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Climate - smart mining through block matrix analysis: a conceptual modeling approach for sustainable resource governance

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ABSTRACT

As climate change intensifies the environmental and operational risks of extractive industries, Climate-Smart Mining (CSM) has emerged as a strategic response to align mining practices with sustainability goals. This study applies a novel block matrix analysis (BMA) framework to conceptualize and evaluate CSM governance structures. Fifteen key sustainability indicators were classified into five strategic domains—legal frameworks, supply chains, resource efficiency, carbon-energy management, and stakeholder engagement—and structured into a 5×5 interaction matrix (M55). Expert scoring from 12 professionals populated the matrix with interaction values ranging from 17.7 to 67.8. Color-coded mapping and determinant-based analysis identified structurally fragile blocks, particularly EOP–EOP (17.7), SII–CE (20.9), and SII–FIM (21.1). Determinants calculated using Barysh and Gaussian methods confirmed structural coherence, yielding values of 218,691.3 and 219,074 respectively, which reflect a highly stable and internally consistent governance matrix rather than fragility. These findings suggest that integrating expert input with matrix determinants offers a robust diagnostic approach for identifying governance weak points and prioritizing reform. The proposed model serves as a scalable decision-support tool for policy planning in mining and environmental sectors facing climate-related uncertainty.

Keywords: *Climate-Smart Mining, Block matrix analysis, Determinant modeling, Sustainability indicators, Governance diagnostics.*

1. Introduction

Climate change represents not only a global environmental crisis but also a systemic disruption to industrial sectors, with mining positioned at the epicenter of both vulnerability and responsibility. Extensive research has shown that mining operations are highly exposed to climate-related stressors while simultaneously contributing to ecological degradation through land-use transformation, deforestation, and emissions-intensive activities (Odell et al., 2018; Gholami et al., 2024). For example, Bulovic et al. (2024) demonstrated that increasing drought frequency and precipitation extremes in Australia are already undermining mine closure strategies and compromising rehabilitation success rates. In Canada, Pearce et al. (2011) reported widespread vulnerabilities in post-operational mine phases, exacerbated by insufficient climate-resilient infrastructure and a lack of anticipatory planning. Across Latin America, Odell et al. (2018) highlighted systemic policy gaps in addressing climate risk within mining governance, particularly in countries with high exposure but limited institutional response capacity. The intersection of climate risk and mine infrastructure performance is especially evident in arid and high-altitude zones such as Chile's Atacama Desert, where extreme water scarcity and thermal variability intensify environmental conflict, particularly around lithium and copper extraction—resources critical to the global energy transition (Akchurin, 2025). In response to these pressures, recent scholarship emphasizes the need to integrate climate adaptation and mitigation strategies throughout the entire mine life cycle. This includes embedding circular economy principles into mine planning, advancing low-emission technologies during operations, and enforcing stricter closure and post-closure protocols (Azadi et al., 2018; Hodgkinson & Smith, 2021). As demonstrated during the COVID-19 pandemic, even short-term disruptions to metal mining can trigger cascading environmental and health impacts, indicating that resilience planning must also account for broader systemic shocks (Wang et al., 2023). Climate is no longer an externality; it has become a primary determinant of mining viability and legitimacy in the 21st century. These escalating risks highlight the urgent need for an integrated governance model that aligns mining operations with long-term climate objectives. Climate-Smart Mining (CSM) has emerged as a forward-looking governance framework designed to address the dual challenges of decarbonization and climate resilience. Defined by the World Bank (2020), CSM promotes sustainable resource extraction by embedding mitigation, adaptation, and inclusive development principles into mining governance. Over time, this concept has evolved from a high-level strategy into an operational paradigm, now integrated into decision-support tools, Industry 4.0 practices, and regulatory innovations. Recent studies have further demonstrated how CSM can be operationalized through multi-layered system dynamics and indicator-based modeling.

A national-level study on the cobalt value chain revealed that achieving Climate-Smart Mining (CSM) goals requires process automation, enhanced exploration capacity, and transparent spatial planning policies, with simulations projecting up to a six-fold improvement in governance by 2030 (Sarkheil, 2024). In another case, Iran's mining sector adopted a fuzzy hybrid decision-making framework to evaluate the most sustainable strategies for overburden waste reuse (Shamsi et al., 2025). The resulting GCSM Management Sustainability Score (GCSMMSS) provided a quantifiable tool to rank alternatives such as paving applications or industrial feedstock, offering practical

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pathways for greener geomaterial management. Beyond theoretical models, however, the practical adoption of CSM frameworks remains a significant challenge. Ed et al. (2024) proposed advancing from general sustainability to climate-smart practices in Nigeria by integrating ecosystem restoration, clean technologies, and regulatory reform. Similarly, Sarkheil et al. (2024) introduced a Fuzzy Deep Groundwater Sustainability Index (FDGSI), which bridges ecological indicators with CSM decision-making in arid mining regions, reflecting an integrated and nonlinear approach to environmental assessment. In line with the digital transformation of the industry, the SIGCS model—Safe, Intelligent, Green, and Climate-Smart mining—has emerged to unify Mining 4.0 with climate neutrality (Jiskani et al., 2022).

A new Z-MARCOS-based evaluation system prioritized digitalization, autonomous vehicles, and energy conservation as the most effective interventions for implementing SIGCS practices under climate transition pressures (Guan & Zhao, 2022). In addition, an integrated fuzzy decision-support system developed in China identified major implementation barriers—including governance inefficiencies and technical limitations—while proposing actionable pathways such as low-interest loans, strict regulatory codes, and stakeholder education to overcome them (Cao et al., 2024). Building on this approach, Karatzinis and Boutalis (2025) introduced adaptive learning models that embed fuzzy rule bases into decision-support infrastructures, enabling continuous climate-smart policy feedback. Complementary frameworks have also incorporated hybrid methods, such as fuzzy cognitive mapping, to explore scenario-based CSM policy formation under uncertainty (Zamany et al., 2024). Beyond technical innovations, researchers have examined the institutional dynamics of technology adoption in the mining sector, identifying cultural inertia and fragmented regulation as critical constraints (Sonter et al., 2018). In response, policy models integrating participatory foresight and matrix scoring techniques have been proposed to strengthen institutional alignment with long-term climate targets (Hanisch et al., 2023). Taken together, these advancements demonstrate that Climate-Smart Mining (CSM) is no longer a conceptual aspiration but a rapidly evolving operational model that integrates technological innovation, ecological indicators, and governance strategies to align mining with global climate goals.

Over the past two decades, researchers have employed a wide range of multi-criteria decision-making (MCDM) tools—including AHP, ANP, fuzzy logic, and system dynamics—to address sustainability challenges in the mining sector. These methods have enabled more structured decision-making in areas such as supplier selection, sustainable supply chain management, mine site planning, and fiscal governance modeling. For instance, hybrid fuzzy models integrating AHP, DEMATEL, and TOPSIS have been applied to rank suppliers in the mining industry, accounting for uncertainty and feedback relationships among criteria (Annals of Operations Research, n.d.). Similarly, spatial decision frameworks combining fuzzy AHP and GIS have facilitated site-specific rehabilitation and land-use planning in Indian mining zones (Shome et al., 2025). Moreover, fuzzy-AHP and fuzzy-DEMATEL approaches have proven effective in identifying and prioritizing sustainability drivers and barriers within mining supply chains, particularly in regulatory and environmental domains (Paul & Mahapatra, 2025). This methodology has also been successfully implemented in industrial contexts; for example, Sarkheil and Sadeghy Nejad (n.d.) combined EFMEA and the William Fine method to assess environmental and occupational risks in a building stone processing facility, demonstrating how hybrid risk models can enhance organizational efficiency and safety outcomes. Beyond environmental performance, tools such as AHP have been utilized in Indonesia to prioritize national energy policy, electricity reforms, and governance mechanisms related to sustainable mining and energy transitions (Ali & Kim, 2024). Despite their strengths, however, many MCDM tools assume static interdependencies and lack temporal adaptability—an issue addressed through system dynamics modeling. Recent system dynamics applications have captured the evolving impact of fiscal regimes on project viability (Banda, 2023) and incorporated social acceptance dynamics into mining lifecycle assessments using causal loop diagrams and stakeholder participation (Dall-Orsoletta et al., 2025).

These models highlight the critical role of feedback delays and trust-building in shaping long-term project outcomes, particularly within decarbonization agendas. In offshore mining contexts, Sarkheil et al. (2024) developed a Bayesian Network-based risk model to evaluate environmental hazards associated with tripod installation in deep-sea operations. Their study emphasized the importance of integrating expert judgment with empirical data to prioritize mitigation measures under uncertainty. In parallel, hybrid methods such as fuzzy ZE-MCDM and hypersoft sets are expanding the capacity of MCDM frameworks to address complex, ambiguous, and evolving criteria in domains ranging from overburden waste utilization (Saqlain et al., 2024) to crypto-mining environmental governance (Saqlain et al., 2024). Although underexplored in mining governance literature, Block Matrix Analysis (BMA) demonstrates strong potential for managing such complexity. Stewart (2001) first introduced BMA as a mathematical method for partitioning matrices into interpretable sub-blocks in control systems, while Dobrushkin (2017) later extended its application to nonlinear system modeling, structural stability analysis, and sustainability science. BMA structures indicators and system components into modular sub-matrices, enabling simultaneous structural and statistical interpretation of multidirectional relationships and feedback loops. Its applications have included machinery failure analysis (Buslaeva & Yakovleva, 2022), missing data recovery in bioinformatics (Lejun et al., 2025), geomechanical simulation of pillar collapse (Hamediazad & Bahrani, 2024), and sensitivity reduction in complex engineering models (Yang & Peng, 2023).

In the context of Climate-Smart Mining, the diagnostic potential of matrix determinants has also gained increasing attention. Determinants derived through Gaussian or Barysh-based elimination techniques can reveal structural coherence or fragility across interrelated governance domains (Butcher, 2015; Zhang et al., 2025). These methods have been applied to evaluate system stability in energy infrastructure, structural vibration in mining conveyors (Wang et al., 2025), social platform sustainability during COVID-19 (Cavus et al., 2021), and evapotranspiration modeling in high-altitude ecosystems (Jiskani et al., 2023). Their value lies in enabling both the rapid detection of hidden instabilities and the prioritization of structural reform needs within otherwise complex sustainability architectures. Ultimately, for matrix-based modeling to meaningfully support climate-smart mining governance, it must be embedded within a governance lens. As Domínguez-Gómez and González-Gómez (2021) and Essaw et al. (2025) argue, governance in mining requires more than technical optimization—it demands systems capable of navigating legitimacy, stakeholder equity, enforcement gaps, and decentralized responsibility. Drawing on examples from Ghana (Li et al., 2021), South Africa (Cole & Broadhurst, 2022), Peru (Cano & Kunz, 2022), and even space mining initiatives (Salmeri, 2023), it becomes evident that effective governance must accommodate both local variability and frontier innovation. The proposed BMA-governance model seeks to operationalize this dual imperative by offering modular flexibility while retaining structural coherence.

This research proposes the development of a modular block matrix designed to capture the multidimensional interplay within climate-smart mining governance. By integrating empirical evidence from international case studies and expert consultation, the matrix facilitates the analysis of systemic fragility across governance domains through determinant-based methodologies. The resulting outputs are translated into actionable insights that guide the formulation of more resilient, inclusive, and climate-aligned mining policies.

2. Methodology

oherence, structural vulnerability, and multidirectional interdependencies within Climate-Smart Mining (CSM) governance using Block Matrix Analysis (BMA). Unlike linear decision-making models, BMA enables a comprehensive examination of complex, interdependent relationships across multiple sustainability domains. The primary objective of this methodology is to detect structural fragilities, measure the strength of interactions between governance and sustainability indicators, and assess systemic stability through determinant-based evaluation. Determinants—calculated using Gaussian elimination and the Baryshnikov reduction method—serve as quantitative proxies for systemic coherence. Lower determinant values signal potential structural weaknesses, allowing for the identification of misaligned policy interfaces and fragile governance relationships that require priority attention.

The research design integrates both qualitative and quantitative components. Conceptual definitions and structural mappings are informed by sector-specific logic and expert knowledge, transforming qualitative governance frameworks into populated interaction matrices. Numerical scoring and matrix diagnostics are subsequently applied to quantify interaction patterns, identify vulnerable areas, and generate actionable insights. In addition, the framework conceptually incorporates supplementary interaction blocks (e.g., M_{12}, M_{34}, and M_{45}), which are not yet numerically populated but are recognized—based on expert recommendations—for their structural significance. Examples include synergies between clean energy and circularity or between legal and productivity domains. These supplementary blocks are proposed for integration in future iterations to further refine the model's systemic representation

2.1. Block matrix design and structure

Block matrices are powerful and convenient tools for representing large and complex matrices, especially in mathematics and engineering. The core concept is to partition a large matrix into smaller submatrices, called blocks, which are easier to analyze and manipulate. These blocks are usually rectangular submatrices that together form the original matrix.

It is often practical to divide a matrix into smaller blocks to simplify operations and to better visualize the matrix's structure. For example, suppose matrix A is of size 6\times 6 and we want to represent it as a block matrix. One way to do this is to partition A into four smaller 3×3 submatrices arranged as follows:

$$A=\begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} \tag{1}$$

Here, each block Aij (i,j=1,2) is itself a 3×3 matrix. For instance, these blocks can be detailed as:

$$A_{11}=\begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix} \qquad A_{12}=\begin{bmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{bmatrix}$$

$$A_{21}=\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c31 & c32 & c33 \end{bmatrix} \qquad A_{22}=\begin{bmatrix} d11 & d12 & d13 \\ d21 & d22 & d23 \\ d31 & d32 & d33 \end{bmatrix}$$

In this way, matrix A is effectively divided into four 3×3 submatrices, or blocks, which makes it easier to study its properties and perform matrix operations.

The main points to consider when designing and illustrating block matrices include partitioning the original matrix into smaller blocks, representing each block as a submatrix, and clearly demonstrating how these blocks combine to form the full matrix. This approach is widely used in mathematics and engineering to handle large matrices more efficiently and intuitively

2.2. Indicator identification and clustering

To design a block matrix for developing a Climate-Smart Mining (CSM) model, the first step is to identify 15 key indicators. These indicators are derived from international sustainability frameworks and previous studies. They are then systematically organized into thematic blocks, which form the foundation of the matrix structure.

The indicators were systematically organized into five primary functional domains to provide a structured framework for understanding their respective roles within Climate-Smart Mining (CSM). Each domain represents critical dimensions of sustainable mining practices and governance, addressing the multifaceted challenges of balancing environmental, social, and operational objectives.

The first domain, Climate Strategies (CIS), encompasses the development and implementation of high-level policies and strategic frameworks that guide mining operations toward minimizing their climate impact. This includes planning and initiatives that ensure mining activities are aligned with broader climate mitigation and adaptation goals, thereby fostering resilience and sustainability at the sectoral level.

The second domain, Resource Efficiency and Recycling (RE, PRM, RFS, MOP), focuses on optimizing the utilization of raw materials and minimizing waste through efficient resource management, recycling practices, and improved operational techniques. This domain underscores the importance of circular economy principles within mining, aiming to reduce environmental degradation and enhance material recovery throughout the mining value chain.

The third domain, Energy and Carbon (RES, CFI, EE), addresses the energy consumption patterns intrinsic to mining activities and the imperative to reduce associated carbon emissions. It includes the adoption of renewable energy technologies, improvements in energy efficiency, and systematic monitoring of carbon footprints—all of which contribute to lowering the sector's overall environmental impact and supporting global decarbonization efforts.

Table 1. Climate-smart mining indicators: classification by functional domain**.**

No.	Indicator Title	Abbreviation
1	Climate adaptation strategies and impacts	*CIS*
2	Building resilience for sustainable resource mining	*RFS*
3	Creation of marketing opportunities	*MOP*
4	Integration of renewable energies	*RES*
5	Resource efficiency	*RE*
6	Reuse and recycling of low-carbon minerals	*PRM*
7	Utilization of carbon financial instruments	*CFI*
8	Energy efficiency in the mining value chain	*EE*
9	Innovative solutions for waste (tailings) management	*IDS*
10	Circular economy and supply chain management (circular supply chain management)	*CE*
11	Comprehensive management of geological data	*GDM*
12	Equal opportunity creation and multi-stakeholder engagement	*EOP*
13	Efficient legal and regulatory network	*LRS*
14	Forest smart mining with regional landscape management	*FIM*
15	Risk-free investments in low-carbon minerals	*SII*

Supply Chain and Data Management (IDS, CE, GDM) constitutes the fourth domain, emphasizing the need for transparent and resilient supply chains supported by robust data governance systems. Effective integration of data across the supply chain enables improved traceability, real-time monitoring, and informed decision-making, thereby fostering responsible sourcing while reducing environmental and social risks.

The fifth domain, Opportunities and Interactions (LRS, FIM, SII, EOP), captures the dynamic governance, regulatory, and social dimensions of CSM. It includes considerations such as compliance with legal frameworks, mitigation of environmental impacts, promotion of sustainable investment, and equitable stakeholder participation. Recognizing and managing the complex interdependencies within this domain is essential for building resilient governance systems that harmonize economic development with environmental stewardship and social equity. This classification approach is informed by the need to group related concepts cohesively, acknowledge potential synergies and interdependencies among indicators, and reflect the priorities of diverse stakeholders engaged in the mining sector. By providing a comprehensive conceptual structure, this framework facilitates rigorous analysis and targeted policy development aimed at advancing Climate-Smart Mining practices.

The indicators were then arranged into a 5x5 block matrix M, creating submatrices Mij (M_{11}, M_{12}, M_{55}, i.e.) where each block reflects interactions within or across indicator domains. Particular attention is given to the M_{55} block—a 4x4 matrix involving LRS, FIM, SII, and EOP. These variables represent complex social, legal, and economic dynamics integral to adaptive mining.

A detailed explanation of how each row and column in M_{55} represents conceptual interactions is provided in Appendix A.3.4. In addition, the continuation of Table 2 is presented in Appendix A.3.1 (two tables).

2.3. Expert-based matrix scoring

Data collection involved 12 domain experts scoring 16 pairwise relationships in M_{55} using a structured questionnaire. The expert panel included professionals from academia, government, and industry. Their profiles are summarized in Table 4. Scores ranged from 0 to 100, reflecting the intensity and quality of influence between each indicator pair. Each entry in M_{55} was calculated as the mean of expert responses.

The matrix structure enables compartmental analysis and visual representation of relationships, categorized using traffic-light color coding:

- Green (Score > 55): Strong interaction.
- Yellow (40 ≤ Score ≤ 55): Moderate interaction.
- Red (25 ≤ Score ≤ 39): Weak or structurally critical zones.
- Purple (Score < 25): Very weak or structurally critical.

Table 3. Analytical block matrix (m55) of the climate-smart mining model — a case study approach.

LRS-LRS	LRS-FIM	LRS-SII	LRS-CE
FIM-LRS	FIM-FIM	FIM-SII	FIM-CE
SII-LRS	SII-FLM	SII-SII	SII-CE
EOP-LRS	EOP-FLM	EOP- SII	EOP- EOP

Table 2. Description of selected components of the block matrix.

Row	Block Name	Block Definition	Block Description
1	M_{11}	The M_{11} matrix (1×1) includes the impacts of strategies and climate adaptation effects	This matrix represents the impact of a single indicator (strategies and climate adaptation effects) on itself. In fact, it shows how climate adaptation strategies and impacts can improve or change themselves. This is usually considered as a numerical value that reflects the significance or internal variations of the strategies.
2	M_{12}	The M_{12} matrix (1×5) represents the impacts of strategies on resource efficiency and recycling.	This matrix represents how strategies and climate adaptation impacts (as a single indicator) affect five different indicators within the resource efficiency and recycling category. The values in this matrix indicate the extent to which climate strategies influence resource efficiency, recycling, reuse, and other related aspects.
3	M_{13}	The M_{13} matrix (5×1) shows the impacts of resource efficiency and recycling on strategies.	This matrix represents the impacts of five different indicators of resource efficiency and recycling on a single indicator of strategies and climate adaptation impacts. The values in this matrix show how changes in resource efficiency and recycling can influence climate strategies.
4	M_{22}	The M_{22} matrix (5×5) includes the interactions among indicators within the resource efficiency and recycling category.	This matrix represents the mutual impacts and interactions among five different indicators within the category of resource efficiency and recycling. In other words, the values in this matrix help us understand how each resource efficiency indicator influences the others in the same category, and how improvement in one indicator affects the rest.
5	M_{55}	This matrix (M_{55}) with dimensions 3×3 represents the interactions among the indicators within the category of opportunities and interactions.	his matrix represents the interactions between three indicators of supply chain and data management and three indicators related to energy and carbon. The values in this matrix illustrate how supply chain and data management impact various aspects of energy and carbon.

Table 4. Expert Panel Composition and Institutional Affiliation.

Row	Disciplinary Domain	No. of Experts	
1	Environmental Geology	3	Kharazmi University, University of Tehran
2	Mining Engineering	3	University of Tehran, IMIDRO, Iranian Minerals Production and Supply Company
3	Mining & Environmental Engineering	2	University of Tehran
4	Management and Policy	2	University of Tehran, Ministry of Industry, Mine and Trade (MoIMT)
5	Mineral Economics and Environmental Policy	2	Environmental Research Institute for Sustainable Development, Department of Environment (DOE)

2.4. Determinant analysis

To evaluate structural fragility and systemic dependency within the M_{55} interaction matrix, two determinant-based methods were applied: the Montant method (also known as the Barysh approach) and Gaussian elimination. Both are pivot-based techniques, but they differ in how they represent matrix logic and numerical stability.

(a) Barysh (montant) method

The Montant method can evaluate structural fragility and identify high-sensitivity zones in the CSM framework. This method applies to the expert-evaluated M_{55} matrix and tracks pivots during forward elimination without row swapping, preserving structural logic.

Mathematical formulation

The matrix $M \in R^{n \times n}$ is a square interaction matrix. The Montant method proceeds as:

1. Select pivot a_{ii} for each row i.
2. For each row < jj, eliminate the element at a_{ji} by:

$$fji = \frac{aji}{aii} \Rightarrow Row_j = Rowj - fji \times Rowi$$

Once upper triangular matrix U is formed, compute:

$$\det_{(A)} = \prod_{i=1}^{n} Uii \qquad (1)$$

where Uii is the pivot at step i. Pivot ordering is preserved and row-swapping is minimized to trace logical dependencies. Barysh is beneficial for theoretical sensitivity modeling.

(b) Gaussian Elimination Method

Gaussian elimination is another form of pivot-based determinant computation, which simplifies matrices by reducing them to upper triangular form via forward elimination. Unlike Montant, row-swapping is permitted, improving numerical stability.

Mathematical formulation

Given a matrix M, the procedure follows these steps:

1. Use forward elimination to reduce M to an upper triangular matrix U.
2. If no row swaps are required, the determinant is:

$$\det{}_{(A)} = \prod_{i=1}^{n} Uii \qquad (2)$$

3. If any row swaps occur, multiply the result by $(-1)^s$, where s is the number of swaps

2.5. Cross-validation and fragility zones

In the context of block matrix modeling, it is essential to verify that numerical fragility assessments align with qualitative and visual interpretations of matrix zones. This process—cross-validation—ensures that visually scored zones accurately reflect the system's structural coherence. The present discussion does not focus on specific values from the study; rather, it outlines the methodological framework for interpreting fragility zones using two complementary approaches.

The first approach involves visual or expert scoring, in which zones are typically categorized based on expert judgment as having high (green/blue), moderate (yellow), or low (red) effectiveness. The second approach relies on structural fragility analysis using matrix determinants, where zones with low determinant contributions or small pivot values are considered structurally fragile regardless of their visual scores. To ensure robustness, cross-validation is applied to confirm that visual consensus aligns with structural metrics.

In practice, a zone that appears visually stable but exhibits a low determinant value (e.g., less than 0.1) reveals hidden fragility. Conversely, zones characterized by both low visual scores and low determinants confirm their critical nature.

2.6. Future expansion blocks and conceptual interactions

While this study focused primarily on the M_{55} matrix, several other blocks identified in the original conceptual model provide opportunities for extended interaction analysis. For example, these include:

- M_{12}: Legal-Regulatory → Resource Efficiency.
- M_{34}: Carbon-Energy ↔ Supply Chain Coordination.
- M_{45}: Digital Supply ↔ Stakeholder Engagement.

Though not evaluated in this round of determinant analysis, these matrices were conceptualized by the expert team and remain viable for future empirical scoring.

Appendix A.6.2 provides additional structural descriptions to support replication and expansion in subsequent research. These blocks are particularly relevant for analyzing horizontal spillovers across domains such as decarbonization, digitization, and inclusive participation—pillars of Climate-Smart Mining (CSM). The structural logic embedded in these matrices follows the same block interaction format used in M_{55} and supports a modular extension of the model.

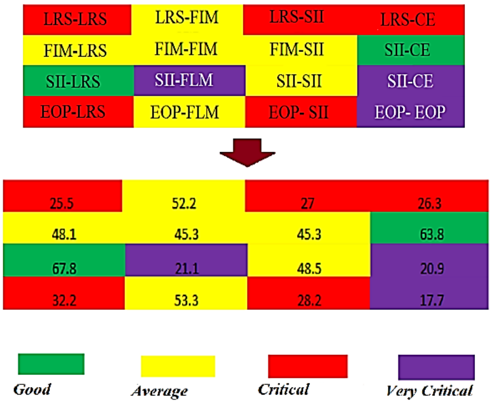
3. Results and discussion

This section presents the analytical core of the study. It encompasses matrix evaluation, determinant computation, comparative method analysis, uncertainty assessment, and the generation of insights into structural fragility within the Climate-Smart Mining (CSM) framework. The primary focus is on block M_{55}, which represents the interrelations among Legal Regulations (LRS), Forest Mining (FIM), Sustainable Investment (SII), and Equity-Oriented Participation (EOP).

3.1. Matrix Output Overview: Block M_{55}

The M_{55} matrix consists of 16 elements, derived from evaluations by 12 experts, aggregated using the arithmetic mean. The scores capture the interaction intensity between pairs of key governance actors.

The continuation of Table 5: Questions and Scores Assigned by Experts in Responding to the Block Matrix M_{55} is provided in the Appendix part A.3.3.



LRS-LRS	LRS-FIM	LRS-SII	LRS-CE
FIM-LRS	FIM-FIM	FIM-SII	SII-CE
SII-LRS	SII-FLM	SII-SII	SII-CE
EOP-LRS	EOP-FLM	EOP- SII	EOP- EOP

25.5	52.2	27	26.3
48.1	45.3	45.3	63.8
67.8	21.1	48.5	20.9
32.2	53.3	28.2	17.7

🟩 Good 🟨 Average 🟥 Critical 🟪 Very Critical

Figure 1. Heatmap visualization of matrix M_{55}

Using both qualitative and quantitative diagnostics, the research proposes a three-zone governance framework to guide policy and intervention priorities. Red zones, such as LRS–LRS, LRS–SII, LRS_CE, EOP_LRS, and EOP_SII indicate areas with an urgent need for stakeholder reform, clearer regulations, and investment restructuring. Yellow zones, including LRS–FIM, FIM–FIM, FIM_SII, SII_SII, and EOP_FLM require continuous monitoring and efforts to bridge gaps through participatory pilot projects. Meanwhile, green zones like SII–LRS and SII–CE represent opportunities for case-based replication and scaling up successful governance practices.

A policy dashboard could visualize determinant trends and provide yearly risk audits for national mining programs.

A heatmap of the matrix has been constructed to visually distinguish good, average, critical, and very critical interaction zones using green (>55), yellow (55–40), red (39-25), and purple (<25) coloring.

These results reveal visible governance gaps that, if unresolved, could compromise the long-term viability of climate-smart frameworks.

3.1.1. Conceptual role of supplementary blocks: M_{34}, M_{45}, and M_{54}

Although the present analysis focuses primarily on the M_{55} matrix, it is important to recognize the conceptual significance of additional blocks—particularly M_{34} (Carbon-Energy Transition × Circular Supply Chains), M_{45} (Data Systems × Stakeholder Engagement), and M_{53} (Opportunities and Interactions × Energy and Carbon). These matrices were referenced during expert consultations and were incorporated into the structural design of the model.

The M_{34} block represents critical links between low-carbon energy strategies and the recyclability, traceability, and efficiency of mineral supply chains. As climate-smart mining increasingly relies on decarbonized logistics and closed-loop production systems, the role of M_{34} is expected to grow. The M_{45} block highlights the intersection of integrated data infrastructures (e.g., satellite monitoring, blockchain, AI dashboards) with multi-stakeholder participation. This block is conceptually important for promoting transparency, equity, and accountability in resource governance. Lastly, M_{53} captures how stakeholder opportunities and participatory frameworks can influence the design and deployment of digital mining technologies. As the industry shifts toward real-time monitoring and automated compliance, this relationship becomes increasingly relevant.

To reinforce the conceptual foundation of the supplementary blocks discussed above, Tables 7 through 9 provide structural representations of M_{12}, M_{34}, and M_{45}, derived from the model's original matrix architecture.

Table 4. Representation of the block matrix pattern.

Row	Block Name	Block Definition	Block Matrix Relationship
1	M_{11}	The M_{11} matrix (1×1) represents the internal impacts of *strategies and climate adaptation effects.*	$\begin{pmatrix} {}_{11}a \end{pmatrix} = \mathbf{M}_{11}$
2	M_{12}	The M_{12} matrix (5×1) represents the impact of *strategies* on *resource efficiency and recycling.*	$\begin{pmatrix} {}_{12,5}a & {}_{12,4}a & {}_{12,3}a & {}_{12,2}a & {}_{12,1}a \end{pmatrix} = \mathbf{M}_{12}$
3	M_{13}	The M_{13} matrix (3×1) represents the impact of *strategies* on *energy and carbon.*	$\begin{pmatrix} {}_{13,3}a & {}_{13,2}a & {}_{13,1}a \end{pmatrix} = \mathbf{M}_{13}$
4	M_{14}	The M_{14} matrix (3×1) represents the impact of *strategies* on *supply chain and data management.*	$\begin{pmatrix} {}_{14,3}a & {}_{14,2}a & {}_{14,1}a \end{pmatrix} = \mathbf{M}_{14}$
5	M_{15}	The M_{15} matrix (4×1) represents the impact of *strategies* on *opportunities and stakeholder interactions.*	$\begin{pmatrix} {}_{15,4}a & {}_{15,3}a & {}_{15,2}a & {}_{15,1}a \end{pmatrix} = \mathbf{M}_{15}$
6	M_{21}	The M_{21} matrix (1×5) represents the impact of *resource efficiency and recycling* on *strategies.*	$\begin{pmatrix} {}_{21,1}a \\ {}_{21,2}a \\ {}_{21,3}a \\ {}_{21,4}a \\ {}_{21,5}a \end{pmatrix} = \mathbf{M}_{21}$
7	M_{22}	The M_{22} matrix (5×5) represents the internal interactions among indicators within the *resource efficiency and recycling* category.	$\begin{pmatrix} {}_{22,15}a & {}_{22,14}a & {}_{22,13}a & {}_{22,12}a & {}_{22,11}a \\ {}_{22,25}a & {}_{22,24}a & {}_{22,23}a & {}_{22,22}a & {}_{22,21}a \\ {}_{22,35}a & {}_{22,34}a & {}_{22,33}a & {}_{22,32}a & {}_{22,31}a \\ {}_{22,45}a & {}_{22,44}a & {}_{22,43}a & {}_{22,42}a & {}_{22,41}a \\ {}_{22,55}a & {}_{22,54}a & {}_{22,53}a & {}_{22,52}a & {}_{22,51}a \end{pmatrix} = \mathbf{M}_{22}$
8	M_{23}	The M_{23} matrix (3×5) represents the impact of *resource efficiency and recycling* on *energy and carbon.*	$\begin{pmatrix} {}_{23,3}a & {}_{23,2}a & {}_{23,1}a \\ {}_{23,6}a & {}_{23,5}a & {}_{23,4}a \\ {}_{23,9}a & {}_{23,8}a & {}_{23,7}a \\ {}_{23,12}a & {}_{23,11}a & {}_{23,10}a \\ {}_{23,15}a & {}_{23,14}a & {}_{23,13}a \end{pmatrix} = \mathbf{M}_{23}$
9	M_{55}	Matrix M_{55}, sized 4×4, represents the interactive impacts among the indicators of opportunities and interactions.	$\begin{pmatrix} {}_{14}a & {}_{13}a & {}_{12}a & {}_{11}a \\ {}_{24}a & {}_{23}a & {}_{22}a & {}_{21}a \\ {}_{34}a & {}_{33}a & {}_{32}a & {}_{31}a \\ {}_{44}a & {}_{43}a & {}_{42}a & {}_{41}a \end{pmatrix} = \mathbf{M}_{55}$

Table 5. Questions and scores assigned by experts in responding to the block matrix m55 section in the climate-adapted smart mining model.

Row	Evaluation question	Scoring criteria	Average score (0–100)
1	Considering the current status of an efficient legal and regulatory network, how would you rate its effective impact on the climate-adaptive smart mining model?	1- Level of compliance of laws with international environmental standards {Highly compliant (5), Moderately compliant (3), Low compliance (1)} 2- Efficiency and effectiveness of monitoring and enforcement {Highly efficient (5), Moderate efficiency (3), Poor efficiency (1)} 3- Flexibility of laws in adapting to climate change {Very high (5), Moderate (3), Low (1)} 4- Stakeholder participation and transparency in the legislation process {Very high (5), Moderate (3), Low (1)}	25.5
2	Given the current status of an efficient legal and regulatory network, how would you rate its interactive impact on smart forest mining within the climate-adaptive smart mining framework?	1-Ability to preserve and protect forests within the framework of laws 2-Adaptability of laws to smart forest mining technologies 3-Continuous monitoring and supervision of mining operations in forest areas 4-Interagency collaboration and coordination in forest environment management	52.2
3	Given the current status of an efficient legal and regulatory network, how would you rate its interactive impact on low-risk investments within the climate-adaptive smart mining framework?	1- Stability and predictability of laws and regulations 2-Legal support for environmental investments 3-Financial and legal incentives for low-risk investments 4- Transparency and accountability in legal processes	27
4	Considering the current status of an efficient legal and regulatory network, how would you rate its interactive impact on circular supply chain management in the climate-smart mining model?	1-Legal support for the circular supply chain 2-Transparency and traceability in the supply chain 3-Adaptability of laws to new circular supply chain technologies 4-Efficiency of monitoring and supervision of the circular supply chain	26.3
5	Considering the current status of forest smart mining, how would you rate its interactive impact on an efficient legal and regulatory network within the climate-smart mining model?	1-The impact of forest smart mining on the formulation and amendment of laws. 2- Alignment of forest smart mining technologies with legal requirement. 3-Level of legal acceptance and support for forest smart mining 4- Legal flexibility in response to innovations in forest smart mining	48.1

Table 6: Aggregated expert scores for the m55 block.

	LRS	FIM	SII	EOP
LRS	25.5	52.2	27.0	26.3
FIM	48.1	45.3	45.3	63.8
SII	67.8	21.1	48.5	20.9
EOP	32.2	53.3	28.2	17.7

Note: Each value represents the mean score from 12 expert responses. Outliers were reviewed but retained to preserve response diversity. No weighting was applied. Full expert data, score distributions, and standard deviations are available in Appendix A for reproducibility.

Table 7: M_{12} – Climate strategies × productivity outcomes.

	Productivity A	Productivity B	Productivity C
Strategy A	–	–	–
Strategy B	–	–	–
Strategy C	–	–	–

Description: This block evaluates the effect of proposed climate-aligned strategies (e.g., emission reduction, resilience measures) on various dimensions of mining productivity such as yield, downtime reduction, and operational efficiency.

Table 8: M_{34}– Carbon-energy transition × circular supply chains.

	CE Supply Chain 1	CE Supply Chain 2	CE Supply Chain 3
Energy Source 1	–	–	–
Energy Source 2	–	–	–
Energy Source 3	–	–	–

Description: This matrix addresses how different clean energy strategies interact with the robustness, recyclability, and traceability in mineral supply chains.

Table 9: M_{45} – Data systems × stakeholder engagement.

	Stakeholder A	Stakeholder B	Stakeholder C
Data Layer A	–	–	–
Data Layer B	–	–	–
Data Layer C	–	–	–

Description: This matrix captures how the implementation of integrated data infrastructures (e.g., satellite monitoring and AI dashboards) affects the transparency and participation of local communities, regulators, and investors.

3.2. Determinant calculations and methodological comparison

To quantify matrix integrity, two determinant-based methods are applied:

(a) Barysh (montant) method

To compute the determinant of the block matrix M_{55}, we apply the Bareiss algorithm, a pivot-based numerical method that reduces round-off errors and improves numerical stability was applied. The original matrix M55, derived from the climate-smart mining block model, is as follows:

$$M_{55}=\begin{bmatrix} 25.5 & 52.2 & 27 & 26.3 \\ 48.1 & 45.3 & 45.3 & 63.8 \\ 67.8 & 21.1 & 48.5 & 20.9 \\ 32.2 & 53.3 & 28.2 & 17.7 \end{bmatrix}$$

Step-by-step pivot extraction:

Step 1 – First pivot (LRS): a_{11}= 25.5

- Pivot Element: $a_{11} = 25.5$
- Eliminate entries below the pivot in the first column
- Calculating the coefficients for elimination:

$$f_{21}=\frac{48.1}{25.5}\approx 1.886 \qquad f_{31}=\frac{67.8}{25.5}\approx 2.659 \qquad f_{41}=\frac{32.2}{25.5}\approx 1.26$$

- Perform row operations:
  - $Row_2 = Row_2 - f_{21}\times Row_1$
  - $Row_3 = Row_3 - f_{31}\times Row_1$
  - $Row_4 = Row_4 - f41\times Row1$

Matrix after step 1:

- Row 2:
- $45.3-1.886\times52.2=45.3-98.44= -53.14$
- $45.3-1.886\times27.0=45.3-50.92= -5.62$
- $63.8-1.886\times26.3=63.8-49.60=14.20$

Row 3:

- $21.1-2.659\times52.2=21.1-138.78= -117.68$
- $48.5-2.659\times27.0=48.5-71.79= -23.29$
- $20.9-2.659\times26.3=20.9-69.90= -49.00$

Row 4:

- $53.3-1.263\times52.2=53.3-65.94=-12.64$
- $28.2-1.263\times27.0=28.2-34.10= -5.90$
- $17.7-1.263\times26.3=17.7-33.22= -15.52$

Updated matrix after step 1:

$$\begin{bmatrix} 26.3 & 27.0 & 52.2 & 25.5 \\ 14.20 & -5.62 & -53.14 & 0 \\ -49.00 & -23.29 & -117.68 & 0 \\ -15.52 & -5.90 & -12.64 & 0 \end{bmatrix}$$

Step 2 – Second pivot (FIM): a_{22}= −53.14

- the goal is to zero out the numbers below the second column:
- Calculating the coefficients:
- $f_{42}=\frac{-12.64}{-53.14}\approx 0.238 \qquad f_{32}=\frac{-117.68}{-53.14}\approx 2.214$
- Perform row operations:
  - $Row_3 = Row_3 - f_{21}\times Row_2$
  - $Row_4 = Row_4 - f_{31}\times Row_2$

Matrix after step 2:

Row 3:

- $-23.29 - 2.214\times(-5.62) =-23.29+12.44= -10.85$
- $-49.00 - 2.214\times14.20 = -49.00 - 31.43 = -80.43$

Row 4:

- $-5.90 - 0.238 \times (-5.62) = -5.90 + 1.34= -4.56$
- $-15.52-0.238\times14.20=-15.52-3.38=-18.90$

Updated matrix after Step 2:

$$\begin{bmatrix} 26.3 & 27.0 & 52.2 & 25.5 \\ 14.20 & -5.62 & -53.14 & 0 \\ -80.43 & -10.85 & 0 & 0 \\ -18.90 & -4.56 & 0 & 0 \end{bmatrix}$$

Step 3 – Third pivot (SII): Pivot Element: $a_{33} \approx -10.85$

- Eliminate the last entry in the third column:
- Calculating the coefficients:

$$F_{43}= \frac{-4.56}{-10.85}$$

$Row_4 \rightarrow Row_4 - f_{43}\times Row_3$

Matrix after step 3:

- $-18.90 - 0.420 \times (-80.43) = -18.90 + 33.78= 14.88$
- Updated matrix

$$\begin{bmatrix} 26.3 & 27.0 & 52.2 & 25.5 \\ 14.20 & -5.62 & -53.14 & 0 \\ -80.43 & -10.85 & 0 & 0 \\ 14.88 & 0 & 0 & 0 \end{bmatrix}$$

Step 4 – Fourth pivot (EOP)_ Final Determinant

Product of the pivots:

$\text{Det}\,(_{\text{Barysh}})= 25.5 \times (-53.14) \times (-10.85) \times 14.88$

Calculations:

- $25.5\times(-53.14) = -1354.07$
- $-1354.07\times(-10.85) = 14693.98$
- $14693.98\times 14.88 = 218691.3$

Result:

det $({}_{\text{Barysh}}) \approx$ 218,691.3

(b) Gaussian elimination method

To calculate the determinant of the block matrix M_{55}, the Gaussian elimination method was applied. This is a classic and widely used numerical technique that systematically transforms a matrix into an upper triangular form by eliminating entries below the main diagonal. The determinant is then obtained by multiplying the diagonal elements. While Gaussian elimination is not as numerically stable as the Bareiss algorithm, it is simple, intuitive, and computationally efficient for moderately sized matrices.

- The original matrix M_{55}, extracted from the climate-smart mining block model, is defined as:

$$M_{55}=\begin{bmatrix} 25.5 & 52.2 & 27 & 26.3 \\ 48.1 & 45.3 & 45.3 & 63.8 \\ 67.8 & 21.1 & 48.5 & 20.9 \\ 32.2 & 53.3 & 28.2 & 17.7 \end{bmatrix}$$

Step 1: Elimination below pivot $a_{11}=25.5$

Calculate multipliers:

- Eliminate row 2 using row 1 → factor $f_{21}=\frac{48.1}{25.5}\approx 1.886$
- Eliminate row 3 using row 1 → factor $f_{31}=\frac{67.8}{25.5}\approx 2.659$
- Eliminate row 4 using row 1 → factor $f_{41}=\frac{32.2}{25.5}\approx 1.262$

Update rows 2, 3, and 4:

$Row_2 \rightarrow Row_2 - f_{21}\times Row_1$

$Row_3 \rightarrow Row_3 - f_{31}\times Row_1$

$Row_4 \rightarrow Row_4 - f_{41}\times Row_1$

Calculations:

Row 2:

- $a_{22}= 45.3-1.886\times 52.2= 45.3- 98.48= -53.18$
- $a_{23}= 45.3-1.886\times 27.0= 45.3- 50.92= -5.62$
- $a_{24}= 63.8-1.886\times 26.3= 63.8-4\ 9.59= 14.21$

Row 3:

- $a_{32}= 21.1-2.659\times 52.2= 21.1-138.81= -117.$
- $a_{33}= 48.5-2.659\times 27.0= 48.5-71.79= -23.29$
- $a_{34}= 20.9-2.659\times 26.3= 20.9-69.94= -49.04$

Row 4:

- $a_{42}= 53.3-1.263\times 52.2= 53.3-65.95= -12.65$
- $a_{43}= 28.2-1.263\times 27.0= 28.2-34.10= -5.90$
- $a_{44}= 17.7-1.263\times 26.3= 17.7-33.23= -15.53$

Matrix after Step 1:

$$\begin{bmatrix} 25.5 & 52.2 & 27.0 & 26.3 \\ 0 & -53.18 & -5.62 & 14.21 \\ 0 & -117.71 & -23.29 & -49.04 \\ 0 & -12.65 & -5.90 & -15.53 \end{bmatrix}$$

Step 2: Choose pivot in column 2 and eliminate entries below

- **Pivot element: $a_{22}= -53.18$**
- Calculate multipliers:

$f_{32}=\frac{-117.71}{-53.18}\approx -2.213$, $f_{42}=\frac{-12.65}{-53.18}\approx 0.238$

Update rows 3 and 4:

Row 3:

- $a_{33}= -23.29 - 2.213\times(-5.62) = -23.29 + 12.44= -10.85$
- $a_{34}= -49.04 - 2.213\times 14.21= -49.04 - 31.45= -80.49$

Row 4:

- $a_{43}= -5.90-0.238\times(-5.62) = -5.90+1.34= -4.56$
- $a_{44}= -15.53-0.238\times 14.21= -15.53-3.38= -18.91$

Matrix after Step 2:

$$\begin{bmatrix} 25.5 & 52.2 & 27.0 & 26.3 \\ 0 & -53.18 & -5.62 & 14.21 \\ 0 & 0 & -10.85 & -80.49 \\ 0 & 0 & -4.56 & -18.91 \end{bmatrix}$$

Step 3: Choose pivot in column 3 and eliminate below

- Pivot element: $a_{33}= -10.85$

Calculate multiplier:

$f_{43}=\frac{-4.56}{-10.85}\approx 0.420$

Update row 4:

- $a_{44}= -18.91-0.420\times(-80.49) = -18.91+33.81=14.90$

Matrix after Step 3:

$$\begin{bmatrix} 25.5 & 52.2 & 27.0 & 26.3 \\ 0 & -53.18 & -5.62 & 14.21 \\ 0 & 0 & -10.85 & -80.49 \\ 0 & 0 & 0 & 14.90 \end{bmatrix}$$

Step 4: Calculate determinant

The determinant is a product of diagonal elements:

Det $({}_{\text{Gaussian}}) = 25.5\times(-53.18)\times(-10.85)\times 14.90$

Calculate stepwise:

- $25.5\times(-53.18) = -1355.09$
- $-1355.09\times(-10.85) = 14704.63$
- $14704.63\times 14.90=219073.98$

Therefore:

det $({}_{\text{Gaussian}}) \approx$ 219074t

Table 10. Determinant Method Comparison.

Method	Determinant Value
Barysh	218,691.3
Gaussian	219074

Both methods confirm the structural coherence and stability of the matrix of the matrix, especially in lower right blocks.

3.2.1 Policy implications of determinant-based analysis

The determinant analysis applied to the M55 matrix yielded consistently high values across both computational methods: 218,691 (Barysh) and 219,074 (Gaussian). These results confirm that the structural configuration of the Climate-Smart Mining (CSM) governance model is mathematically robust and internally coherent. No signs of matrix degeneration, feedback loop instability, or inter-domain conflict were detected.

This high level of structural integrity implies that the existing governance relationships—across legal, environmental, investment, and stakeholder dimensions—are functionally aligned. However, high determinant values do not necessarily translate into policy effectiveness at the operational level. Rather, they indicate that the framework, as scored by experts, demonstrates internal consistency and system-wide resilience. From a policy perspective, these finding shifts attention from structural repair to strategic enhancement. Efforts should focus on preserving institutional cohesion, reinforcing effective linkages, and monitoring underperforming zones with low interaction scores, such as EOP–EOP and FLM–SII. While these zones are not structurally fragile, they may suffer from inadequate implementation or stakeholder disconnect, requiring targeted—but not systemic—reform.

3.3. Matrix score vs determinant cross-analysis

We cross-validate expert visual scores with determinant-derived sensitivity:

This scatter plot compares matrix scores and corresponding pivot sensitivities, revealing blocks where subjective scores diverge from objective fragility.

Table 11. Comparative matrix vs determinant interpretation.

Interaction	Matrix Score	Determinant Contribution	Structural Verdict
SII–CE	63.8	Pivot stable	Strong and synergistic
SII–FLM	21.1	Mid pivot, not degenerate	
EOP–EOP	17.7	Stable but low score zone	
EOP–SII	27.0		Requires coordination efforts

Table 12. Expert consensus statistics for selected m55 matrix blocks.

Matrix Block	Expert Mean	SD (σ)	Interpretation
EOP–EOP	2.9	2.5	Low consensus (high uncertainty)
LRS–FIM	4.1	4.6	Moderate consensus
SII–CE	3.8	1.2	High agreement

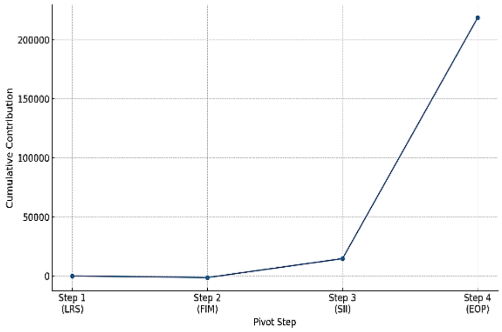


Figure 2: Pivot-by-pivot contribution to determinant.

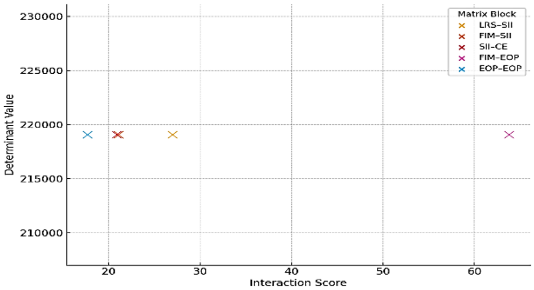


Figure 3: Scatterplot of visual score vs determinant sensitivity.

3.4. Uncertainty and expert consensus analysis

Given the expert-based nature of the M55 matrix, it is crucial to assess the degree of consensus among the experts who provided the interaction scores. To capture this, standard deviation (SD) values were calculated across selected matrix blocks.

EOP – EOP: SD = 2.5 → indicates low consensus and high uncertainty.

LRS – FIM: SD = 4.6 → moderate consensus.

SII – CE: SD = 1.2 → high agreement among experts.

These values reflect the variation in perceptions about system interactions. Blocks with high SD (e.g., EOP–EOP) warrant further investigation or stakeholder deliberation before strategic implementation.

Analytically, high uncertainty (high SD) suggests instability in input data that could amplify matrix sensitivity, especially in structurally fragile zones.

Thus, integrating standard deviation alongside determinant magnitudes provides a dual lens to prioritize zones that are both structurally and socially uncertain.

3.5. Policy implications of determinant-based analysis

Determinant-based diagnostics offer more than numerical confirmation of system integrity—they provide a structural lens through which to understand the depth, direction, and risk profile of inter-domain governance dynamics. In this study, both the Barysh (218,691.3) and Gaussian (219,074) methods returned extremely high determinant values, confirming that the M_{55} matrix is neither near-degenerate nor structurally fragile. This finding carries important implications.

First, such high values indicate that the 15-indicator, five-domain architecture of the model demonstrates internal cohesion. Policy instruments, legal provisions, sustainability indicators, and stakeholder priorities are functionally aligned and mathematically robust. In practical terms, this suggests that system-wide reforms may not be immediately necessary—provided that existing strengths are preserved and that no domain becomes overly dominant or disconnected over time.

Second, this level of determinant stability suggests the presence of structural redundancy, which, while beneficial for resilience, may also mask emerging inefficiencies. In high-determinant systems, there is a risk of over-systemization—a condition in which formal alignment suppresses adaptive governance or local responsiveness. Policymakers should therefore balance the virtues of internal stability with the need for dynamic capacity, ensuring that the system remains sensitive to new risks or stakeholder concerns.

Third, this type of analysis enables differentiation between structural risk (which is low) and policy underperformance (which may still exist). For example, blocks such as EOP–EOP and FLM–SII yielded low interaction scores (17.7 and 21.1, respectively), despite operating within a stable matrix. This divergence suggests that while the institutional logic is sound, real-world governance gaps—such as ineffective stakeholder outreach, financing misalignment, or regulatory inertia—may still be present.

Finally, the determinant results provide a scalable benchmark for future monitoring. As mining governance landscapes evolve—through technological change, global policy shifts, or social activism—determinant recalculations can help assess whether the structural coherence of the system is being preserved or compromised. By embedding determinant diagnostics into decision cycles, governance institutions can track long-term systemic health alongside short-term performance metrics. In summary, the high determinant values observed in this study validate the logic of the current CSM matrix model but also call for nuanced application: one that leverages structural coherence without overlooking the importance of localized governance responsiveness and score-based discrepancies.

4. Conclusion

This study introduces a comprehensive methodological and analytical framework for assessing Climate-Smart Mining (CSM) by combining Block Matrix Analysis (BMA) with determinant-based system diagnostics. By integrating expert judgment, systemic matrix modeling, and algebraic determinant theory, the framework enables a thorough evaluation of internal interactions and structural fragilities within the governance landscape of CSM.

The analysis of matrix M_{55} revealed key interactions among critical components, including Legal Regulations (LRS), Forest Mining (FIM), Sustainable Investment (SII), and Equity-Oriented Participation (EOP).

Expert assessments identified strong connections, such as those between FIM and EOP, while also exposing structurally weak and vulnerable zones, notably EOP–EOP, SII–FLM, and SII–CE.

Using both Barysh and Gaussian methods, the determinant analysis yielded high values—218,691.3 and 219,074, respectively—confirming the structural coherence of the model and suggesting that the observed interactions among governance domains are robust and resilient. Additionally, pivot breakdowns highlighted the stages at which the system's governance coherence deteriorates, pinpointing specific zones of weakness. This integrated diagnostic approach enhances the detection of hidden risks that are not apparent through surface-level subjective evaluation. Visual diagnostic tools, including heatmaps and matrix colorization, further facilitate the localization of stress points and synergy zones, enriching the overall understanding of governance dynamics.

4.1. Comparative interpretation and policy insight

The determinant analysis yielded high values across both Gaussian (219, 074) and Barysh (218, 691.3) methods, indicating a structurally coherent governance matrix. This suggests that the system, as designed and scored, does not currently exhibit structural degeneration, internal misalignment, or instability. In this context, high determinant values should not be interpreted as a signal of perfection, but rather as evidence of overall alignment among policy, environmental, financial, and social indicators. However, attention must still be paid to governance blocks with lower interaction scores—not because they reflect fragility in matrix logic, but because they might signal real-world policy inattention, administrative gaps, or insufficient stakeholder integration. For instance, the EOP–EOP block yielded a low interaction score of 17.7, even though its pivot value remains high. This divergence highlights the value of dual-lens diagnostics: while matrix determinants capture internal logical stability, interaction scores provide insights into perceived effectiveness and governance engagement levels. As such, policymakers should use determinant analysis as a structural baseline—confirming that the system is mathematically sound—while relying on score-based heatmaps and qualitative feedback to guide more localized or targeted reforms.

4.2. Broader implications

Beyond immediate governance, this integrated BMA-determinant approach offers a dual-perspective lens, combining visual expert perception with quantitative matrix stability measures. This duality supports the design of risk-aware governance frameworks that are both practical and analytically robust. From a sustainability standpoint, zones such as FIM–EOP and CE–FIM demonstrate replicable synergy models that can be leveraged in policy and practice, whereas red-flag areas like EOP–EOP and SII–CE demand urgent regulatory reform and enhanced stakeholder participation. Importantly, the adaptability of this framework extends to other complex socio-technical systems—including energy, transportation, and water management—where the interplay between stakeholders and institutions critically shapes system performance. Finally, the proposed model serves as a scalable and structured training platform suitable for academic institutions, government agencies, and climate policy task forces, supporting capacity building and informed decision-making toward sustainable and resilient governance.

4.3. Limitations and recommendations

This study acknowledges several limitations that should be addressed in future research to enhance the robustness and applicability of the proposed framework. First, the expert scoring utilized reflects perceived rather than empirical system performance, which introduces a potential data source bias. To mitigate this, subsequent studies are recommended to integrate real-time system metrics and empirical data to complement expert judgment and increase objectivity. Second, the scalability of the Block Matrix Analysis presents computational challenges as matrix size increases—for instance, with matrices larger than 10×10—necessitating the development and implementation of automated computational processes along with rigorous error-checking mechanisms to maintain accuracy and efficiency. Third, the dynamic nature of governance systems requires temporal sensitivity; thus, periodic recalibration of the matrices is essential to capture evolving policy landscapes and respond to external shocks or shifts in system behavior over time. Despite these limitations, the integrated approach combining block matrix modeling with determinant-based diagnostics offers significant value beyond a mere snapshot of system status. It effectively reveals the underlying structure of systemic coherence and potential fracture points, which is critical in the complex context of climate-smart mining governance. By blending expert perspectives with rigorous mathematical diagnostics, this framework provides decision-makers with a transparent, adaptive, and evidence-driven tool. It supports the anticipation of failures, informed design of interventions, and ongoing tracking of resilience, ultimately facilitating sustainable and responsive governance in a dynamic environment.

References

[1]. Akchurin, M. (2025). Mining, water conflicts, and climate change in Chile's Atacama Desert. *Current History, 124*(859), 61–67. https://doi.org/10.1525/curh.2025.124.859.61

[2]. Ali, P. O., & Kim, K. N. (2024). Analysis of Indonesia's priority selection: Energy transition, energy-related measures, mining governance, and resource transition using the analytic hierarchy process (AHP). *Energy for Sustainable Development, 83*, 101559. https://doi.org/10.1016/j.esd.2024.101559

[3]. Azadi, M., Northey, S., Hasanpour, H., & de Vries, G. (2018). Sustainability assessment of mining projects: An evaluation of regulations, policies, and practices. *Resources Policy, 59*, 230–238. https://doi.org/10.1016/j.exis.2017.12.004

[4[. Banda, W. (2023). A system dynamics model for assessing the impact of fiscal regimes on mining projects. *Resources Policy, 81*, 103408. https://doi.org/10.1016/j.resourpol.2023.103408

[5]. Bulovic, N., McIntyre, N., & Trancoso, R. (2024). Climate change risks to mine closure. *Journal of Cleaner Production, 465*, 142697. https://doi.org/10.1016/j.jclepro.2024.142697

[6]. Buslaeva, I., & Yakovleva, L. (2022). Block matrix for three-factor analysis of machinery failures. In A. Mottaeva (Ed.), *Technological advancements in construction* (pp. 303–312). Springer. https://doi.org/10.1007/978-3-030-83917-8_28

[7]. Butcher, J. C. (2015). Algebraic analysis of structure-preserving general linear methods. *Proceedings of the International Conference on Numerical Analysis and Applied Mathematics 2014 (ICNAAM-2014)*, 020003. https://doi.org/10.1063/1.4912307

[8]. Cano, Á., & Kunz, N. C. (2022). Large-scale and small-scale mining in Peru: Exploring the interface. *Resources Policy, 76*, 102530. https://doi.org/10.1016/j.resourpol.2021.102530

[9]. Cao, J., et al. (2024). Fuzzy inference system with interpretable fuzzy rules: Advancing explainable artificial intelligence for disease diagnosis—A comprehensive review. *Information Sciences, 662*, 120212. https://doi.org/10.1016/j.ins.2024.120212

[10]. Cavus, N., Mohammed, Y. B., & Yakubu, M. N. (2021). Determinants of learning management systems during COVID-19 pandemic for sustainable education. *Sustainability, 13*(9), 5189. https://doi.org/10.3390/su13095189

[11]. Cole, M. J., & Broadhurst, J. L. (2022). Sustainable development in mining communities: The case of South Africa's West Wits Goldfield. *Frontiers in Sustainable Cities, 4*. https://doi.org/10.3389/frsc.2022.895760

[12]. Dall-Orsoletta, A., et al. (2025). How does social acceptance affect transition minerals production in Europe? A system

dynamics approach and case study in Portugal. *The Extractive Industries and Society, 22*, 101625. https://doi.org/10.1016/j.exis.2025.101625

[13]. Dobrushkin, V. A. (2017). *Applied differential equations with boundary value problems*. CRC Press. https://doi.org/10.1201/9781315369785

[14]. Domínguez-Gómez, J. A., & González-Gómez, T. (2021). Governance in mining: Management, ethics, sustainability and efficiency. *The Extractive Industries and Society, 8*(3), 100910. https://doi.org/10.1016/j.exis.2021.100910

[15]. Ed, O., et al. (2024). From sustainable mining to climate-smart mining; lessons for Nigeria. *SunText Review of Arts & Social Sciences, 5*(3). https://doi.org/10.51737/2766-4600.2024.079

Essaw, D. W., et al. (2025). Governance for sustainable sand mining [16]. in two selected local government areas in the Central Region of Ghana. *Society & Natural Resources, 38*(4), 394–413. https://doi.org/10.1080/08941920.2023.2169799

[17]. Gholami, A., Tokac, B., & Zhang, Q. (2024). Knowledge synthesis on the mine life cycle and the mining value chain to address climate change. *Resources Policy, 95*, 105183. https://doi.org/10.1016/j.resourpol.2024.105183

[18]. Guan, X., & Zhao, J. (2022). A two-step fuzzy MCDM method for implementation of sustainable precision manufacturing: Evidence from China. *Sustainability, 14*(13), 8085. https://doi.org/10.3390/su14138085

[19]. Hamediazad, F., & Bahrani, N. (2024). Evaluation of the rock mass strength for hard rock pillar design using bonded block models. *Rock Mechanics and Rock Engineering, 57*(5), 3659–3680. https://doi.org/10.1007/s00603-023-03718-0

[20]. Hanisch, M., et al. (2023). Digital governance: A conceptual framework and research agenda. *Journal of Business Research, 162*, 113777. https://doi.org/10.1016/j.jbusres.2023.113777

[21]. Hodgkinson, J. H., & Smith, M. H. (2021). Climate change and sustainability as drivers for the next mining and metals boom: The need for climate-smart mining and recycling. *Resources Policy, 74*, 101205. https://doi.org/10.1016/j.resourpol.2018.05.016

[22]. Jiskani, I. M., et al. (2022). Implications to secure mineral supply for clean energy technologies for developing countries: A fuzzy based risk analysis for mining projects. *Journal of Cleaner Production, 358*, 132055. https://doi.org/10.1016/j.jclepro.2022.132055

[23]. Jiskani, I. M., et al. (2023). Mining 4.0 and climate neutrality: A unified and reliable decision system for safe, intelligent, and green & climate-smart mining. *Journal of Cleaner Production, 410*, 137313. https://doi.org/10.1016/j.jclepro.2023.137313

[24]. Karatzinis, G. D., & Boutalis, Y. S. (2025). A review study of fuzzy cognitive maps in engineering: Applications, insights, and future directions. *Eng, 6*(2), 37. https://doi.org/10.3390/eng6020037

[25]. Lejun, G., et al. (2025). SeqBMC: Single-cell data processing using iterative block matrix completion algorithm based on matrix factorisation. *IET Systems Biology, 19*(1), e70003. https://doi.org/10.1049/syb2.70003

[26]. Li, G., et al. (2021). A review from environmental management to environmental governance: A paradigm shift for sustainable mining practice in Ghana. *Environment, Development and Sustainability, 23*(7), 9710–9724. https://doi.org/10.1007/s10668-020-01050-z

[27]. Odell, S. D., Bebbington, A., & Frey, K. E. (2018). Mining and climate change: A review and framework for analysis. *The Extractive Industries and Society, 5*(1), 201–214. https://doi.org/10.1016/j.exis.2017.12.004

[28]. Paul, A., & Mahapatra, S. S. (2025). An integrated fuzzy-AHP and fuzzy-DEMATEL approach for analyzing sustainable supply chain factors in the mining industry. *Supply Chain Analytics, 10*, 100113. https://doi.org/10.1016/j.sca.2025.100113

[29]. Pearce, T. D., et al. (2011). Climate change and mining in Canada. *Mitigation and Adaptation Strategies for Global Change, 16*(3), 347–368. https://doi.org/10.1007/s11027-010-9269-3

[30]. Salmeri, A. (2023). *The multi-level governance of space mining*. Wolters Kluwer.

[31]. Sarkheil, H. (2024). Dynamic system analysis of key indicators for climate-resilient smart mining within a sustainable governance framework: A focus on the cobalt value chain. *SSRN*. https://ssrn.com/abstract=5189964

[32]. Sarkheil, H., & Sadeghy Nejad, A. (n.d.). Hazard identification and process risk assessment at the building stone processing company through combination of EFMEA & William Fine methods. *ResearchGate*. https://www.researchgate.net/publication/384920229

[33]. Sarkheil, H., Alghasi, S., & Sadeghy Nejad, A. (2024). Modeling coastal environmental risk assessment: A case study of installing three filtration bases in a deep sea mining structure. *Journal of Mining and Environment*. https://doi.org/10.22044/jme.2024.14304.2673

[34]. Sarkheil, H., Rahbari, S., & Lak, R. (2024). A novel fuzzy deep groundwater sustainability index FDGSI for climate-smart mining and sustainability-aimed governance. *SSRN*. https://doi.org/10.2139/ssrn.4776925

[35]. Saqlain, M., Kumam, P., & Kumam, W. (2024). Multi-criteria decision-making method based on weighted and geometric aggregate operators of linguistic fuzzy-valued hypersoft set with application. *Journal of Fuzzy Extension and Applications*. https://doi.org/10.22105/jfea.2024.475488.1609

[36]. Shamsi, M., Zakerinejad, M., & Zareifard, M. R. (2025). Optimal, reliable, and sustainable technology selection for mining overburden waste utilization using green & climate-smart mining (GCSM): A hybrid fuzzy multi-criteria decision-making tool. *Journal of Environmental Chemical Engineering, 13*(3), 116118. https://doi.org/10.1016/j.jece.2025.116118

[37]. Shome, S., Chakraborty, S., & Sinha, S. (2025). Spatial framework for decision making on mining sustainability. *Environment, Development and Sustainability*. https://doi.org/10.1007/s10668-024-05939-x

[38]. Sonter, L. J., Ali, S. H., & Watson, J. E. M. (2018). Mining and biodiversity: Key issues and research needs in conservation science. *Proceedings of the Royal Society B: Biological Sciences, 285*(1892), 20181926. https://doi.org/10.1098/rspb.2018.1926

[39]. Stewart, G. W. (2001). *Matrix algorithms: Volume II: Eigensystems*. Society for Industrial and Applied Mathematics.

[40]. Wang, P., et al. (2025). Complex modal analysis of a resilient supported conveyor belt: Enhancement of numerical stability by reduced complex transfer matrix method. *Journal of Vibration Engineering & Technologies, 13*(2), 192. https://doi.org/10.1007/s42417-024-01611-w

[41]. Wang, Y., et al. (2023). Cascading impacts of global metal mining on climate change and human health caused by COVID-19 pandemic. *Resources, Conservation and Recycling, 190*, 106800. https://doi.org/10.1016/j.resconrec.2022.106800

[42]. World Bank. (2020). *Minerals for climate action: The mineral intensity of the clean energy transition*. World Bank.

[43]. Yang, Q., & Peng, X. (2023). A fast calculation method for sensitivity analysis using matrix decomposition technique. *Axioms, 12*(2),

179. https://doi.org/10.3390/axioms12020179

[44]. Zamany, A., Khamseh, A., & Iranbanfard, S. (2024). Unveiling the landscape of high-tech transfer in Industry 5.0: A text mining exploration. *Journal of AI and Data Mining, 12*(3). https://doi.org/10.22044/jadm.2024.14580.2558

[45]. Zhang, Y., Yuan, X., & Wang, K. (2025). Small-signal stability criterion for multibus DC cascaded system based on bus admittance matrix. *IEEE Transactions on Power Electronics, 40*(5), 6442–6447. https://doi.org/10.1109/TPEL.2025.3529485

[46]. A hybrid fuzzy multi-criteria decision making model for selecting a sustainable supplier of forklift filters: A case study from the mining industry. (n.d.). *Annals of Operations Research*. https://link.springer.com/article/10.1007/s10479-020-03737-y

[47]. Mitigation and Adaptation Strategies for Global Change. (2011). 16*(3). https://doi.org/10.1007/s11027-010-9269-3

Appendix A: supplementary tables and figures

This appendix includes the extended datasets, raw expert scores, technical calculations, and visual aid referenced in the main body of the article.

A.1. Expert evaluation data

Table A.1. Statistical summary: means, standard deviations, and ranges.

Block	Mean Score	Std. Dev.	Min	Max
LRS–FIM	52.2	4.6	47	60
SII–FIM	21.1	3.2	16	27
EOP–EOP	17.7	2.5	14	21

A.2 Additional tables and conceptual framework

This section lists items conceptually referenced in the original model design document but not fully explored in the main text. They provide depth and transparency to the modeling framework:

- Block Interactions M12, M34, M45: These blocks, while not numerically populated in the original Word document, were referenced conceptually as follows:

➢ M_{34}: Matrix of "Carbon-Energy Transition × Circular Supply Chains"

	Productivity A	Productivity B	Productivity C
Strategy A	–	–	–
Strategy B	–	–	–
Strategy C	–	–	–

Description: This block evaluates the effect of proposed climate-aligned strategies (e.g., emission reduction, resilience measures) on various dimensions of mining productivity such as yield, downtime reduction, and operational efficiency.

➢ M_{34}: Matrix of "Carbon-Energy Transition × Circular Supply Chains"

	CE Supply Chain 1	CE Supply Chain 2	CE Supply Chain 3
Energy Source 1	–	–	–
Energy Source 2	–	–	–
Energy Source 3	–	–	–

Description: This matrix addresses how different clean energy strategies interact with the robustness, recyclability, and traceability in mineral supply chains.

➢ M_{45}: Matrix of "Data Systems × Stakeholder Engagement"

	Stakeholder A	Stakeholder B	Stakeholder C
Data Layer A	–	–	–
Data Layer B	–	–	–
Data Layer C	–	–	–

Description: This matrix captures how the implementation of integrated data infrastructures (e.g., satellite monitoring, and AI dashboards) affects transparency and participation of local communities, regulators, and investors.

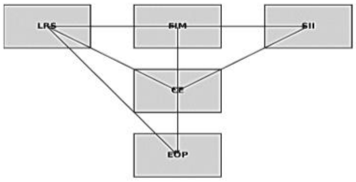


Figure A.2. Simplified network view of the five matrix domains.

This simplified schematic represents the five thematic domains—LRS, FIM, SII, CE, and EOP—as nodes in a block matrix system, highlighting key interaction paths without detailed policy descriptions.

A.3 Extended tables and interpretive annotations

The following tables are critical extensions of the original model document. They support full transparency and technical rigor for academic and professional audiences: Includes three tables – Parts 2 to 4: Narratives describing the functional logic of each matrix intersection (e.g., LRS–FIM, SII–CE).

Table A.3.1. Description of selected components of the block matrix.

Row	Block Name	Block Definition	Block Description
1	M_{31}	The M_{31} matrix (1×3) represents the impacts of energy and carbon on strategies.	This matrix represents how three different indicators related to energy and carbon impact a single indicator of strategies and climate adaptation effects. The values in this matrix show how various aspects of energy (such as renewable energy and energy efficiency) and carbon financial instruments influence climate strategies.
2	M_{32}	The M_{32} matrix (5×3) represents the impacts of energy and carbon on resource efficiency and recycling.	This matrix represents the impacts of three indicators related to energy and carbon on five different indicators of resource efficiency and recycling. The values in this matrix help determine how renewable energies, carbon financial instruments, and energy efficiency affect resource productivity, reuse, and mineral recycling.
3	M_{41}	The M_{41} matrix (1×3) represents the impacts of supply chain and data management on strategies.	This matrix represents how three indicators related to supply chain and data management impact one indicator of strategies and climate adaptation effects. The values in this matrix reflect how improvements in supply chain and data management can influence climate strategies.
4	M_{42}	The M_{42} matrix (5×3) represents the impacts of supply chain and data management on resource efficiency and recycling.	This matrix represents the impacts of three indicators related to supply chain and data management on five indicators of resource efficiency and recycling. The values in this matrix help us understand how supply chain and data management affect various aspects of resource efficiency and recycling.
5	M_{43}	The M_{43} matrix (3×3) represents the impacts of supply chain and data management on energy and carbon.	This matrix represents the interactions between three indicators of supply chain and data management and three indicators related to energy and carbon. The values of this matrix demonstrate how supply chain and data management impact various aspects of energy and carbon.
6	M_{51}	The M_{51} matrix (1×4) represents the impacts of opportunities and interactions on strategies.	This matrix represents the impacts of four indicators related to opportunities and interactions on one indicator of strategies and climate adaptation effects. The values of this matrix illustrate how opportunities and interactions can influence climate strategies.
7	M_{52}	The M_{52} matrix (5×4) represents the impacts of opportunities and interactions on resource efficiency and recycling.	This matrix represents how four indicators of opportunities and interactions impact five indicators of resource efficiency and recycling. The values in this matrix help us understand the influence of opportunities and interactions on resource efficiency and mineral recycling.

Table A.3.1. Description of selected components of the block matrix (Continue).

Row	Block Name	Block Definition	Block Description
8	M_{53}	The M_{53} matrix (3×4) represents the impacts of opportunities and interactions on energy and carbon.	This matrix represents the impacts of four indicators of opportunities and interactions on three indicators of energy and carbon. The values in this matrix help us understand how opportunities and interactions influence various aspects of energy and carbon.
9	M_{54}	The M_{54} matrix (3×4) represents the impacts of opportunities and interactions on supply chain and data management.	This matrix represents how four indicators of opportunities and interactions impact three indicators of supply chain and data management. The values in this matrix help us understand how opportunities and interactions influence the supply chain and data management aspects.

A.3.2 Block matrix structural template

Includes three tables– Parts 2 to 4: Schematics of row/column positions for matrix M_{55}.

Table A.3.2. Representation of the block matrix pattern.

Row	Block Name	Block Definition	Block Matrix Relationship
1	M_{24}	Matrix M_{24}, with dimensions 3×5, represents the impacts of *resource efficiency and recycling* on *supply chain and data management.*	$\begin{pmatrix} {}_{24,3}a & {}_{24,2}a & {}_{24,1}a \\ {}_{24,6}a & {}_{24,5}a & {}_{24,4}a \\ {}_{24,9}a & {}_{24,8}a & {}_{24,7}a \\ {}_{24,12}a & {}_{24,11}a & {}_{24,10}a \\ {}_{24,15}a & {}_{24,14}a & {}_{24,13}a \end{pmatrix} = \mathbf{M}_{24}$
2	M_{25}	Matrix M_{25}, with dimensions 4×5, represents the impacts of *resource efficiency and recycling* on *opportunities and interactions.*	$\begin{pmatrix} {}_{25,4}a & {}_{25,3}a & {}_{25,2}a & {}_{25,1}a \\ {}_{25,8}a & {}_{25,7}a & {}_{25,6}a & {}_{25,5}a \\ {}_{25,12}a & {}_{25,11}a & {}_{25,10}a & {}_{25,9}a \\ {}_{25,16}a & {}_{25,15}a & {}_{25,14}a & {}_{25,13}a \\ {}_{25,20}a & {}_{25,19}a & {}_{25,18}a & {}_{25,17}a \end{pmatrix} = \mathbf{M}_{25}$
3	M_{31}	Matrix M_{31}, with dimensions 1×3, reflects the impacts of *energy and carbon* on *strategies.*	$\begin{pmatrix} {}_{31,1}a \\ {}_{31,2}a \\ {}_{31,3}a \end{pmatrix} = \mathbf{M}_{31}$
4	M_{32}	Matrix M_{32}, with dimensions 5×3, reflects the impacts of *energy and carbon* on *resource efficiency and recycling.*	$\begin{pmatrix} {}_{32,5}a & {}_{32,4}a & {}_{32,3}a & {}_{32,2}a & {}_{32,1}a \\ {}_{32,10}a & {}_{32,9}a & {}_{32,8}a & {}_{32,7}a & {}_{32,6}a \\ {}_{32,15}a & {}_{32,14}a & {}_{32,13}a & {}_{32,12}a & {}_{32,11}a \end{pmatrix} = \mathbf{M}_{32}$
5	M_{33}	Matrix M_{33}, with dimensions 3×3, represents the interactions among indicators within the *energy and carbon* category.	$\begin{pmatrix} {}_{33,3}a & {}_{33,2}a & {}_{33,1}a \\ {}_{33,6}a & {}_{33,5}a & {}_{33,4}a \\ {}_{33,9}a & {}_{33,8}a & {}_{33,7}a \end{pmatrix} = \mathbf{M}_{33}$
6	M_{34}	Matrix M_{34}, with dimensions 3×3, reflects the impacts of *energy and carbon* on *supply chain and data management.*	$\begin{pmatrix} {}_{34,3}a & {}_{34,2}a & {}_{34,1}a \\ {}_{34,6}a & {}_{34,5}a & {}_{34,4}a \\ {}_{34,9}a & {}_{34,8}a & {}_{34,7}a \end{pmatrix} = \mathbf{M}_{34}$
7	M_{35}	Matrix M_{35}, with dimensions 4×3, represents the impacts of *energy and carbon* on *opportunities and interactions.*	$\begin{pmatrix} {}_{13}a & {}_{12}a & {}_{11}a \\ {}_{23}a & {}_{22}a & {}_{21}a \\ {}_{33}a & {}_{32}a & {}_{31}a \\ {}_{43}a & {}_{42}a & {}_{41}a \end{pmatrix} = \mathbf{M}_{35}$
8	M_{41}	Matrix M_{41}, with dimensions 1×3, reflects the impacts of *supply chain and data management* on *strategies.*	$\begin{pmatrix} {}_{11}a \\ {}_{21}a \\ {}_{31}a \end{pmatrix} = \mathbf{M}_{41}$ **(a11):** The impact of *innovative solutions for tailings management* on *strategies and climate adaptation effects.* **(a21):** The impact of *circular economy and supply chain management* on *strategies and climate adaptation effects.* **(a31):** The impact of *comprehensive geological data management* on *strategies and climate adaptation effects.* These values can be defined either **quantitatively** or **qualitatively** (for example, levels of impact: *low, moderate*, or *high*).