact rock material and discontinuities [1], both of which strongly influence its strength and deformability [2]. Compared with intact rock, the mechanical properties of fractured rock masses vary widely and are difficult to predict with confidence. This variability creates challenges from the initial stage of site investigation through to the final design of geotechnical engineering structures.

Fractured rock masses are inherently anisotropic, with properties that differ by direction [3]. Such randomness must be carefully addressed in design analyses. The resulting uncertainty is a key factor in geotechnical projects, as it complicates reliable estimation of rock mass properties [4]. Uncertainty reflects limited knowledge about system parameters, while variability describes the natural spread of values within a system. Although uncertainty can sometimes be reduced by collecting more data, it must still be quantified and explicitly considered during the design process.

This uncertainty must be explicitly quantified so that it can be consistently considered during the design process. The choice of an appropriate quantification method depends on both the amount of available data and the degree of confidence in understanding the system. Recognizing and managing uncertainty allows engineers to achieve more robust and resilient designs for structures constructed on or within these complex geological formations.

Given the variable nature of fractured rock masses, it is not feasible to estimate their directional properties using limited datasets [5]. Many conventional methods, particularly empirical approaches, provide only a single output value for each parameter and therefore fail to reflect the inherent uncertainties of the rock mass. Experimental measurement of variability is also difficult, since tests typically produce only one deterministic value [6]. In practice, mean values are often used in design analyses [7-8], which can lead to either overestimation or underestimation of uncertainty. For this reason, it is essential to incorporate both estimation uncertainty and natural variability into the analysis.

Stochastic approaches provide a more realistic alternative because they consider full distributions of data rather than single values [9-10]. These methods are commonly used to handle uncertainty associated with fracture geometry [11-12] and mechanical properties [13]. Although some studies have examined stochastic analysis of discontinuities [14-15] and rock mass parameters [16-17], relatively few have focused on the combined stochastic evaluation of both strength and deformability in fractured rock masses.

Monte Carlo simulation (MCS) is one of the most widely applied stochastic tools [18-22]. It uses random sampling from probability distributions to estimate parameter ranges and capture variability. MCS has been successfully applied in various geotechnical problems [23-24],

* Corresponding author. *E-mail address:* noriyan@kashanu.ac.ir (M. Noorian-Bidgoli).

offering a probabilistic representation that improves reliability-based design compared with deterministic methods.

Statistical approaches [25-32] and probabilistic methods [33-35] provide efficient alternatives to purely deterministic techniques for estimating the mechanical properties of rock masses. Unlike deterministic analyses, these methods explicitly incorporate the effects of uncertainty, making them particularly valuable for reliability-based design. By accounting for different sources of variability, they offer a more realistic framework for characterizing rock properties and improving the safety and robustness of geotechnical designs.

Nevertheless, certain uncertainties in geomechanics cannot be adequately addressed by probability theory alone. Fuzzy logic provides a flexible alternative by allowing parameters to be expressed as fuzzy sets [36]. This approach has been increasingly applied to rock mechanics problems, including prediction of UCS, modulus of elasticity, and other geotechnical properties [37-44]. However, few studies have combined fuzzy systems with stochastic simulation to quantify uncertainty in fractured rock mass characterization.

In this study, we apply fuzzy logic to model the uncertainty of seven key parameters—Poisson's ratio, Young's modulus, UCS, cohesion, friction angle, and the Hoek–Brown constants m and s. These fuzzy-defined parameters are then incorporated into Monte Carlo simulations to generate confidence intervals and quantify uncertainty. This integrated fuzzy–Monte Carlo framework provides one of the first systematic approaches for reducing uncertainty in anisotropic fractured rock mass characterization.

2. Background

2.1. Fuzzy theory

The term "fuzzy" refers to information that is ambiguous or uncertain. Fuzzy logic is based on degrees of truth and can be used to infer knowledge from incomplete or imprecise data. Fuzzy sets, first introduced by Zadeh (1965) [45], are widely applied to represent uncertainty in complex systems for which conventional crisp sets are inadequate.

In fuzzy sets, membership functions define the degree to which an element belongs to a category, with values ranging from 0 (non-membership) to 1 (full membership). Unlike crisp sets, which have sharp boundaries, fuzzy sets allow smooth transitions, making them particularly useful for modeling geotechnical systems characterized by uncertainty.

Several types of membership functions are available in fuzzy sets, but the triangular distribution is among the most widely used. It is valued for its simple mathematical form, ease of implementation, and computational efficiency, which are practical advantages in engineering applications where datasets are limited and uncertainty is high. In geotechnical and rock engineering, the linear form of the triangular membership function also enables transparent representation of expert judgment and sparse datasets, while still capturing the essential variability of parameters.

Previous studies have shown that triangular functions can reliably approximate uncertainty ranges in rock mechanics and slope stability problems, making them a practical and defensible choice for the present study (e.g., Fayek, 2020 [46]; Madanda, 2023 [47]). More recent research has further confirmed their suitability in geotechnical applications. For example, Zhou (2023) [48] and Hosseini (2024) [49] demonstrated that triangular membership functions provide a transparent and computationally efficient way to capture uncertainty in rock mass characterization and other geotechnical modeling tasks. Together, these studies highlight the continued relevance and robustness of triangular functions in fuzzy-based uncertainty analysis for geotechnical engineering.

The triangular membership function is defined by three parameters: a (lower bound), b (most likely value), and c (upper bound). Figure 1 illustrates a typical triangular membership function. In this figure, the x-axis represents the input from the process, and the y-axis represents the corresponding fuzzy value. Parameters a and c serve as the lower and upper boundaries on the x-axis, respectively, and their corresponding fuzzy values are zero (μ(x)=0). Parameter b represents the center membership, with a corresponding fuzzy value of 1 (μ(x)=1).

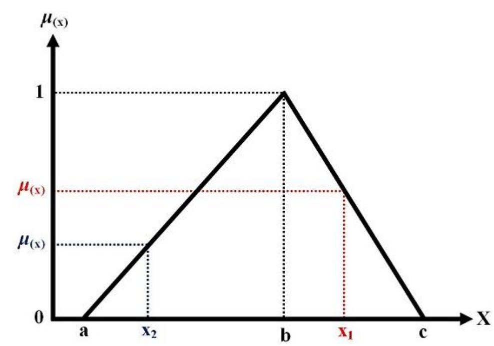

Figure 1. The boundary of the triangular membership function.

The characteristic function $\mu(x)$ of a fuzzy set M defines the membership of a value x within the set and can be depicted using a triangular membership function.

$$\mu_M(x) = \begin{cases} 1 & if\ x \in M \\ 0 & otherwise \end{cases} \tag{1}$$

This mathematical representation of the triangular membership function can be described as follows:

$$\mu_{(x;a,b,c)} = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & c \leq x \end{cases} \tag{2}$$

According to this definition, if x = b, the input has full membership in the given set (μ(x) = 1). If x is less than a or greater than c, it does not belong to the fuzzy set, and its membership value is μ(x) = 0. If x lies between a and b, or between b and c, its membership value varies between 0 and 1.

2.2. Monte Carlo simulation

Monte Carlo simulation (MCS) is a powerful stochastic tool for analyzing systems with random inputs. Instead of relying on single deterministic values, MCS repeatedly samples from probability distributions to generate a range of possible outcomes. This process allows uncertainty in the input variables to be propagated through to the model outputs.

Monte Carlo simulation (MCS) operates on the principle that each random sample may fall anywhere within an input distribution, but values from regions with higher probability are more likely to be selected. With a sufficiently large number of iterations, MCS can closely reproduce the input distributions. This capability allows hundreds or thousands of "what-if" scenarios to be simulated computationally, thereby capturing a wide range of parameter combinations and quantifying the resulting statistical distributions [50]. The process generally involves four steps:

(1) Random value selection: A random value is chosen within the specified range of estimates, and an output is computed based on this input.
(2) Result recording: The outcome is stored, and the process is repeated many times using different random values.
(3) Random-number generation: A random-number generator produces uniformly distributed values over the interval [0,1]. These numbers provide the foundation for repeated stochastic sampling.
(4) Distribution transformation: The generated random numbers are then transformed into non-uniform values to reflect the

probability distribution of the parameter being modeled.

The simulation procedure generally consists of three stages:

- Modeling: Input parameters are represented as probability density functions.
- Sampling: Random values are drawn from the specified distributions.
- Computation: Statistics of the resulting outputs are calculated, providing probabilistic ranges.

A schematic overview of the uncertainty evaluation process using MCS is provided in Fig. 2. By accounting for the random nature of certain variables and leveraging computational power, Monte Carlo simulation offers a robust means of exploring complex stochastic systems and understanding the underlying uncertainties in various scenarios.

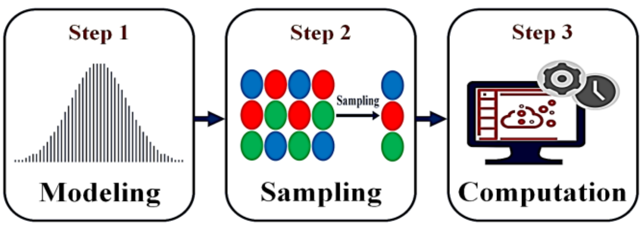

Figure 2. Schematic representation of the uncertainty evaluation with Monte Carlo procedure.

In conventional forecasting models, fixed values are typically used for estimation. However, Monte Carlo Simulation (MCS) employs a more dynamic approach by using a range of estimated values as inputs, resulting in a range of values as outputs. This method creates a more realistic representation of a simulation model.

Through sufficient iterations, MCS approximates the underlying probability distributions and captures the variability of system behavior. Its ability to represent "what-if" scenarios makes it a valuable tool in geotechnical engineering, where uncertainty is unavoidable.

3. Uncertainty analysis of fractured rock mass

The overall workflow of this study is illustrated in Figure 3. It shows the step-by-step integration of fuzzy set theory and Monte Carlo simulation to incorporate uncertainty into geomechanical parameters. Starting from raw input data, the procedure involves constructing fuzzy sets, assigning degrees of membership (DoM), defining parameter intervals, and deriving statistical characteristics (mean and standard deviation). These statistics are then used for random sampling, leading to the estimation of 95% confidence intervals. The framework highlights the logical dependency among steps, particularly the role of DoM in shaping parameter distributions, and ensures transparency and repeatability in the uncertainty quantification process.

3.1. Database

The database used in this research originates from the author's earlier work [3, 51] and is based on the fractured rock mass at Sellafield, UK. It includes seven key geomechanical parameters obtained through systematic numerical experiments.

The dataset was generated from the directional variation of a large number of stochastic discrete fracture network (DFN) models, built using the discrete element method (DEM). The fracture geometry and rock properties were defined from site investigation mapping to ensure realistic conditions.

The parameters considered are Poisson's ratio, Young's modulus, uniaxial compressive strength (UCS), cohesion, friction angle, and the Hoek–Brown constants m and s. Table 1 summarizes their statistical characteristics, including minimum, maximum, mean, and standard deviation values.

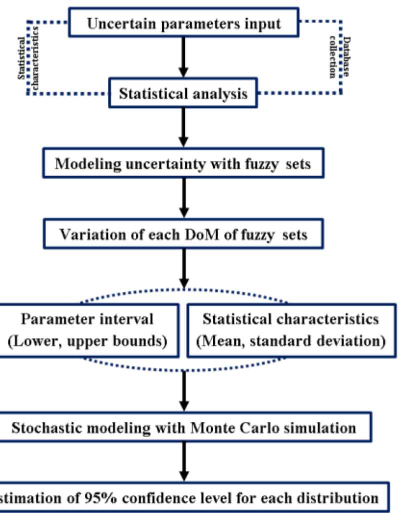

Figure 3. Flowchart of the proposed combined methodology utilizing both fuzzy system and Monte Carlo simulation.

Table 1. Statistical characteristic of strength and deformation parameters of fractured rock mass.

Parameters	Unit	Statistical characteristics			
		Min	Max	Mean	Std.
Poisson's ratio	-	0.52	0.61	0.56	0.03047
Young's modulus	GPa	0.0307	0.043	0.0368	0.00490
UCS	MPa	41.0	52.79	45.38	5.47745
Cohesion	MPa	0.0363	0.1583	0.0989	0.04730
Friction Angle	°	25.32	29.25	27.18	1.58113
m	-	0.0332	0.0592	0.0461	0.012
S	-	1.51E-06	9.84E-06	4.80E-06	3.6E-06

3.2. Modeling uncertainty with fuzzy sets

The statistical values in Table 1 were converted into fuzzy sets to capture parameter uncertainty. In this approach, the mean value was taken as the most likely (center) point, while the minimum and maximum defined the lower and upper bounds of the fuzzy set. Figure 4 shows the resulting fuzzy sets for the parameters.

Each fuzzy number was then divided into intervals according to its degree of membership (DoM). For every DoM, an interval defined by the lower (a) and upper (c) bounds was obtained, representing the uncertainty range for that parameter. Table 2 lists the values of these fuzzy intervals at different DoM levels.

3.3. Monte Carlo simulation

Stochastic modeling provides a means of quantifying the expected random variation in a system by using an established model framework. In this stage of the study, Monte Carlo simulation was applied to each fuzzy-defined parameter set to generate random samples, from which confidence intervals for the parameters were predicted.

To calculate these confidence intervals, random numbers were generated according to the assumed statistical distribution of each parameter. Following Hoek's foundational recommendation (1998) [52], the normal distribution was selected for geotechnical characterization, as it is commonly applied when the true underlying distribution of a variable is not known. More recent studies have complemented this classical approach by emphasizing the importance of reliability-based frameworks in soil and rock engineering [53] and by providing an updated forty-year review of the Hoek–Brown failure criterion for jointed rock masses [54]. Together, these contributions highlight the need to combine traditional guidelines with modern probabilistic methods to achieve more robust and reliable designs.

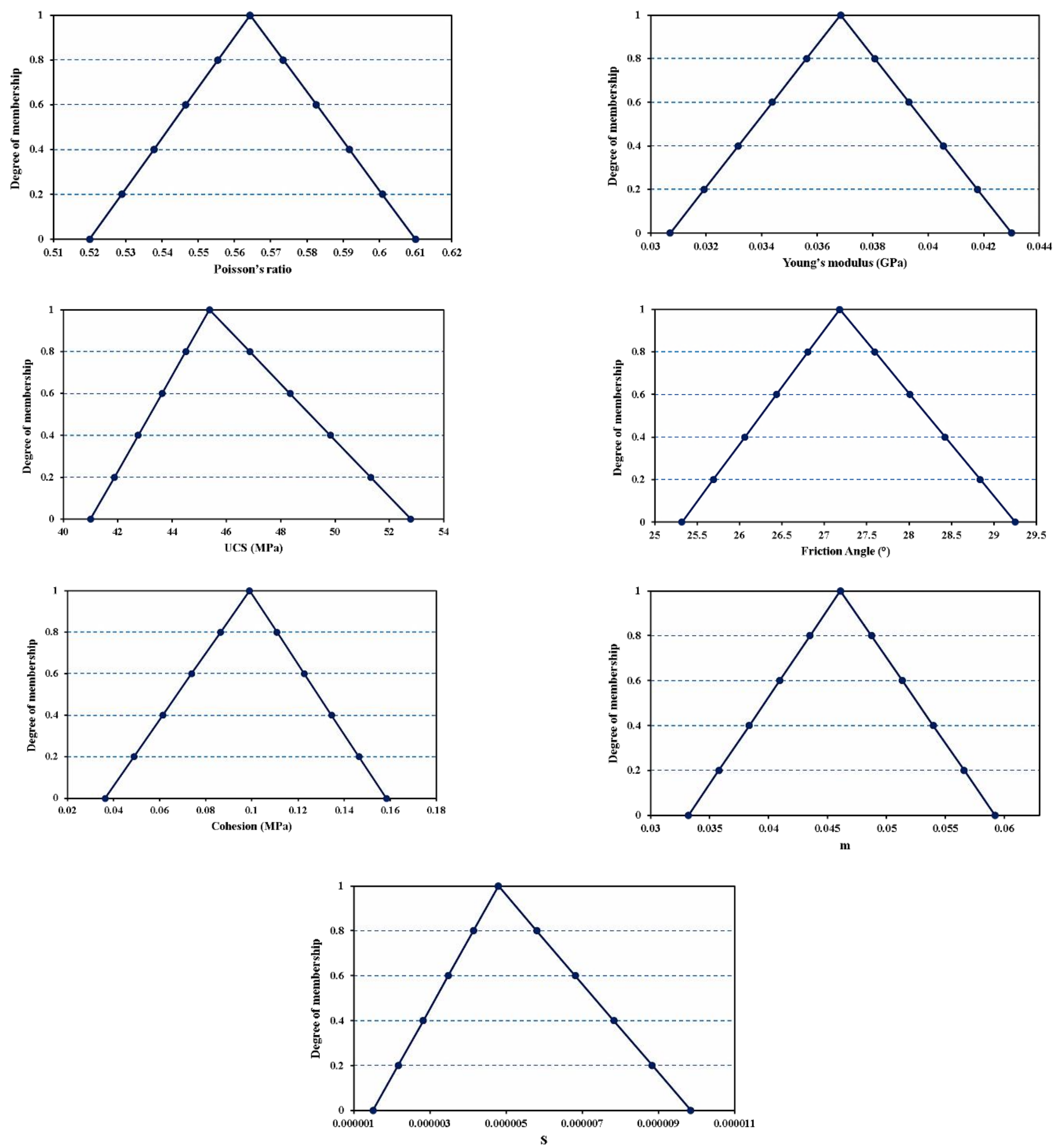

Figure 4. Schematic representation of the uncertainty evaluation with fuzzy sets.

Table 2. Variation of each degree of membership of fuzzy sets.

Parameters	Boundary	Degree of membership (DoM)					
		0	0.2	0.4	0.6	0.8	1
Poisson's ratio	Lower	0.52	0.529	0.538	0.547	0.5555	0.56
	Upper	0.61	0.601	0.592	0.583	0.573	0.56
Young's modulus (GPa)	Lower	0.0307	0.0319	0.0332	0.0344	0.0356	0.0368
	Upper	0.0430	0.0418	0.0405	0.0393	0.0381	0.0368
UCS (MPa)	Lower	41	41.88	42.75	43.63	44.51	45.38
	Upper	52.79	51.31	49.83	48.34	46.86	45.38
Cohesion (MPa)	Lower	0.0363	0.0489	0.0614	0.0739	0.0864	0.0989
	Upper	0.1583	0.1464	0.1346	0.1227	0.1108	0.0989
Friction Angle (°)	Lower	25.32	25.69	26.06	26.44	26.81	27.18
	Upper	29.25	28.84	28.42	28.01	27.59	27.18
m	Lower	0.0332	0.0358	0.0384	0.0409	0.0435	0.0461
	Upper	0.0592	0.0566	0.0540	0.0513	0.0487	0.0461
S	Lower	1.51E-06	2.17E-06	2.82E-06	3.48E-06	4.14E-06	4.80E-06
	Upper	9.84E-06	8.83E-06	7.82E-06	6.81E-06	5.80E-06	4.80E-06

The normal distribution requires two input parameters: the mean and the standard deviation. As summarized in Table 2, the central value of each fuzzy set (corresponding to DoM = 1) was taken as the mean, while the square root of the variance was used as the standard deviation (data dispersion). Because the lower (a) and upper (c) bounds differ across DoM levels, each degree of membership was associated with a unique standard deviation. For every DoM level, these bounds were used to calculate the standard deviation required for Monte Carlo simulation. Assuming a symmetric triangular membership function, the standard deviation (SD) for each DoM was determined using the following equation:

$$\frac{(c-a)}{2\sqrt{6}} = \mathrm{DOM}^{\mathrm{SD}} \quad (3)$$

Here, c and a represent the upper and lower bounds of the fuzzy set interval, respectively. The denominator in the equation is derived from the variance formula of a symmetric triangular distribution. This formulation links each degree of membership (DoM) to a specific range of uncertainty, allowing probabilistic simulation to be carried out directly from the fuzzy-defined parameter bounds. To evaluate this effect, the influence of the standard deviation—defined as the spread between the upper and lower bounds at each DoM—was examined when establishing the 95% confidence intervals. For this analysis, DoM levels of 0.2, 0.4, 0.6, and 0.8 were selected. Subsequently, 25000 random numbers were generated for each parameter's mean and standard deviation within the selected DoM levels. This Monte Carlo procedure was implemented in Microsoft Excel using the built-in functions RAND() and NORM.INV() to produce random samples consistent with the statistical properties of the fuzzy intervals. The resulting simulated normal distributions for all strength and deformation parameters are shown in Fig. 5. As illustrated in Fig. 5, the 95% confidence intervals for each parameter distribution were determined, and the results are summarized in Table 3. The table shows that as the DoM increases from 0.2 to 0.8, the width of the 95% confidence interval becomes progressively narrower. This trend indicates that higher membership levels lead to reduced uncertainty and provide more precise estimates of the geomechanical parameters.

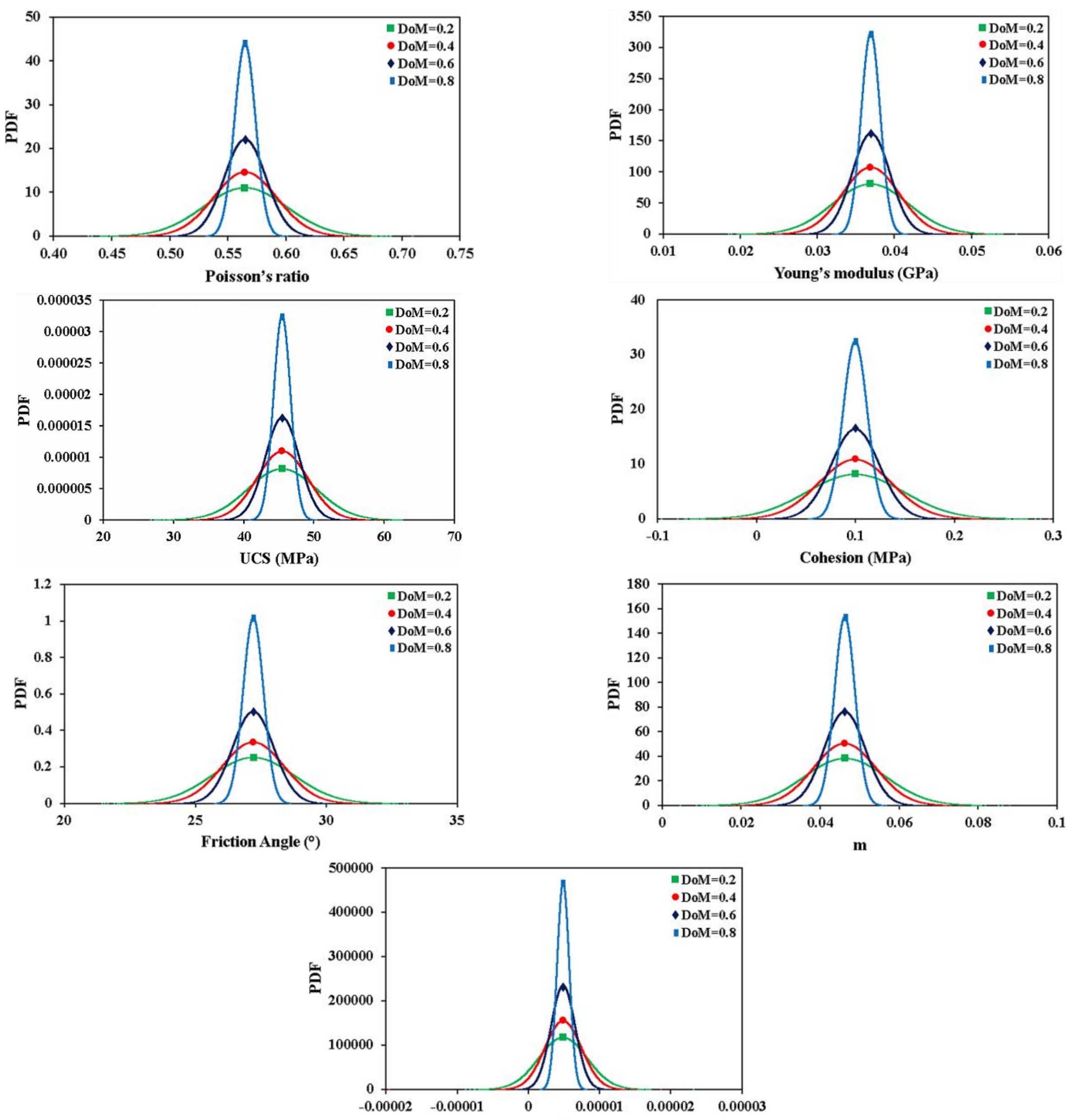

Figure 5. The simulated normal distribution for geotechnical parameters.

Table 3. Variation of each degree of membership of fuzzy sets.

Parameters	95% Confidence level	Degree of membership (DOM)			
		0.2	0.4	0.6	0.8
Poisson's ratio	Lower	0.56387	0.56403	0.56412	0.56419
	Upper	0.56476	0.56471	0.56457	0.56441
Young's modulus (GPa)	Lower	0.03676	0.03681	0.03686	0.03683
	Upper	0.03689	0.03690	0.03692	0.03686
UCS (MPa)	Lower	45.32459	45.37327	45.37498	45.36551
	Upper	45.44597	45.46352	45.43542	45.39586
Cohesion (MPa)	Lower	0.09854	0.09844	0.09880	0.09886
	Upper	0.09975	0.09935	0.09940	0.09916
Friction Angle (°)	Lower	27.1531	27.1605	27.1652	27.1747
	Upper	27.1922	27.1899	27.1848	27.1844
m	Lower	0.04603	0.04598	0.045998	0.04605
	Upper	0.04629	0.04617	0.04613	0.04612
S	Lower	4.722E-06	4.767E-06	4.768E-06	4.784E-06
	Upper	4.806E-06	4.830E-06	4.810E-06	4.805E-06

A comparison between Table 3 (results of the proposed fuzzy–Monte Carlo method) and Table 1 (deterministic results) highlights the superiority of the developed approach, especially in handling parameters strongly affected by uncertainty. By incorporating expert judgment and site-specific conditions, the method accounts for a wider range of potential parameter values—defined by the degrees of membership (DoM) of the fuzzy sets—and thereby reduces the risks associated with uncertainty in geotechnical projects.

The results also show that, compared with deterministic estimates, the fuzzy–Monte Carlo framework provides more reliable evaluations of geomechanical parameters, significantly lowering overall project uncertainty. This makes the procedure particularly valuable during the design stage of geotechnical structures, where considering parameter ranges with higher confidence levels can improve the reliability of numerical models and reduce the risks of both unsafe and overly conservative designs.

To further quantify uncertainty, two statistical indices were introduced for each geomechanical parameter at different DoM levels: the 95% confidence interval width (CIW) and the relative uncertainty (RU), defined as the ratio of CIW to the mean value. These metrics provide an objective and comparable measure of uncertainty across parameters and membership levels. The results, summarized in Table 4, clearly demonstrate that higher DoM values are consistently associated with smaller CI widths and lower RU values. This confirms the robustness of the fuzzy–Monte Carlo framework in capturing parameter variability and reducing predictive uncertainty.

To assess the robustness of the proposed uncertainty quantification framework, a sensitivity analysis was performed by varying the Degree of Membership (DoM) levels. In addition to the primary DoM values (0.2, 0.4, 0.6, and 0.8), intermediate levels of 0.1, 0.3, 0.5, 0.7, and 0.9 were also examined. For each DoM level, both the 95% confidence interval width (CIW) and the relative uncertainty (RU) were calculated. Figure 6 illustrates how the CIW changes across different DoM levels for three representative parameters: uniaxial compressive strength (UCS), friction angle, and the Hoek–Brown constant m. The results show a clear trend—higher DoM levels consistently yield narrower confidence intervals, reflecting reduced uncertainty and more precise parameter estimates. This pattern highlights the importance of selecting suitable membership levels when applying the fuzzy–Monte Carlo framework. Furthermore, the sensitivity analysis demonstrated that the method produces stable and consistent outcomes regardless of the chosen DoM values, thereby reinforcing the reliability of the proposed uncertainty evaluation process.

It should be emphasized that the narrow confidence intervals reported in Table 3 result from two main factors:

1. Bounded fuzzy sets: The fuzzy-based definition of parameter ranges inherently restricts the spread of values at each Degree of Membership (DoM). This, in turn, constrains the input domain of the Monte Carlo simulations.

2. Normal distribution with small standard deviations: As derived from the fuzzy bounds—particularly at higher DoM levels—the standard deviations are relatively small because the [c–a] intervals are narrow. This leads directly to tighter confidence limits.

To examine the influence of distributional assumptions, a sensitivity analysis was conducted using different probability distributions (normal, log-normal, and uniform) for the UCS parameter. As shown in Fig. 7, the confidence intervals widen considerably when log-normal or uniform distributions are applied. This confirms that the choice of distribution has a critical effect on uncertainty estimates. The narrow bounds from the normal distribution reflect the limited variability imposed by the fuzzy ranges, whereas the log-normal and uniform assumptions capture a broader spectrum of potential outcomes.

These findings highlight the need to select probability distributions thoughtfully, considering the geological context and the characteristics of the parameter being modeled, rather than relying on a default assumption. Ultimately, this comparison underscores the importance of distribution selection in probabilistic geomechanical modeling and its implications for risk assessment in rock engineering.

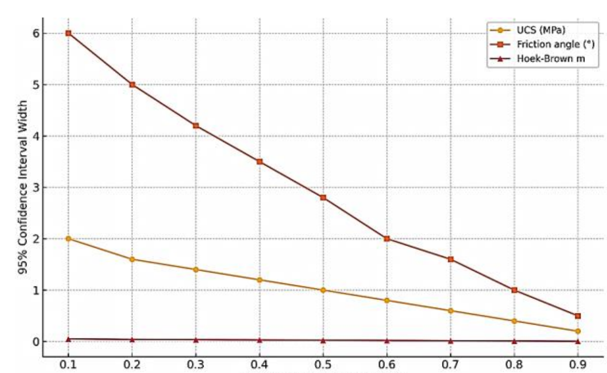

Figure 6. Sensitivity of 95% confidence interval width to Degree of Membership (DoM) for selected geomechanical parameters.

The sensitivity analysis further demonstrated that the choice of probability distribution has a notable influence on the resulting confidence intervals. While the normal distribution was selected in this study in accordance with Hoek's guideline and for consistency with prior geotechnical practice, the comparison with log-normal and uniform distributions (Fig. 7) revealed that wider ranges of variability may be captured when alternative assumptions are employed. This

Table 4. Quantitative uncertainty metrics for geomechanical parameters across different Degrees of Membership (DoM).

Parameter	DoM	Mean	95% CI Width	Relative Uncertainty
Poisson's ratio	0.2	0.56	0.08	0.1429
	0.4	0.56	0.06	0.1071
	0.6	0.56	0.04	0.0714
	0.8	0.56	0.02	0.0357
Young's modulus (GPa)	0.2	0.0368	0.012	0.3261
	0.4	0.0368	0.009	0.2446
	0.6	0.0368	0.006	0.163
	0.8	0.0368	0.003	0.0815
UCS (MPa)	0.2	45.38	1.6	0.0353
	0.4	45.38	1.2	0.0264
	0.6	45.38	0.8	0.0176
	0.8	45.38	0.4	0.0088
Cohesion (MPa)	0.2	0.0989	0.1	1.0111
	0.4	0.0989	0.08	0.8089
	0.6	0.0989	0.06	0.6067
	0.8	0.0989	0.03	0.3033
Friction angle (°)	0.2	27.18	4	0.1472
	0.4	27.18	3	0.1104
	0.6	27.18	2	0.0736
	0.8	27.18	1	0.0368
m	0.2	0.0461	0.03	0.6508
	0.4	0.0461	0.02	0.4338
	0.6	0.0461	0.015	0.3254
	0.8	0.0461	0.007	0.1518
s	0.2	4.8E-06	0.000006	1.25
	0.4	4.8E-06	0.000004	0.8333
	0.6	4.8E-06	0.000003	0.625
	0.8	4.8E-06	0.000001	0.2083

finding indicates that no single distribution is universally adequate, and that the selection should be carefully guided by the geological context and the nature of the available dataset. For future applications, we recommend testing multiple plausible probability distributions to evaluate the robustness of uncertainty estimates and to better inform risk-aware geotechnical design.

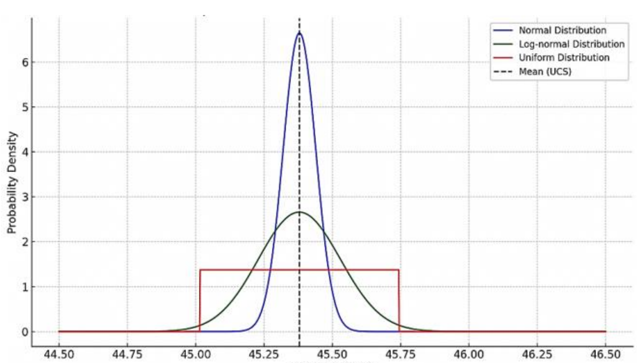

Figure 7. Comparison of probability distributions applied to uniaxial compressive strength (UCS) for uncertainty analysis.

4. Conclusions

In geotechnical and geological engineering, the mechanical characterization of fractured rock masses remains a major challenge because of inherent uncertainties. This study introduced a combined framework that integrates fuzzy logic with Monte Carlo simulation to predict the mechanical properties of anisotropic fractured rock masses using project-specific data.

In the proposed approach, the fuzzy system was employed to represent the strength and deformation parameters as fuzzy sets, while Monte Carlo simulation was used to generate predictive probability distributions. Seven key parameters—Poisson's ratio, Young's modulus, uniaxial compressive strength (UCS), cohesion, friction angle, and the Hoek–Brown constants m and s—were modeled as fuzzy variables within the analysis.

The results demonstrate that the procedure is particularly effective for parameters strongly affected by uncertainty. Its application can substantially reduce risks associated with parameter estimation in fractured rock masses, making it a reliable alternative to conventional empirical methods. Furthermore, by providing probabilities and confidence intervals, the framework enhances the prediction of geomechanical parameters and strengthens the foundation for reliability-based rock engineering design.

The key conclusions of this study can be summarized as follows:

- At least one representative dataset is required to ensure reliable parameter estimation.
- Quantifying uncertainty is essential in stochastic analyses, as it improves the reliability of strength and deformation parameter estimates. Reliability-based design methods play a central role in achieving this goal.
- Accurate probabilistic estimation depends strongly on the choice of statistical distribution, particularly when uncertainty in the dataset is not fully constrained.
- The stochastic simulation approach effectively captures uncertainty in input data, with the computed variability closely reflecting the ambiguity inherent in the parameters.
- By providing minimum and maximum bounds, the simulation framework enables both optimistic and conservative scenarios to be considered, supporting safer and more informed design decisions.
- Given the randomness of fracture occurrence in rock masses, a statistical methodology based on stochastic simulations offers a more realistic representation than conventional deterministic approaches.

The reliability of the proposed method is directly linked to the quality of the input data, and its application could be extended to a wide range of geotechnical problems. Incorporating risk analysis alongside this procedure may provide an additional tool for reducing project uncertainty. While the narrow confidence intervals obtained under the normal distribution reflect the conservative nature of fuzzy-defined bounds, the sensitivity analysis showed that log-normal and uniform distributions produce much wider intervals. Therefore, future applications of this methodology should consider multiple distributional assumptions to capture a broader spectrum of possible variability in rock mass parameters.

Competing interest

The author has not received any funding or financial support for this research, and there are no financial or personal relationships that could be perceived as influencing the results or interpretation of this study.

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